PROJECT SUMMARY: INFORMATION GEOMETRY AND MARKOV PROCESSES

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The PI will use techniques of information geometry to study Markov processes, especially those describing collections of randomly interacting particles. In applications to biology these 'particles' could be organisms with different genotypes. As usual, these Markov processes obey a differential equation: the master equation. But in the limit of large numbers, the expected number of things of each kind obeys a differential equation of its own: the 'rate equation'. In biology this equation includes, as special cases, various well-known models of predator-prey interactions.

The stability of equilibria for the rate equation is often proved by showing that a function akin to entropy increases. In biology, this technique includes Fisher's fundamental theorem on how the mean fitness of a population increases through natural selection. However, this technique only works for a certain special class of rate equations. Using recent work on information geometry, the PI and two graduate students will construct a more general theory of quantities that increase in Markov processes describing many-body interactions, as described either by the rate or master equation.

This opens the door to a deeper understanding of natural selection. The PI and his students will show that mean fitness is only one of many quantities that increase with the passage of time in evolution as described by a Markov process.

The **intellectual merit of the proposed activity** is that it will use ideas from information geometry and Markov process theory to better understand evolution in biology. In particular, the relation between 'entropy increase' and 'increase in mean fitness through natural selection' will be clarified.

The **broader impacts resulting from the proposed activity** include shedding new light on the role of random processes in mathematical biology. Through talks and his well-known blog, the PI will publicize the results of this research, explaining the probability theory to biologists and the biology to probability theorists. The two graduate students, after doing theses on this subject, will be in a position to serve as a bridge between these communities.