Why Mathematics is Boring

John C. Baez

Department of Mathematics
University of California
Riverside CA 92521
email: baez@math.ucr.edu

January 22, 2025

Abstract

Storytellers have many strategies for luring in their audience and keeping them interested. These include standardized narrative structures, vivid characters, breaking down long stories into episodes, and subtle methods of reminding the readers of facts they may have forgotten. The typical style of writing mathematics systematically avoids these strategies, since the explicit goal is "proving a fact" rather than "telling a story". Readers are left to provide their own narrative framework, which they do privately, in conversations, or in colloquium talks. As a result, even expert mathematicians find papers—especially those outside their own field—boring and difficult to understand. This impedes the development of mathematics.

Introduction

I don't really think mathematics is boring. I hope you don't either. But I can't count the number of times I've launched into reading a math paper, dewy-eyed and eager to learn, only to have my enthusiasm slowly crushed by pages and pages of bad writing. There are many ways a research paper on mathematics can be bad. Here I want to focus on just one: it can be dull. This happens when it neglects the human dimension.

The reader's interest is a delicate thing. It can die at any moment. Yet properly fed, and encouraged, it can grow to a powerful force. Clarity, well-organized prose, saying just enough at just the right time—these are tremendously important. You can learn these virtues from good math writers. But it also makes sense to look to story-tellers. After all, these are people whose whole business is keeping us interested.

Everyone loves a good story. We have been telling and listening to stories for untold millennia. Stories are one of our basic ways of understanding the world. I believe that when we read a piece of mathematics, part of us is reading it as a highly refined and sublimated sort of story, with characters and a plot, conflict and resolution.

If this is true, maybe we should consider some tips for short story writers, taken from a typical online guide [2], and see how they can be applied—in transmuted form—to the writing of mathematics. These tips go straight to the heart of what gets people interested, and keeps them interested.

Write a Catchy First Paragraph

We are constantly encountering pieces of writing. We don't bother to finish reading most of them. Once books were rare and precious. Now there is always too much to read. We efficiently cull out most of the material vying for our attention. Often we base our decision on the first sentence or two. Thus, writers of short stories learn the importance of quickly grabbing the reader's attention. In a catchy story, each sentence makes the reader want to read the next. The first few sentences bear the brunt of this responsibility.

I like to put it this way. Of the people who see your math paper, 90% will read only the title. Of those who continue, 90% will read only the abstract. Of those who go still further, 90% will read only the introduction, and then quit. Thus, it pays to put a huge amount of energy into making the front end of your paper clear and enticing. This can reduce those 90% figures (which I made up)

to about 80%, leading ultimately to an eightfold increase in the number of people who read beyond your paper's introduction.

So, don't start your paper like this:

> Let M be a complete Riemannian manifold, G a compact Lie group and $P \to M$ a principal G-bundle.

Instead, try something more like this:

> One of the main problems in gauge theory is understanding the geometry of the space of solutions of the Yang–Mills equations on a Riemannian manifold.

Then quickly start explaining what progress you've made. At this early stage, avoid bringing in any notation or terminology that is not absolutely necessary—bearing in mind that much of what you've come to consider "absolutely necessary" is actually not. A good exercise is to imagine you're explaining your paper to a colleague in an elevator. The prose in your first paragraph should be more formal than that, and better thought out, but this exercise will help you remember what is truly essential.

Avoid packing the first paragraph with references. Most of them can come a bit later. You should also not repeat material that is in the abstract, because most likely the reader can see that just a few inches up the page. The first paragraph is an opportunity to provide a different way in to the paper.

Use Setting and Context

In a short story, the reader is usually "located" as an observer to some scene of action, with a definite point of view — perhaps in a room somewhere, perhaps in some character's mind, or whatever. The story should quickly and unobtrusively establish this context.

In a typical math paper, setting the scene is usually done in the introduction. This section explains the main results in more detail than the abstract, and put these results in their historical and mathematical context.

It's hard to appreciate a piece of mathematics without the necessary background. At the very simplest level, we need to understand all the *words*: mathematics bristles with technical terminology. So the introduction to a math paper should set the scene as simply as possible. This may require "watering down" the results being described—stating corollaries or special cases instead of the full theorems in their maximal generality. Sometimes you may even need to leave out technical conditions required for the results to really be true. In that case, you must warn the reader that you're doing so.

Readers often return over and over to the introduction for guidance as they struggle to understand a paper. So, ideally your introduction will not only set the stage, but serve as a guide to the main concepts and results. Someone reading it should get a good idea of what your paper is about. Even if they don't read your paper today, this may encourage them to return to it later.

I often structure my introductions as follows:

- A catchy first paragraph that sets the stage and states the main result as clearly and simply as possible.
- Then a review of the context, introducing the most important concepts and notation, and weaving in some references to earlier work. Sometimes this review can explicitly take the form of a narrative, especially if the problem has an interesting history.
- Then a more precise statement of the results, taking advantage of the background and notation introduced so far.

- Finally, a listing of the key features of each section of the paper, with links to these sections. This serves as a road-map that readers can use to find their way around the paper. They may come back to it repeatedly.

Note that with this structure, the key results, already tersely stated in the abstract, are presented more vividly in the first paragraph and then in more detail near the end of the introduction. If each of these explanations is a bit different, the reader gets three chances to understand the results before seeing them worked out in the body of the paper. Each explanation should build on the previous one, revealing new aspects but clearly talking about the same thing.

Develop Your Characters

If a mathematics paper is secretly like a story, the "characters" are the mathematical entities involved. Some of these characters are more important than others; there are usually just a few heroes—and sometimes villains.

We can see the importance of developing characters by our tendency to singularize the plural. When we prove a theorem about a class of spaces X_n depending on some number n, we often tell the reader to "fix" n: that is, pick one, without saying which. Then we talk as if we were dealing with a *particular* space: a representative of the class under discussion. While this sort of move has been thoroughly analyzed by logicians, the art of good story-telling is also at work here. It is harder to keep in mind a class of entities than a particular representative of that class. Even authors of the crudest sort of politically engaged fiction, seeking to depict the "plight of the working class", know enough to tell their story about a particular member, not the whole class all at once.

For your paper to be enjoyable, the main characters must be introduced in a way that marks them as special and highlights their already known properties: their "personality". Don't be afraid to say some things about them that the reader may already know—and when the hero arrives, there should be a little flourish of trumpets, like:

> Now we come to a key player: the group of deck transformations.

Create Conflict and Tension

The "conflict" in a mathematics paper is usually the struggle to understand—often manifested in the struggle to prove something. As Piet Hein noted,

> Problems worthy of attack
> prove their worth by fighting back.

The most famous conjectures gain their interest from the way *truths resist being known*, forcing us into hard work and brand new insights. So if the results in your paper are harder to prove or less complete than you'd like, don't feel too bad—played right, it can give your paper a touch of drama.

Alas, mathematicians are often too eager to play down the difficulties they faced. This not only makes math boring, it can make it harder to understand. A clever idea often seems unmotivated and mysterious until one sees the problems it managed to overcome or circumvent. If every step in the final writeup is easy to understand, there may be little scope for conflict and tension. But this is rare.

Of course, you should not give a detailed blow-by-blow account of every wrong turn you made. That would be a fundamental mistake. Just as fiction is not usually an account of the author's life, a math paper should typically not be the history of the mathematician's work. You are free to use whatever structure works best! Conflict and tension are only useful insofar as they lead to a clear and exciting paper.

Find a Resolution

The conclusion of a math paper should set our feelings at rest by assuring us that the problems that have been solved have indeed been solved, while reminding us of those that have *not* yet been solved.

All too often a math paper will end abruptly right after the main result has been proved. This is unpleasant, like lowering the curtain and turning on bright lights the instant after a movie reaches its climax. Are you really so eager to leave? Don't be afraid to linger with the reader for a while and talk with them about a few topics that didn't fit into the main flow of the argument. They will also enjoy hearing about open problems they could try to solve.

Conclusion

Some of the tricks I'm suggesting here take practice to implement well. They should not be overdone. I'm *not* saying that a good math paper should consciously remind readers of a story. Ideally the narrative flow will be almost invisible, affecting readers in a subliminal way: they will merely feel that your paper is clear, enjoyable, and carries them smoothly from the title to the conclusion.

Acknowledgements

This paper was originally intended to become a contribution to a book, which I recommend for further insights on the role of narrative in mathematics [1]. I thank Mike Shulman and Mark Meckes for some useful suggestions.

References

[1] A. Doxiadis and B. Mazur, eds., *Circles Disturbed: the Interplay of Mathematics and Narrative*, Princeton U. Press, Princeton, 2012.

[2] K. Kennedy, Short stories: 10 tips for novice creative writers, available at http://jerz.setonhill.edu/writing/creative/shortstory/.