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John Baez <johnb@ucr.edu>

chern connection

9 messages

JAMES DOLAN <james.dolan1@students.mq.edu.au> Thu, Feb 24, 2022 at 11:07 AM
To: john.baez@ucr.edu

i don't think i'm saying this correctly yet!! (about how to think of the chern connection as determining the lifting of [the action of the period lattice on the contractible base] to [an action of the refinement-group on the trivial total space] ....)

i'm still struggling to get the formulas to work. it should be so easy ....

JAMES DOLAN <james.dolan1@students.mq.edu.au> Thu, Feb 24, 2022 at 12:18 PM
To: john.baez@ucr.edu

i'm hoping that the last glitch to fix here is that if you try to perform the descent _after_ taking the associated vector bundle of the principal bundle, then it gets messy; hopefully if i just take care to perform the descent _before_ passing to the associated bundle then it will work more straightforwardly ....

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Fri, Feb 25, 2022 at 12:16 PM
To: john.baez@ucr.edu

still just thinking outloud here ....

at the moment it almost seems like the way to get the formulas to work is to modify the automorphy equation so that instead of obtaining the "automorphy term" as a "degenerate symplectic pairing" between vector (living in the universal cover of the torus) and period (living in the period lattice), we should instead use the "hermitianization" of that pairing.

at the moment this seems pretty much just a kludge (if it works at all); somehow though it should actually make conceptual-geometric sense in terms of the associated hermitian line bundle of a principal u(1)-bundle, and/or the (proto-)chern connection thereon ....

...
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John Baez <john.baez@ucr.edu> Fri, Feb 25, 2022 at 1:29 PM
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>

Hi -

> at the moment it almost seems like the way to get the formulas to work is to modify the automorphy equation so that instead of obtaining the "automorphy term" as a "degenerate symplectic pairing" between vector (living in the universal cover of the torus) and period (living in the period lattice), we should instead use the "hermitianization" of that pairing.

That reminds me of this from Birkenhake and Lange's book *Abelian Varieties:*

The following lemma shows that the alternating forms of Proposition 2.1.6 are just the imaginary parts of hermitian forms. Recall that a *hermitian form on* V is a map $H: V \times V \to \mathbb{C}$, which is $\mathbb{C}$-linear in the first argument and satisfies $H(v, w) = \overline{H(w, v)}$ for all $v, w \in V$.

Lemma 2.1.7. *There is a 1-1-correspondence between the set of hermitian forms H on V and the set of real valued alternating forms E on V satisfying* $E(iv, iw) = E(v, w)$*, given by*

$$E(v, w) = \operatorname{Im} H(v, w) \quad \textit{and} \quad H(v, w) = E(iv, w) + iE(v, w)$$

for all $v, w \in V$.

The formula for H here looks like the "hermitianization" of the antisymmetric bilinear form E. They go ahead and use this stuff to do exactly the kind of stuff you're doing - get factors of automorphy.

Best,
jb

JAMES DOLAN <james.dolan1@students.mq.edu.au> Fri, Feb 25, 2022 at 2:01 PM
To: John Baez <baez@math.ucr.edu>

ok, thanks. though i'm slightly too afraid to look at it yet!

i'll look really soon, as soon as i get a bit more of a chance to work it out for myself. it really feels like one of those things i need to figure out for myself ....

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Mon, Feb 28, 2022 at 1:03 PM
To: John Baez <baez@math.ucr.edu>

"The formula for H here looks like the "hermitianization" of the antisymmetric bilinear form E. They go ahead and use this stuff to do exactly the kind of stuff you're doing - get factors of automorphy."

yes, i think i'm understanding this stuff much better now; i probably wouldn't mind if we spent the whole discussion this afternoon (assuming it actually happens) just going over this, fixing all my mistakes from last time ....

though there's also lots of other stuff to talk about, time permitting; for example that [endomorphism rings of abelan varieties] stuff ....

....
On Fri, Feb 25, 2022 at 4:29 PM John Baez <john.baez@ucr.edu> wrote:
[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au> Mon, Feb 28, 2022 at 9:34 PM
To: John Baez <baez@math.ucr.edu>

a vague thought which i'd meant to throw into the discussion on monday but which i forgot to include:

the way in which we're separating the chern connection of a holomorphic hermitian line bundle into "phase" and "amplitude" components, and the way in which this might generalize to higher-dimensional holomorphic hermitian vector bundles, vaguely reminds me of rumors i've heard about grothendieck's approach to the riemann-roch theorem; and particularly the role of maximal compact subgroups and flag varieties in that approach. or something like that ....

....

On Mon, Feb 28, 2022 at 4:40 PM JAMES DOLAN <james.dolan1@students.mq.edu.au> wrote:

> sorry i'm late ! ....
>
> On Mon, Feb 28, 2022 at 4:17 PM John Baez <john.baez@ucr.edu> wrote:
>
> > On Mon, Feb 28, 2022 at 1:03 PM JAMES DOLAN <james.dolan1@students.mq.edu.au> wrote:
> >
> > > "The formula for H here looks like the "hermitianization" of the antisymmetric bilinear form E. They go ahead and use this stuff to do exactly the kind of stuff you're doing - get factors of automorphy."
> > >
> > > yes, i think i'm understanding this stuff much better now; i probably wouldn't mind if we spent the whole discussion this afternoon (assuming it actually happens) just going over this, fixing all my mistakes from last time ....
> >
> > That's fine with me. I'm expecting you at the new usual place at 1:30 my time, 4:30 yours:
> >
> > https://ucr.zoom.us/j/7727771354
> >
> > Best,
> > jb

JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Mar 2, 2022 at 9:24 AM
To: John Baez <baez@math.ucr.edu>

so during our most recent monday zoom discussion, still focusing to a great extent on neron-severi groups and thus on the discrete aspect of the classification of holomorphic line bundles, you brought up the issue of the continuous aspect of that classification, or in other words the picard variety and the other connected components of the picard group. i mostly tried to sweep that issue under the carpet for now, suggesting that it's easy and straightforward to deal with (at least for abelian varieties).

however i'm finding now that the issue is more interesting than i'd realized, so i hope to really clarify how it enters into the discussion of "descent from the universal cover" for abelian varieties; and i'm also hoping that this eventually leads in some way to the general topic of moduli stacks of vector bundles (and particularly of how to relate this to exponentiation of 2-rig spectrums!), and of dual abelian varieties as perhaps more or less a special case of this.

....

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John Baez <john.baez@ucr.edu> Wed, Mar 2, 2022 at 11:50 AM
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>
Cc: John Baez <baez@math.ucr.edu>

Hi -

> so during our most recent monday zoom discussion, still focusing to a great extent on neron-severi groups and thus on the discrete aspect of the classification of holomorphic line bundles, you brought up the issue of the continuous aspect of that classification, or in other words the picard variety and the other connected components of the picard group. i mostly tried to sweep that issue under the carpet for now, suggesting that it's easy and straightforward to deal with (at least for abelian varieties).
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> into the discussion of "descent from the universal cover" for abelian varieties; and i'm also hoping that this eventually leads in some way to the general topic of moduli stacks of vector bundles (and particularly of how to relate this to exponentiation of 2-rig spectrums!), and of dual abelian varieties as perhaps more or less a special case of this.

Sounds great! I'd like to better understand how these moduli stacks have a discrete aspect coming from the topological classification of vector bundles, followed by a more continuous aspect coming from the holomomorphic classification.

There should be a lot of nice abstract nonsense about this, but here's a concrete puzzle I don't know the answer to: are the connected components of the moduli space of vector bundles over a variety completely explained by the topological classification? Or can there be a non-connected moduli space of holomorphic structures on a specific topological vector bundle?

I'd already be happy to understand this for line bundles over an arbitrary variety; we understand it pretty well for abelian varieties now though we could make it nicer.

I'm also interested in how 2-rigs of topological vector bundles differ from 2-rigs of holomorphic vector bundles; for any variety there's a map from the latter to the former, but can we think about this in a purely 2-riggish way?

Best,
jb
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