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John Baez <johnb@ucr.edu>

the figure-eight puzzle

26 messages

JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 3:09 AM
To: john.baez@ucr.edu

so, i really want to clear up that puzzle that's been bugging me: elliptic curves are famously groups; the premier example of an elliptic curve ("bernoulli's lemniscate" of 1694 according to steven kleiman) is a sort of "figure-8"; but the figure-8 is not a group; so what gives?

of course this puzzle must have a very simple resolution but as usual i feel i need to figure it out for myself in order to really understand what's going on, so please don't give the answer away just yet.

i'm still trying to get it to work out somehow that a "real elliptic curve" is the fixed-point-set of an anti-holomorphic involution on a complex elliptic curve and that if you allow the involution to have poor compatibility with the group structure then the fixed-point-set could be as bad as a figure-eight; however i'm still having trouble to get it to actually work so far. so let me think about it a bit more!

John Baez <john.baez@ucr.edu> Wed, Jun 8, 2022 at 12:56 PM
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>

Hi -

Yesterday (before getting your email) I asked around on Google about Bernoulli's lemniscate. Apparently it's not an elliptic curve but rather a genus 0-curve with 3 double points.

I'm still confused about why it triggered so much work on elliptic integrals and complex multiplication but apparently that has something to do with the formula for the arc length along the lemniscate: the integrand is a square root of a quartic rational function.

The clearest article about the lemniscate is

- When is a curve an octahedron?

This points out that the lemniscate in homogenized form has double points at (i,1,0), (-i,1,0) and (0,0,1), where the last one is the visible double point in the visible figure 8.

Another really interesting historical account is

- Some milestones of lemniscatomy

This explains that Gauss generalized his famous result about which regular polygons are constructible to a very similar result about chopping up the lemniscate into pieces of equal arc length using ruler and compass! VERY similar: the exact same prime numbers appear, etc. I'm kind of hoping this is part of some sort of Jugendtraumisch analogy between cyclotomic fields and elliptic curves or something, and the end of the paper seems to hint at such things, but I don't understand it.

Best,
jb

[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 2:36 PM
To: John Baez <baez@math.ucr.edu>

ok that's very interesting but now i'm trying to figure out how to deal with spoilers in your emails! i guess i'll have to work out some way to anticipate when you're about to give an answer away ....

anyway, lots of what you're saying makes straightforward sense (for one thing the story about how "elliptic curve" got it's name via arc-length of ellipses (which are basically genus zero curves in some sense i think) is pretty easy to understand and it sounds offhand like the lemniscate story is just a minor variation on that ....) but there are also some confusing level-slips in what you're saying about "equi-division points" on the lemniscate, vs equi-division points on the elliptic curve itself, in a jugendtraum context .... more later ....

this seems like another possibility:

https://math.uchicago.edu/~tghyde/Cox,%20Hyde%20--%20Galois%20theory%20on%20the%20lemniscate.pdf

by the way it's entirely possible that a more careful reading of kleiman's exposition would have made the situation clear; i could already tell from my glance at it that what they said about "arc-length" didn't really fit with my guess that the lemniscate was itself an elliptic curve, but i decided to try as my first assumption that perhaps kleiman was speaking carelessly ....

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 3:01 PM
To: John Baez <baez@math.ucr.edu>

me: "anyway, lots of what you're saying makes straightforward sense (for one thing the story about how "elliptic curve" got it's name via arc-length of ellipses (which are basically genus zero curves in some sense i think) is pretty easy to understand and it sounds offhand like the lemniscate story is just a minor variation on that ....) but there are also some confusing level-slips in what you're saying about "equi-division points" on the lemniscate, vs equi-division points on the elliptic curve itself, in a jugendtraum context .... more later ...."

so let me say a bit more about this, as it might be relevant .... again, based in part on some somewhat wild guesses ....

so, let's think about relationships between "elliptic curves" and "ellipses", with a view towards trying trying to decide whether there's basically just one important such relationship, vs maybe two contrasting such relationships.

so, the obvious (to me) relationship between elliptic curves and ellipses is that roughly speaking elliptic curves can be thought of as geometric orientations between lattices and binary quadratic forms, and "ellipses" are almost the same thing as "binary quadratic forms".

on the other hand, another relationship between "elliptic curves" and "ellipses" is the relationship allegedly responsible for the similarity in names: that an "elliptic integral" is just the indefinite integral for arc-length along an ellipse, which turns out to be multi-valued in the complex domain as a function of the limit of integration given by the moving endpoint of the arc; and that the local functional inverse of that multi-valued function is the doubly-periodic "elliptic function" that wants to live on an elliptic curve. (or something like that.)

so my first question here is: are these two relationships secretly and/or not-so-secretly really the same relationship? offhand i seem to remember that they _are_ basically the same relationship, but i need to stop and think about it for a bit ....

....
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John Baez <john.baez@ucr.edu> Wed, Jun 8, 2022 at 3:22 PM

Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>
Cc: John Baez <baez@math.ucr.edu>

Hi -

> ok that's very interesting but now i'm trying to figure out how to deal with spoilers in your emails! i guess i'll have to work out some way to anticipate when you're about to give an answer away ....

I would hope that everything I say gives the answer to *some* question away, so maybe I should warn you when I might actually say something interesting.

> this seems like another possibility:
>
> https://math.uchicago.edu/~tghyde/Cox,%20Hyde%20--%20Galois%20theory%20on%20the%20lemniscate.pdf

That's interesting. They describe an elliptic curve E there, somehow associated to the lemniscate, but in a not-very-obvious way involving the lemniscate constant, a number that acts a lot like pi. (I think the arclength of a standard-sized lemniscate is 2 times the lemniscate constant.)

I think there should be some much more conceptual way to understand all this stuff, so I don't think I really spoiled your fun.

Best,
jb
[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 3:29 PM
To: John Baez <baez@math.ucr.edu>

me: "but there are also some confusing level-slips in what you're saying about "equi-division points" on the lemniscate, vs equi-division points on the elliptic curve itself, in a jugendtraum context .... more later ...."

so it seems like we ought to try to clarify the relationship between ["equi-division points" on an ellipse and/or on a "lemniscate"] on the one hand, and ["torsion points" (which we should also try to think of as some sort of "equi-division points" in some contexts) on a corresponding elliptic curve] ....

it also seems like i'm suggesting that maybe "lemniscates" and "ellipses" are birationally equivalent over the complex numbers, maybe in an arc-length preserving way or something like that ?? .... ?? and that you can maybe think of elliptic curves as also being "lemniscatic curves" relating to lemniscates in roughly the same way that they also relate to ellipses .... ??

except i'm really not sure yet whether this is on the right track .... there seems to be a straightforward story about how genus 1 elliptic curves relate to genus 0 ellipses, but when i try to figure out where to fit "lemniscates" into the picture, i'm unsure whether the lemniscates should be more like the elliptic curves or more like the ellipses, or else maybe just more like themselves. lemniscates being "genus 0" may be a clue but i'm not sure how to read it because i'm not sure how "genus" works for curves with singularities.

i'm also wondering if there's another significant elementary and/or obvious lesson for me somewhere here: that if a real curve looks like it has a double point then that can't just go away in the complex picture ....

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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 3:40 PM
To: John Baez <baez@math.ucr.edu>

you: "I would hope that everything I say gives the answer to some question away,"

sure, it's just that in this case the question was the one about which i said "of course this puzzle must have a very simple resolution but as usual i feel i need to figure it out for myself in order to really understand what's going on, so please don't give the answer away just yet". but seriously, don't worry about this if it's going to make it harder for you to talk with me; i'll just have to work out ways to deal with it.

you: "That's interesting. They describe an elliptic curve E there, somehow associated to the lemniscate, but in a not-very-obvious way involving the lemniscate constant, a number that acts a lot like pi. (I think the arclength of a standard-sized lemniscate is 2 times the lemniscate constant.)

I think there should be some much more conceptual way to understand all this stuff, so I don't think I really spoiled your fun."

yes, this is all very interesting and suggestive .... i need to think about it more ....

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 3:43 PM
To: John Baez <baez@math.ucr.edu>

of course one of the things we should think about is how "arc length of a circle" fits in as a "degenerate" case of "arc length of an ellipse" ....

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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 4:49 PM
To: John Baez <baez@math.ucr.edu>

you: "Another really interesting historical account is: Some milestones of lemniscatomy"

ok, i didn't realize till just now that i'm supposed to parse "lemniscatomy" more like "cyclotomy" than like "lobotomy" or "anatomy" .... well, or i guess you're telling me that the salient commonality is "cutting" or "dissecting" ....

....
[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 4:56 PM
To: John Baez <baez@math.ucr.edu>

oh ok, like "a-tom" ....

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 9:21 PM
To: John Baez <baez@math.ucr.edu>

so here's one calculation i plan to try: take the field C(z) of meromorphic functions on the riemann sphere, and try to find a nice set of field generators for the subfield of those functions that take on the same value at 0 as at infinity, same at 1 as at -1, and same at i as at -i; to see if this gives anything like bernoulli's lemniscate.

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 9:26 PM
To: John Baez <baez@math.ucr.edu>

by the way, there's another possible approach we could try towards all this "lemniscate" stuff: we could try ignoring it in favor of concentrating on other approaches to elliptic curves; on the theory that those other approaches might be conceptually simpler.

however at the moment i'm inclined to pursue the lemniscate ideas a bit further, more or less along the lines you seem to be suggesting, trying to connect it with jugendtraum ideas and so forth.

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Wed, Jun 8, 2022 at 11:21 PM
To: John Baez <baez@math.ucr.edu>

me: "so here's one calculation i plan to try: take the field C(z) of meromorphic functions on the riemann sphere, and try to find a nice set of field generators for the subfield of those functions that take on the same value at 0 as at infinity, same at 1 as at -1, and same at i as at -i; to see if this gives anything like bernoulli's lemniscate."

blecchh, what i just wrote there was conceptually illiterate!! the birational geometry of fields of meromorphic functions on curves is incapable of seeing singularities such as double points, so the plan sketched out above makes no sense.

to try to get the plan to make more sense i should re-phrase it in terms of homogeneous coordinate algebras instead of in terms of fields of meromorphic functions ....

....
[Quoted text hidden]

John Baez <john.baez@ucr.edu> Wed, Jun 8, 2022 at 11:34 PM
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>
Cc: John Baez <baez@math.ucr.edu>

On Wed, Jun 8, 2022 at 4:49 PM JAMES DOLAN <james.dolan1@students.mq.edu.au> wrote:

> you: "Another really interesting historical account is: Some milestones of lemniscatomy"
>
> ok, i didn't realize till just now that i'm supposed to parse "lemniscatomy" more like "cyclotomy" than like "lobotomy" or "anatomy" .... well, or i guess you're telling me that the salient commonality is "cutting" or "dissecting" ....

Yes, I was annoyed by that "a" myself - I guess the "a" in "lemniscate" trumped the "o" in "cyclotomy".

Best,
jb

John Baez <john.baez@ucr.edu> Wed, Jun 8, 2022 at 11:49 PM
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>
Cc: John Baez <baez@math.ucr.edu>

Hi -

Here's a connection between tori and the lemniscate of Bernoulli that might or might not be related to the connection between elliptic curves and the lemniscate of Bernoulli: i

If you slice the usual sort of "round torus" in R^3 with a plane in such a way that the intersection is a figure 8, you get the lemniscate of Bernoulli. A picture might help:

https://mobile.twitter.com/matthen2/status/1243424349446483968

But whether this "round torus" is at all related to an elliptic curve - beyond having the right topology - seems a bit unlikely to me! Simon Burton was asking about this once.

Best,
jb
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Sat, Jun 11, 2022 at 1:58 AM
To: John Baez <baez@math.ucr.edu>

wait a minute, is _that_ what archimedes plutonium was saying all that time ?

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Sat, Jun 11, 2022 at 2:17 AM
To: John Baez <baez@math.ucr.edu>

me:

"by the way, there's another possible approach we could try towards all this "lemniscate" stuff: we could try ignoring it in favor of concentrating on other approaches to elliptic curves; on the theory that those other approaches might be conceptually simpler.

however at the moment i'm inclined to pursue the lemniscate ideas a bit further, more or less along the lines you seem to be suggesting, trying to connect it with jugendtraum ideas and so forth."

i'm steering a bit more now towards ignoring (or at least soft-pedaling) the lemniscate ideas, on the grounds that:

1. i'm seeing more evidence now that the relationship between elliptic curves and lemniscates is mostly just an obfuscated replay of the relationship between elliptic curves and ellipses. thus i suspect that this "lemniscatomy" business (dealing with dissecting lemniscates into portions of equal arc-length, and relating this to torsion points on a corresponding elliptic curve) is paralleled by a conceptually more straightforward "ellipsotomy" story (even if i don't find any google-hits on "ellipsotomy" yet).

2. the only way i got interested in lemniscates in the first place was through a mistaken reading of some stuff steven kleiman wrote.

so, in my next follow-up emails, i'm planning to pursue a lot of the conceptual themes that you've been bringing up in connection with lemniscates, but with the context changed form leminscates to ellipses.

....
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JAMES DOLAN <james.dolan1@students.mq.edu.au> Sat, Jun 11, 2022 at 2:51 AM
To: John Baez <baez@math.ucr.edu>

so, lesson #1 about elliptic curves tends to be: don't get mixed up between elliptic curves and ellipses; yes there's a historical relationship between them but they're very different kinds of objects.

whereas here i am now taking a skeptical attitude toward lesson #1; that is, i want to see how far i can and/or should get in conflating "elliptic curve" with "ellipse".

my vague idea to start with here is something like this: to try to construe an elliptic curve as the 1-parameter symmetry group of the arc-length structure on an ellipse.

there's a mix of sophistication-levels bumping into each other here:

1: a lowbrow viewpoint according to which an algebraic variety is a set with structure

2: a middlebrow viewpoint (the so-called "functorial viewpoint") according to which an algebraic variety is a set-valued functor on a category of commutative rings

3: a highbrow viewpoint according to which an algebraic variety is the spectrum of models of a theory in the belief-doctrine of total 2-rigs.

at the lowbrow level we'll try to see a map from a real elliptic curve to a real ellipse which is in some sense close to being a structure-preserving bijection, taking torsion points on the elliptic curve to "equi-subdivision" points on the ellipse wrt arc-length. to get started on this it's probably helpful to start writing down formulas expressing arc-length on an explicitly given ellipse in indefinite-integral form ....

[more later, hopefully]

....
[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au>
To: John Baez <baez@math.ucr.edu>
Sat, Jun 11, 2022 at 3:07 AM

so proceeding naively ....

i'm picturing an ellipse in the (x,y)-plane with 2 focuses on the x-axis, with the origin halfway between the 2 focuses. i want to construe the part of the ellipse above the x-axis as the graph of a function y = f(x) on an interval, implicitly given by x^2 + k1*y^2 = k2. then i want to write down in indefinite-integral form a formula for the arc-length of the ellipse from x=x1 to x=x2.

i guess i'll try waiting till i'm more awake before writing the formula down ....

....
[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au>
To: John Baez <baez@math.ucr.edu>
Sat, Jun 11, 2022 at 3:17 AM

by the way, i'm trying to encourage us here into believing that the formulas are not bad; that is that when you stare at them just a bit you can see pretty directly that the arc-length along an ellipse leads us to a double cover of the riemann sphere with 4 branch-points (aka "an elliptic curve"), and moreover that the naive geometric picture relates the elliptic curve to the ellipse in a straightforward way.

....
[Quoted text hidden]

JAMES DOLAN <james.dolan1@students.mq.edu.au>
To: John Baez <baez@math.ucr.edu>
Sat, Jun 11, 2022 at 4:32 AM

roughly speaking what i'm suggesting here is that the riemann sphere which is double-covered-with-4-branch-points by an elliptic curve x _is_ essentially morally the ellipse corresponding to x; and that the arc-length structure on the ellipse is encoded in the 4 branch points.

how can it make sense for an "ellipse" to be a "riemann sphere"? an ellipse is a sort of RP^1 while a riemann sphere is a sort of CP^1, so the interplay between R and C here should help in making this conceptual identification ....

the basic ideas here still need to be carefully checked though! they're still based on some fairy wild guesses so far.

....
[Quoted text hidden]

John Baez <john.baez@ucr.edu> Sat, Jun 11, 2022 at 9:17 AM
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>

Hi -

> by the way, i'm trying to encourage us here into believing that the formulas are not bad;

Heh, I think you're mainly trying to encourage yourself.

Indeed I've been slowly digging into the math of the lemniscate, just for the fun of it. I haven't made much progress on the original puzzle yet but all this material must be leading up to some story about the lemniscate and an elliptic curve that's the complex plane mod the lattice generated by (1+i) ϖ and (1-i) ϖ, where ϖ is the "lemniscate constant".

I've been tweeting about this a bit. The tweets are sort of click-baity rather than deep. If you regularly read my tweets let me know and I won't bother showing you stuff from them. But here's one:

I just learned that there's a number ϖ that's a lot like π.

Just as 2π is the circumference of the unit circle, 2ϖ is the perimeter of a curve called Bernoulli's lemniscate.

There's even a whole family of functions resembling trig functions with period 2ϖ!

The **lemniscate constant** ϖ is like a mutant version of the number π:

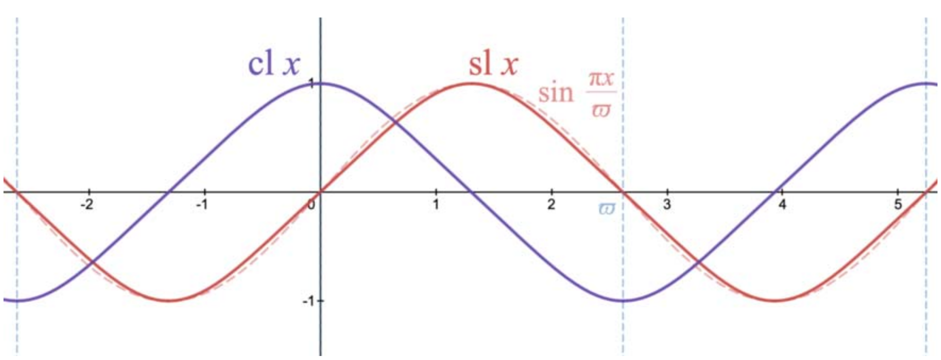
$$\pi = \int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}} \approx 3.14159 \qquad \varpi = \int_{-1}^{1} \frac{dx}{\sqrt{1-x^4}} \approx 2.622057$$

It obeys a lot of similar formulas. For example:

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}} \cdot \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \cdots$$

$$\frac{2}{\varpi} = \sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{2} + \frac{1}{2}\Big/\sqrt{\frac{1}{2}}} \cdot \sqrt{\frac{1}{2} + \frac{1}{2}\Big/\sqrt{\frac{1}{2} + \frac{1}{2}\Big/\sqrt{\frac{1}{2}}}} \cdots$$

The lemniscate sine and cosine functions look a lot like the usual sine and cosine. They have period 2ϖ instead of 2π, so here we compare sin(2πx/ϖ), which also has period 2ϖ. (2/n)



The lemniscate sine and cosine functions obey mutant versions of the usual trig identities! For example:

$\mathrm{sl}^2\, x + \mathrm{cl}^2\, x = 1 - \mathrm{sl}^2\, x\; \mathrm{cl}^2\, x$

and they have derivatives

$\mathrm{sl}' x = - (1 + \mathrm{sl}^2 x) \mathrm{cl}\, x$
$\mathrm{cl}' x = - (1 + \mathrm{cl}^2 x) \mathrm{sl}\, x$

It's easier to define their inverses:

Inverse lemniscate functions

The inverse function of the lemniscate sine function is

$$\operatorname{arcsl} x = \int_0^x \frac{\mathrm{d}t}{\sqrt{1-t^4}}$$

The inverse function of the lemniscate cosine function is

$$\operatorname{arccl} x = \int_x^1 \frac{\mathrm{d}t}{\sqrt{1-t^4}}$$

The lemniscate versions of trig functions are examples of 'elliptic functions'.

The Weierstrass elliptic function and Jacobi elliptic functions [of which the lemniscate trig functions are a special case!] are more familiar.

This page is like looking into a closet and discovering a new world:

https://en.wikipedia.org/wiki/Lemniscate_elliptic_functions

Best,
jb

JAMES DOLAN <james.dolan1@students.mq.edu.au> Sat, Jun 11, 2022 at 11:17 AM
To: John Baez <baez@math.ucr.edu>

you: "If you regularly read my tweets let me know and I won't bother showing you stuff from them."

so far i haven't found twitter to be a useful tool so i haven't bothered to learn how to use it efficiently. if you think it might be useful for me to use it then i could probably learn to use it more efficiently, particularly if i can get a few casual/ish answers from you as to questions like "what is twitter supposed to be good for?"; this is perhaps related to your comment about "click-baity rather than deep".

by the way, googling on the phrase "lemniscatic curve" suggests that some people do espouse "lemniscatic curve" : "lemniscate" :: "elliptic curve" : "ellipse" ....

so far my only wild guess as to why some people might like lemniscatic curves in comparison to elliptic curves is that they think the lemniscate corresponding to a "lemniscatic curve with square period lattice" is more visually distinctive than the ellipse corresponding to an "elliptic curve with square period lattice". here i'm using "lemniscate" in a variable way allowing lemniscates of varying "eccentricity" ....

you: "I haven't made much progress on the original puzzle yet" am i confused here? i thought the "original puzzle" vanished when it turned out i'd misread kleiman .... again, the main thing i'm trying to salvage from my mistake is roughly speaking your suggestion to give an "arc-length" interpretation of jugendtraum ideas, though (mostly) with arc-length of an ellipse rather than of a lemniscate.

(actually of course there _are_ things that i'm confused about, in the near-vicinity of topics in this discussion ....)

....
[Quoted text hidden]

John Baez <john.baez@ucr.edu>
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>
Cc: John Baez <baez@math.ucr.edu>
Sat, Jun 11, 2022 at 12:29 PM

Hi -

On Sat, Jun 11, 2022 at 4:33 AM JAMES DOLAN <james.dolan1@students.mq.edu.au> wrote:

> roughly speaking what i'm suggesting here is that the riemann sphere which is double-covered-with-4-branch-points by an elliptic curve x _is_ essentially morally the ellipse corresponding to x; and that the arc-length structure on the ellipse is encoded in the 4 branch points.

Something like has got to be right. Computing the arclength of an ellipse gives you an integral that you can massage into the integral involving the square root of a quartic. Here's one famous integral of that general sort:

$$F(x;k) = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}.$$

The fact that the square root is in the denominator is not essential here.

Modern folks like us have to think about where the integrand is defined; the square root says we need some sort of double cover of the Riemann sphere, and since the stuff in the square root vanishes at 4 points - 1, -1, k, -k - then this cover has 4 branch points.

So, the integrand here is really a function on an elliptic curve.

Then we have to think about doing the integral...

Best,
jb

John Baez <john.baez@ucr.edu>
Reply-To: baez@math.ucr.edu
To: JAMES DOLAN <james.dolan1@students.mq.edu.au>
Cc: John Baez <baez@math.ucr.edu>
Sat, Jun 11, 2022 at 12:53 PM

Hi -

> if you think it might be useful for me to use it then i could probably learn to use it more efficiently, particularly if i can get a few casual/ish answers from you as to questions like "what is twitter supposed to be good for?"; this is perhaps related to your comment about "click-baity rather than deep".

It may not be useful for you. There are some people who reliably post nice images and movies related to Coxeter groups, etc. - like this:

https://twitter.com/mananself/status/1535657672271437824

And there are other people who reliably tweet good stuff about archaeology, linguistics, etc. etc. etc. But I probably wouldn't bother with twitter if I didn't enjoy having hordes of followers who like my math and physics tweets.

> so far my only wild guess as to why some people might like lemniscatic curves in comparison to elliptic curves is that they think the lemniscate corresponding to a "lemniscatic curve with square period lattice" is more visually distinctive than the ellipse corresponding to an "elliptic curve with square period lattice". here i'm using "lemniscate" in a variable way allowing lemniscates of varying "eccentricity" ....

Elsewhere:

> you: "I haven't made much progress on the original puzzle yet"
>
> am i confused here? i thought the "original puzzle" vanished when it turned out i'd misread kleiman.

I guess by "original puzzle" I meant: what does Bernoulli's lemniscate have to do with elliptic curves?

It's quite possible ellipses will be good enough and you can forget about lemniscates. But there's so much cool stuff about lemniscate elliptic functions, and the history of elliptic functions turned so heavily on them, that I can't help but think there's something good about understanding them.

You don't need to bother! But I'm having a lot of fun reading " Some Milestones of Lemniscatomy", which makes more sense each time. Maybe at some point I'll get to the point of having something worth explaining.

Best,
jb

JAMES DOLAN <james.dolan1@students.mq.edu.au> Sat, Jun 11, 2022 at 1:22 PM
To: John Baez <baez@math.ucr.edu>

you: "Something like has got to be right."

yes, but i'm not sure yet of a best and/or good way to really clarify it ....

naively, i'm just trying to overlay pictures of an ellipse and of an elliptic curve in such a way that "arc-length equi-division points" on the ellipse coincide with torsion points on the elliptic curve (if that's really the right idea!) and to give some less-naive conceptual interpretation to the mappings implicit in such an overlay ....

(i did briefly think about the extent to which any of this generalizes to abelian varieties of higher dimension and/or to curves of higher genus. the apparent shortage of interesting such generalizations again suggests that some of what's going on here isn't very conceptually fundamental; on the other hand i'm interested in the general theme here of the interaction between "highbrow" and "lowbrow" approaches to geometry .... the "highbrow" approaches have the flavor of "carrying extra weights while training, on the theory that it helps you to develop extra strength which will manifest itself after you stop carrying them" ....)

....
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