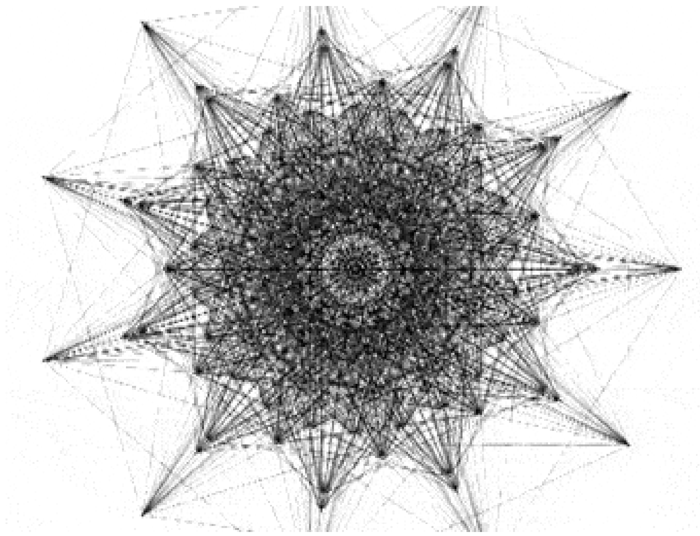
[For my December 2014 diary, go here.]

Diary — January 2015

John Baez

January 1, 2015



Happy New Year!

The Earth has successfully completed another revolution about the Sun! This is a traditional excuse for a moment of reflection, so I'll try that.

It's been an interesting and stressful year for me. I'm struggling to do some more practical things for the health of our planet. I believe global warming is a serious problem and we're facing a mass extinction event. I can't just sit around. But my love for the beauty of pure math and theoretical physics keeps pulling me back to the things I used to think about. I feel torn and frustrated.

With my pals at the Azimuth Project, we've reached the point of understanding a bit about El Niño prediction — I gave a talk about this to about 1000 people at the Neural Information Processing Seminar, a big annual conference on machine learning. We made some good progress. But we've only just dipped our toes into a very deep subject. To go further I'd need to learn a lot more, get serious about programming, and start attending the annual conference on Climate Informatics. I'd need to get better at working with folks in the Azimuth Project, and pull more experts into it. And most of all: I'd need to think harder about climate science and the art of prediction, and come up with some new ideas.

By comparison, it seems easy to come up with new ideas in pure math and theoretical physics — because I spent decades doing it. Unfortunately, it feels a bit pointless. I don't think the world urgently needs to understand more about the fundamental laws of physics, not right now. Someday it will be important. But fundamental physics doesn't hold the 'magic bullet' for the problems we face today. And anyway, we've already got a lot of very smart people banging their heads against that wall. We need something a bit different. I'm in a lucky position where I can afford to thrash around trying to figure out what that is. If 1000 of us try, some will succeed, and we may do a bit better finding our way through the ecological bottleneck.

That's what I tell myself, anyway. But I also just love pure math regardless of whether it's good for anything. So right now I'm pursuing it as a kind of 'hobby'. It helps me relax. I've stepped aside from the great mathematical challenge of our time — developing the theory of infinity-categories and the new world of math this opens up. Instead, I'm thinking about
 ir role in physics: things like the octonions, the group called E_8, and the Leech lattice. I've put enough time into these over the years that I can come up with cute ideas without a massive investment of

effort.... thanks to help from Greg Egan, who is great at proving or disproving my conjectures.

As a kind of middle road, I'm also working with my grad students on 'network theory' — basically, applying category theory to comlex systems made of interacting parts, as we see in biology, chemistry, electrical engineering, and the like. This is not instantly useful; it will take years to develop. But I have a good feeling about it! This might be a place where fancy abstract math can do some good.

So I guess it's a three-pronged approach to life. It gets to be a bit much at times! And then there's the job I actually get paid for: teaching. I may be doing too many things to do any of them well.

But I'm rarely bored. When I was a kid, I was often bored. I didn't know how to do the cool things I dreamt of doing. I hated it. Those days are gone. I'm very happy about that.

In case you're wondering, the image here is the Higman-Sims graph, an exceptional structure lurking in the Leech lattice, animated by David Madore. He writes:

> The Higman-Sims graph is the unique graph with 100 vertices such that each is adjacent to 22 others and no two adjacent vertices have a common neighbor (i.e., the graph has no triangle) and any two non-adjacent vertices have exactly six common neighbors. It has 88704000 automorphism, forming an extension of 2 by the unique simple group of order 44352000 (the Higman-Sims group, a sporadic group).
>
> The Higman-Sims graph occurs inside the 24-dimensional Leech lattice (if X,Y,Z are Leech lattice points at distances 3,3,2 from each other, then there are 100 Leech lattice points at distance 2,2,2 from X,Y,Z, and if we connect those at distance 3 from another, we obtain the Higman-Sims graph).
>
> This animation displays various orthogonal projections of the Higman-Sims graph inside the Leech lattice, chosen so as to reveal an 11-fold symmetry (there is only one conjugacy class of order 11 in the Conway group 70, which is in the Higman-Sims group).

January 4, 2015

The most dangerous animal in the world

An adult male grizzly bear can stand 3 meters tall (almost 10 feet) on its hind legs. A big one can weigh 360 kilograms (almost 800 pounds).

But that's not the really dangerous animal in this picture. A human being won this contest — with a gun.

Luckily it was a dart gun. This bear, near Vancouver, is sedated, about to be tagged by scientists. It will be fine, losing only a bit of its dignity.

Derrick Jensen wrote a book *Thought to Exist in the Wild: Awakening from the Nightmare of Zoos*. Here are some quotes:

> The bear takes seven steps, her claws clicking on concrete. She dips her head, turns, and walks toward the front of the cage. Another dip, another turn, another three steps. When she gets back to where she started, she begins all over. This is what's left of her life.
>
> Outside the cage, people pass by on a sidewalk. Parents stop strollers until they realize there's nothing here to see. A pair of teenagers approach, wearing Walkmans and holding hands; one glance inside is enough, and they're off to the next cage. Still the bear paces; three steps, head dip, turn.
>
> My fingers are wrapped tightly around the metal railing outside the enclosure. I notice they're sore. I look at the silver on the bear.s back, the concave bridge of her nose. I wonder how long she's been here. I release the rail, and as I walk away, the rhythmic clicking of claws on concrete slowly fades.
>
> Unfortunately most of us by now have been to enough zoos to be familiar with the archetype of the creature who has been driven insane by confinement: the bear pacing a precise rectangle; the ostrich incessantly clapping his bill; the elephants rhythmically swaying. But the bear I describe is no archetype. She is a bear. She is a bear who, like all other bears, at one time had desires and preferences all her own, and who may still, beneath the madness.

Or at this point she may not.

[...]

If you see an animal in a zoo, you are in control. You can come, and you can go. The animal cannot. She is at your mercy; the animal is on display for you.

In the wild, the creature is there for her own purposes. She can come, and she can go. So can you. Both of you can display as much of yourselves to the other as you wish. It is a meeting of equals. And that makes all the difference in the world.

One of the great delights of living far from the city is getting to know my nonhuman neighbors — the plants, animals, and others who live here. Although we've occasionally met by chance, I've found that it is usually the animals who determine how and when they reveal themselves to me. The bears, for example, weren't shy, showing me their scat immediately and their bodies soon after, standing on hind legs to put muddy paws on windows and look inside; or offering glimpses of furry rumps that disappeared quickly whenever I approached on a path through the forest; or walking slowly like black ghosts in the deep gray of predawn. Though I am used to their being so forward, it is always a gift when they reveal themselves, as one did recently when he took a swim in the pond in front of me.

Robins, flickers, hummingbirds, and phoebes all present themselves, too. Or rather, like the bear, they present the parts of themselves they want seen. I see robins often, and a couple of times I've seen fragments of blue eggshells long after the babies have left, but I've never seen their nests.

These encounters — these introductions — are on terms chosen by those who were on this land long before I was: they choose the time, place, and duration of our meetings. Like my human neighbors and friends, they show me what they want of themselves, when they want to show it, how they want to show it, and for that I am glad. To demand they show me more — and this is as true for nonhumans as it is for humans — would be unconscionably rude. It would destroy any potential our relationship may once have had. It would be unneighborly.

I am fully aware that even a young bear can kill me. I am also fully aware that humans have coexisted with bears and other wild animals for tens of thousands of years. Nature is not scary. It is not a den of fright and horrors. For almost all of human existence, it has been home, and the wild animals have been our neighbors.

Right now, worldwide, more than 1 million people die each year in road accidents. In the United States alone, there are about forty-two thousand traffic fatalities a year. Yet I am not afraid of cars — though perhaps I should be. Around the world, nearly 2 million people per year are killed through direct violence by other people. Almost 5 million people die each year from smoking. And how many people do bears kill? About one every other year in all of North America.

We are afraid of the wrong things.

[...]

I'm at a zoo. Everywhere I see consoles atop small stands. Each console has a cartoonish design aimed at children, and each has a speaker with a button. When I push the button, I hear a voice begin the singsong: "All the animals in the zoo are eagerly awaiting you". The song ends by reminding the children to be sure to "get in on the fun".

I look at the concrete walls, the glassed-in spaces, the moats, the electrified fences. I see the expressions on the animals' faces, so different from the expressions of the wild animals I've seen. The central conceit of the zoo, and in fact the central conceit of this whole culture, is that all of these "others" have been placed here for us, that they do not have any existence independent of us, that the fish in the oceans are waiting there for us to catch them, that the trees in the forests stand ready for us to cut them down, that the animals in the zoo are there for us to be entertained by them.

> It may be flattering to believe that everything is here to serve you, but in the real world, where real creatures exist and real creatures suffer, it's narcissistic and dangerous to pretend nobody matters but you.

For more of Derrick Jensen's book, see:

- Derrick Jensen, Thought to exist in the wild — awakening from the nightmare of zoos, *Sun*, November 2007.

I got the photo from Sean Sparling's Twitter feed. I do not know who took it.

January 10, 2015

Everyone knows this: $3^2 + 4^2 = 5^2$

Now you know *this*: $3^3 + 4^3 + 5^3 = 6^3$

The Pythagorean theorem says the sum of the squares of the sides of a right triangle is the square of the hippopotamus. For example, there's a right triangle with sides of length 3, 4, and 5, since

$$9 + 16 = 25$$

so

$$3^2 + 4^2 = 5^2$$

We call three integers with these properties a Pythagorean triple. There are infinitely many! For example, the next ones are

$$5^2 + 12^2 = 13^2 \qquad (25 + 144 = 169)$$

$$8^2 + 15^2 = 17^2 \qquad (64 + 225 = 289)$$

There's a nice recipe to get *all* the Pythagorean triples! Just take integers $n < m$ and let

$$\begin{aligned} a &= m^2 - n^2 \\ b &= 2mn \\ c &= m^2 + n^2 \end{aligned}$$

Then you get

$$a^2 + b^2 = c^2$$

This doesn't give all the Pythagorean triples yet — but you can get the rest by taking $a, b,$ and c and multiplying them all by the same number.

All this has been known for a long time — Euclid wrote about it around 300 BC. There's a lot more to say, but not now! Yesterday the guy who fixes my computers, David Scharffenberg, told me that

$$3^3 + 4^3 + 5^3 = 6^3$$

That's great! It looks like a generalization of

$$3^2 + 4^2 = 5^2$$

But it's not really a generalization in any way that I know. As far as I know, this equation is just a wonderfully cute, meaningless *coincidence*. I could be wrong. But in particular,

$$3^4 + 4^4 + 5^4 + 6^4 \neq 7^4$$

When is the sum of 3 cubes a cube? I don't know, but there's a conjecture saying that any number *except* for those of the form $9k + 4$ and $9k - 4$ is the sum of 3 cubes.

Puzzle 1: Why can't numbers of the form $9k + 4$ or $9k - 4$ for some integer k be written as the sum of 3 cubes of integers?

Puzzle 2: The solution to Puzzle 1 involves working modulo 9. Why can't you get more constraints by working modulo other numbers?

For example, 29 is the sum of 3 cubes:

$$3^3 + 1^3 + 1^3 = 29$$

But cubes can be negative! This makes it harder to find all the solutions. For example, we also have

$$4^3 + (-2)^3 + (-3)^3 = 29$$

So, was only rather recently that the number 30 was shown to be the sum of 3 cubes:

$$(-283059965)^3 + (-2218888517)^3 + 2220422932^3 = 30$$

Pine, Yarbrough, Tarrant, and Beck discovered this in 1999 following an approach suggested by Noam Elkies.

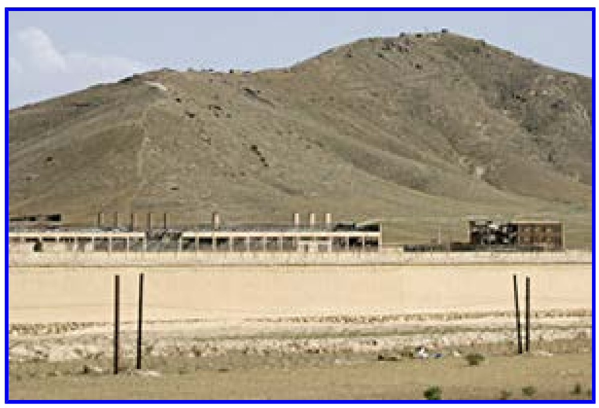
It's still not known if the number 33 is a sum of 3 cubes! But don't bother looking for solutions where the absolute value of one of the three numbers being cubed is less than 100 trillion, because there aren't any.

For more see:

- Michael Beck, Eric Pine, Wayne Tarrant and Kim Yarbrough Jensen, New integer representations as the sum of three cubes, *Mathematics of Computation* **76** (2007), 1683–1690.

January 11, 2015

The Salt Pit



On Dec. 31, 2003, I took a bus from Germany to Macedonia. When we arrived, my nightmare began. Macedonian agents confiscated my passport and detained me for 23 days. I was not allowed to contact anyone, including my wife.

At the end of that time, I was forced to record a video saying I had been treated well. Then I was handcuffed, blindfolded and taken to a building where I was severely beaten. My clothes were sliced from my body with a knife or scissors, and my underwear was forcibly removed. I was thrown to the floor, my hands pulled behind me, a boot placed on my back. I was humiliated.

Eventually my blindfold was removed, and I saw men dressed in black, wearing black ski masks. I did not know their nationality. I was put in a diaper, a belt with chains to my wrists and ankles, earmuffs, eye pads, a blindfold and a hood. I was thrown into a plane, and my legs and arms were spread-eagled and secured to the floor. I felt two injections and became nearly unconscious. I felt the plane take off, land and take off. I learned later that I had been taken to Afghanistan.

Khaled El-Masri wrote this back in 2005, and I added it to my collection of posts about the US-run torture program:

- John Baez Torture.

In Afghanistan, he was interrogated in the Salt Pit, a CIA-run 'black site' shown in the picture above. We are now learning more about this place.

> There, I was beaten again and left in a small, dirty, cold concrete cell. I was extremely thirsty, but there was only a bottle of putrid water in the cell. I was refused fresh water.
>
> That first night I was taken to an interrogation room where I saw men dressed in the same black clothing and ski masks as before. They stripped and photographed me, and took blood and urine samples. I was returned to the cell, where I would remain in solitary confinement for more than four months.

He was interrogated, force-fed, lost 60 pounds. His requests to see a lawyer were ignored. Eventually he was blindfolded, handcuffed, chained to an airplane seat, and taken to Albania, where he was left in the mountains. Eventually he made it back to his home in Germany.

His crime? His name resembled that of the terror suspect Khalid al-Masri.

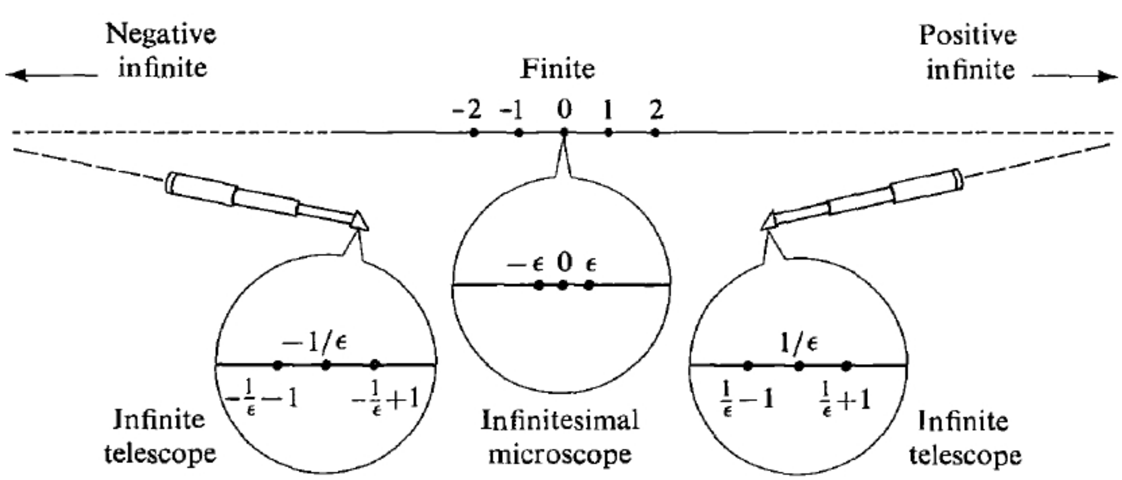
In 2006 as U.S. Federal District Judge dismissed a lawsuit he filed against the CIA, stating that a public trial would "present a grave risk of injury to national security." A Court of Appeals also dismissed the case, and in 2008 so did the U.S. Supreme Court.

In the newly released U.S. Senate report, a supervisor is quoted as saying the Salt Pit was "good for interrogations because it is the closest thing ... to a dungeon." According to the *Los Angeles Times*:

> Guards and interrogators tiptoed through the darkness, carrying headlamps to count detainees packed into two dozen cells. Their lights illuminated prisoners hanging from overhead bars, next to buckets on the floor to catch their waste. One hung there for 17 days.
>
> Another detainee "looked like a dog that had been kenneled," wrote an interrogator. "When the doors to their cells were opened, they cowered," according to CIA documents quoted in the report.
>
> Indeed, reports of sleep and sensory deprivation; of nudity and unhealthful, unsanitary food; of cold showers and ice buckets; and of rough takedowns and mock executions never were reported to supervisors.

The moral? I don't have a moral. But it's curious: anyone in the US who cared has known the rough outlines of what we've been doing for at least 12 years. Read my posts! Yet now some people are acting surprised. Where were they back then?

January 12, 2015



Leibniz based his approach to calculus on infinitesimals - numbers that are bigger than zero but smaller than 1/2, 1/3, 1/4, ... and so on. Many people were uncomfortable with these, so they figured out how to do calculus without infinitesimals.

That's how it's usually taught now.

But it turns out you can do calculus with infinitesimals in a perfectly rigorous way... and in some ways, it's easier! Here's a free online textbook that teaches calculus this way:

- H. Jerome Keisler, *Elementary Calculus*.

The picture here is from this book. There's a tiny little infinitesimal number ϵ, pronounced 'epsilon'. And $1/\epsilon$ is infinitely big! These aren't 'real numbers' in the usual sense. Sometimes they're called hyperreal numbers.

You can calculate the derivative, or rate of change, of a function f by doing

$$\frac{f(x+\epsilon) - f(x)}{\epsilon}$$

and then at the end throwing out terms involving ϵ. For example, suppose

$$f(x) = x^2$$

Then to compute its derivative we do

$$\frac{(x+\epsilon)^2 - x^2}{\epsilon}$$

Working this out, we get

$$\frac{x^2 + 2\epsilon x + \epsilon^2 - x^2}{\epsilon} = \frac{2\epsilon x + \epsilon^2}{\epsilon} = 2x + \epsilon$$

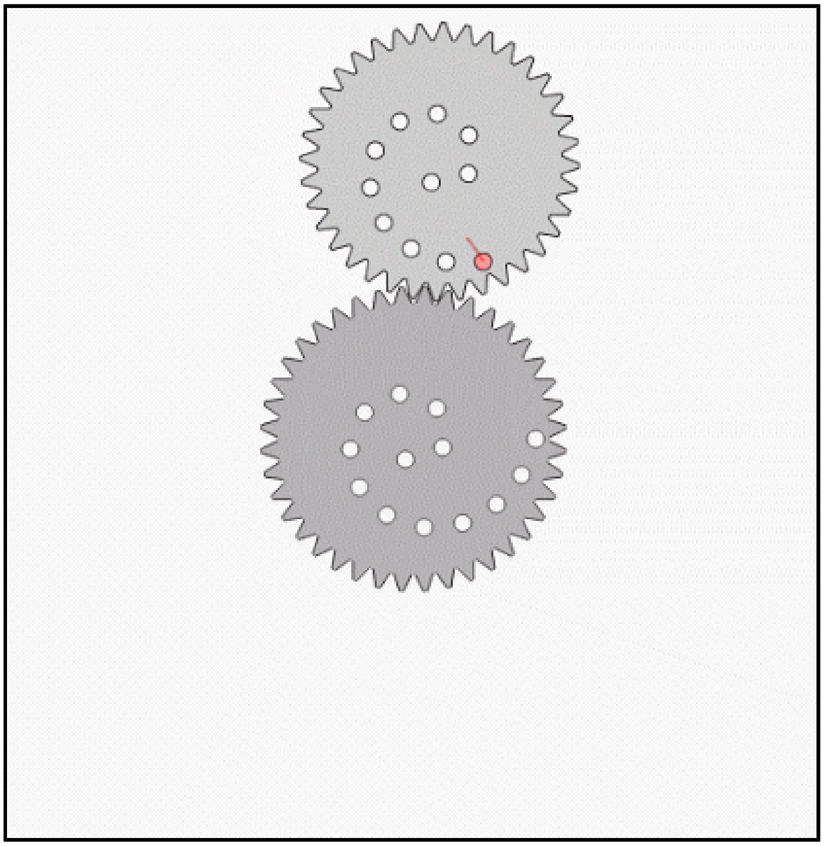
Then, at the end, we throw out the term involving ϵ. So, we get $2x$ This is the rate of change of the function x^2.

The book will teach you calculus this way, from scratch. If you had trouble understanding 'limits' in calculus, you might prefer this way. Or, you might just enjoy seeing another approach.

The details of this subject are infinitely interesting, but I'll just say an infinitesimal amount. In 1961 the logician Abraham Robinson showed that hyperreal numbers are just as consistent as ordinary real numbers, and that the two systems are compatible in a certain precise sense. In 1976, Jerome Keisler, a student of the famous logician Tarski, published this elementary textbook that teaches calculus using hyperreal numbers.

Now it's free, with a Creative Commons copyright!

January 13, 2015

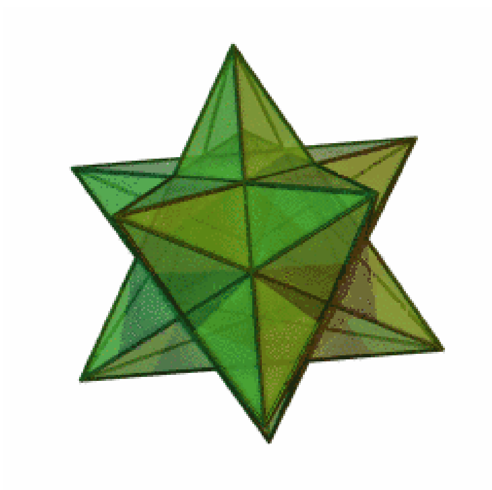


If you ever had a spirograph, or even better if you never had one: now there's a one on your web browser! It's called Inspirograph.

It's easy to use. You just move one gear around the other using your mouse (or finger). I still prefer the actual spirograph: working with actual physical tools is a much more immersive experience than twiddling a computer. People are getting starved for contact with interesting matter. But not everyone has access to a spirograph!

It's written using TypeScript - a typed superset of JavaScript that compiles to plain JavaScript.

January 14, 2015



What if you look for shapes that are as symmetrical as Platonic solids, but where all the faces are stars? Then you'll find this.

If you look carefully, you'll see lots of 5-pointed stars. Each one is a regular pentagram &mdash a 5-pointed star whose corners are a regular pentagon. Each one touches 5 others at each corner, in exactly the same way. So, it's as regular as you might want.

But it's funny in some ways. First, the faces are stars instead of regular polyhedra. Second, the faces intersect each other: that's why you don't see all of any star.

There are 2 polyhedra whose faces are all regular stars, with each face just like every other and each vertex like every other.

This particular one is called the small stellated dodecahedron, because if you remove all the pyramid-shaped pieces you're left with a dodecahedron! Each star lies in the same plane as one of this dodecahedron's faces. So, there are 12 stars in this shape.

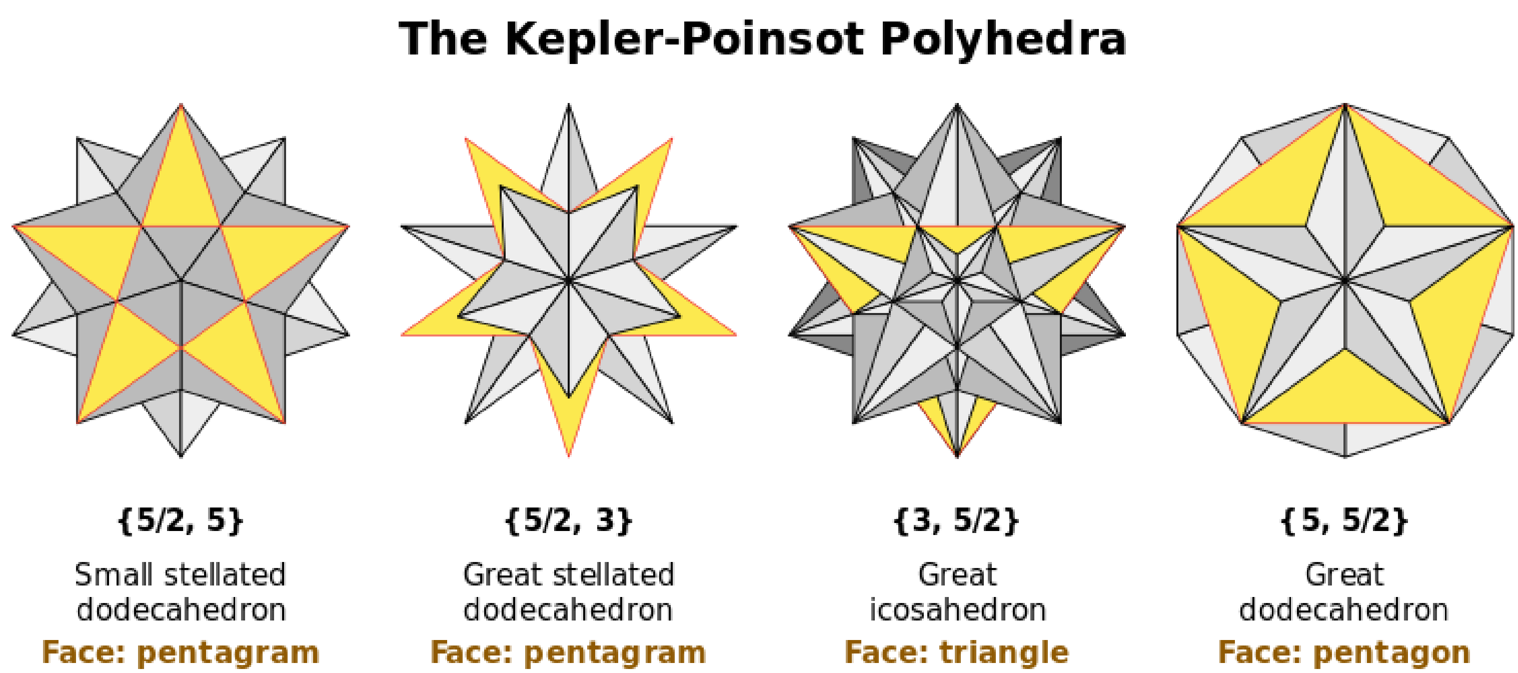
On the other hand, the sharp points of this shape form the corners of an invisible icosahedron! So, there are 20 sharp points.

Puzzle: how many edges does this shape have?

This shape can be seen in a floor mosaic in the Basilica of Saint Mark in Venice, built in 1430. It was rediscovered by Kepler in his work *Harmonice Mundi* in 1619. This book, about the "harmonies of the world", is an amazing mix of geometry, astronomy and music theory — a mystical warmup for his later breakthroughs on the orbits of the planets.

Much later, Escher made himself a wood model of the small stellated dodecahedron, which he drew in two woodcuts called Order and Chaos.

Besides the small stellated dodecahedron, there's another regular polyhedron with star faces: the great stellated dodecahedron. These two and their duals are called the Kepler–Poinsot polyhedra.



While the Kepler–Poinsot polyhedra are beautiful, I've avoided studying them because I don't see how they fit into the theory of Coxeter groups &mdash the study of discrete symmetries that connects Platonic solids, Archimedean solids and hyperbolic honeycombs to deeper strands of math like Lie theory, the study of continuous symmetries. I've been afraid these shapes are merely cute, not deep.

Maybe it's time to find out.

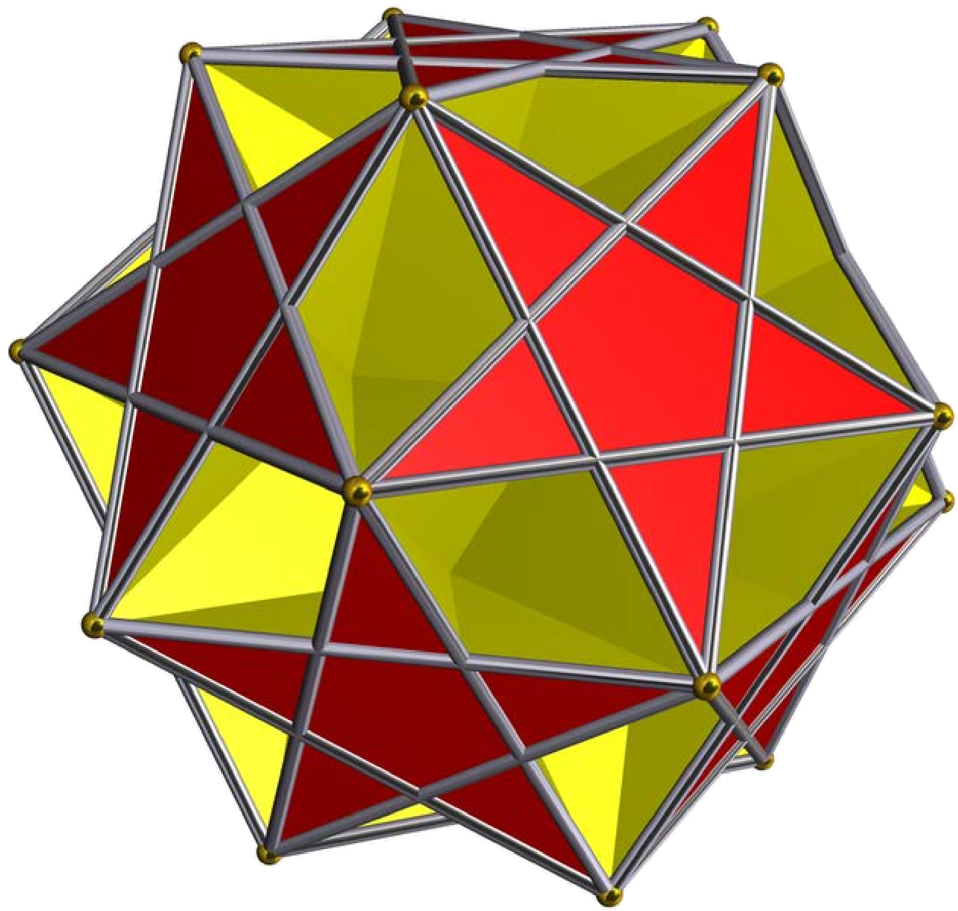
For more, see:

- Small stellated dodecahedron, Wolfram Mathworld.

- Small stellated dodecahedron, Wikipedia.

The Mathworld page has a much better picture of the mosaic in the Basilica of Saint Mark. The rotating image of the small stellated dodecahedron was made by Cyp and placed on Wikicommons with a Creative Commons Attribution-Share Alike 3.0 Unported license. The pictures of Kepler–Poinsot polyhedra were made by Tom Ruen and Júi;lio Reis and put on Wikicommons with the same sort of license.

January 15, 2015



This shape is called a ditrigonal dodecadodecahedron. The term 'ditrigonal' is a bit hard to explain. But it's called a 'dodecadodecahedron' because it has 12 pentagons and 12 pentagrams as faces.

It's easy to see the pentagrams - they're the red stars. But what about the 12 pentagons? That's the yellow stuff.

Do you see how to get this yellow stuff from 12 pentagons? At first I didn't see how. Now I do. The 20 vertices of a dodecahedron lie on 4 parallel planes. 10 form two pentagonal faces of the dodecahedron. The 10 remaining ones will lie on two more parallel planes, forming two larger pentagons. These give 2 of the 12 yellow pentagonal faces hiding in the picture above. We get the rest by slicing the dodecahedron with parallel places in other ways: there are 6 ways, for a total of $6 \times 2 = 12$ yellow pentagonal faces.

This shape is an example of a uniform star polyhedron. A 'uniform polyhedron' has regular polygons as faces, with enough symmetries that every vertex looks like every other. In a 'uniform star polyhedron' we also allow regular stars as faces.

I like how uniform star polyhedra look, but I've never been sure the math of them is deep enough to be worth studying. That may sound snobbish. But you see, a lot of uniform polyhedra come from Coxeter groups. These are discrete symmetry groups that are closely connected to lots of other great math - so these are very interesting. The uniform star polyhedra, on the other hand, don't seem connected to other math in such a strong way. Or maybe I just haven't learned how.

Still, they're pretty. There are 57 of them — not counting an infinite number of prisms and antiprisms, star prisms and star antiprisms. You can see them all here:

- [Uniform star polyhedron](), Wikipedia.

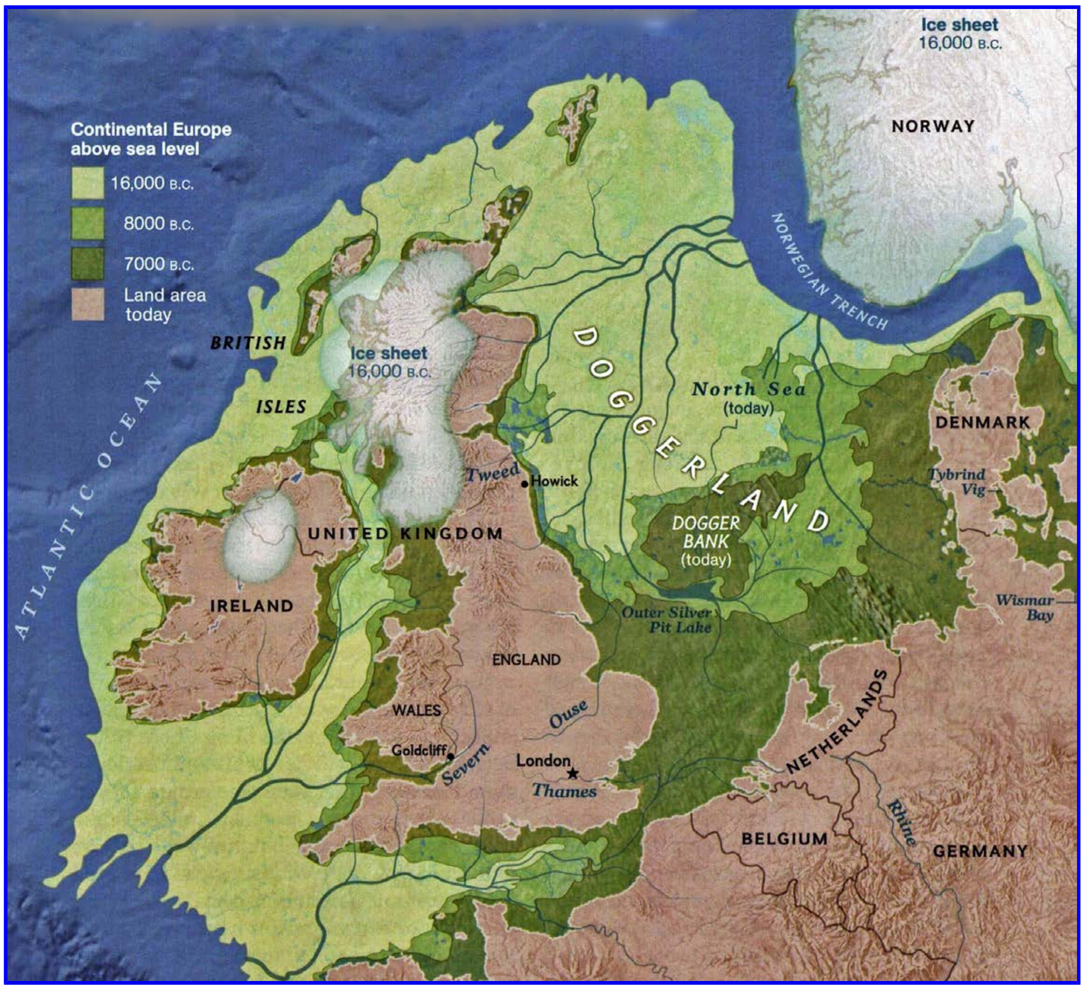
Puzzle 1. Why does it say "57 varieties" on a bottle of Heinz ketchup? Is it really because there are 57 uniform star polyhedra?

Puzzle 2. What's the most important appearance of the number 57 in group theory?

Puzzle 3. Why is this shape called ditrigonal'?

This picture was made by Tom Ruen using Robert Webb's Stella software and put on Wikimedia Commons. Webb demands a link to his website. For answers to the puzzles, read the comments on my G+ post.

January 16, 2015



As recently as 6500 BC, Great Britain was connected to Europe! And if you go back further in your time machine, you'll see a huge plain called Doggerland between Britain and Denmark.

Why? Because the sea level is lower during ice ages. More water is locked up in ice!

The last ice age, the Wisconsin glaciation, reached its peak a bit before 18000 BC. Back then, there were huge ice sheets going down to the Great Lakes and the mouth of the Rhine. The north of Britain was covered with ice, and the south was a polar desert!

The light green stuff in this map shows the land a bit later, in 16000 BC. Back then Doggerland was a wide undulating plain full of complicated meandering river systems.

As the ice age ended, the sea level rose rather quickly. Doggerland shrank to the medium green stuff in 8000 BC and the dark green stuff in 7000 BC. One of the last parts to survive was the Dogger Bank. You can see it on the map if you look close. It was an island until 5000 BC.

A new theory says that Doggerland was flooded by a huge tsunami around 6200 BC, thanks to a submarine landslide off the coast of Norway! It's called the Storegga Slide. There's geological evidence of sediments washed up onto land then. Maybe an earthquake triggered a catastrophic expansion of methane hydrates underwater.

This tsunami would have devastated a rich hunting and fishing ground populated by Mesolithic humans. People of some sort have lived on the British Isles, on and off, for much longer! There are flint tools dating back to 815,000 BC. These would not be made by *Homo sapiens*, since our species only came into existence around 250,000 BC.

But there were *Homo sapiens* in Britain by 40,000 BC, in the middle of the last ice age. And when that ice age ended and treeless tundra slowly turned into forests of birch trees, more of us moved in. Instead of eating reindeer and wild horses, the ancient Britons started eating pigs, elk, deer, wild boar and wild cattle — hunting them with ever more sophisticated stone tools. So by 6200 BC, when the tsunami crashed over Doggerland, there would have been lots of people living quite well.

Puzzle 1: Why is this area called 'Doggerland'?

Puzzle 2: What's a 'dogger'?

Puzzle 3: When did people start building things around the cite now called Stonehenge — how does that fit into the chronology here?

For answers to the puzzles, read the comments on my G+ post.

January 17, 2015

This is Alberto Behar in Greenland with the robotic boat he designed. How fast is Greenland melting due to global warming? Where does the water go? Some people sit around and argue. Others go and find out.

It was very warm in Greenland from July 11th to 13th, 2012. Scientists from NASA traveled by helicopter to study the melting ice. They mapped rivers and streams over 5400 square kilometers of Greenland. They found 523 separate drainage systems — small streams joining to form larger streams and rivers.

The water in every one of these flowed into a moulin! A moulin is a circular, vertical shaft. Water pours down the moulin and goes deep below the surface — sometimes forming a layer between ice and the underlying rock. This layer can help glaciers slide down toward the ocean. And this water reaches the ocean fast.

In the area they studied, a total of between 0.13 and 0.15 cubic kilometers of water were flowing into moulins each day. That's a lot! That would be enough to drain 2.5 centimeters of water off the surface each day.

To study the flow of water, Alberto Behar designed two kinds of remote-controlled boats. The boat shown here, was a drone that measured the depth of the water and how much light it reflected, allowing the researchers to calibrate the depth of the surface water from satellite images. They used this boat on lakes and slow-flowing rivers. But for dangerous, swift-flowing rivers, Behar developed disposable robotic drifters that measured the water's velocity, depth and temperature as they swept downstream.

Just a few days ago, Alberto Behar died in a plane crash. The plane he was flying crashed shortly after he took off from a small airport near NASA’s Jet Propulsion Laboratory in Pasadena, California.

So, his coauthors dedicated their paper on this research to him. Here is is:

- Laurence C. Smith *et al.*, Efficient meltwater drainage through supraglacial streams and rivers on the southwest Greenland ice sheet, *Proc. Nat. Acad. Sci.*.

Check out the cool images and maps. And watch this great movie:

January 18, 2015

This is not a tornado or hurricane! It's a supercell: a thunderstorm with a deep, persistently rotating updraft.

Supercells are one of the least common kinds of thunderstorm — but they can be the most severe! Supercells can happen anywhere — but especially in the Great Plains of America and the Tornado Corridor of Argentina, Uruguay and southern Brazil.

They start when the wind is moving faster at one height than another: this is called wind shear, and it can create a vortex. Thunderstorms often have a strong updraft, and this can tilt the vortex so it's vertical instead of horizontal! This creates a mesocyclone, which you see here. And sometimes the mesocyclone creates tornadoes.

Things always get more complicated and interesting when you study them in detail. I find weather to be a very tricky subject. I've just skimmed the surface; you can learn more at the links.

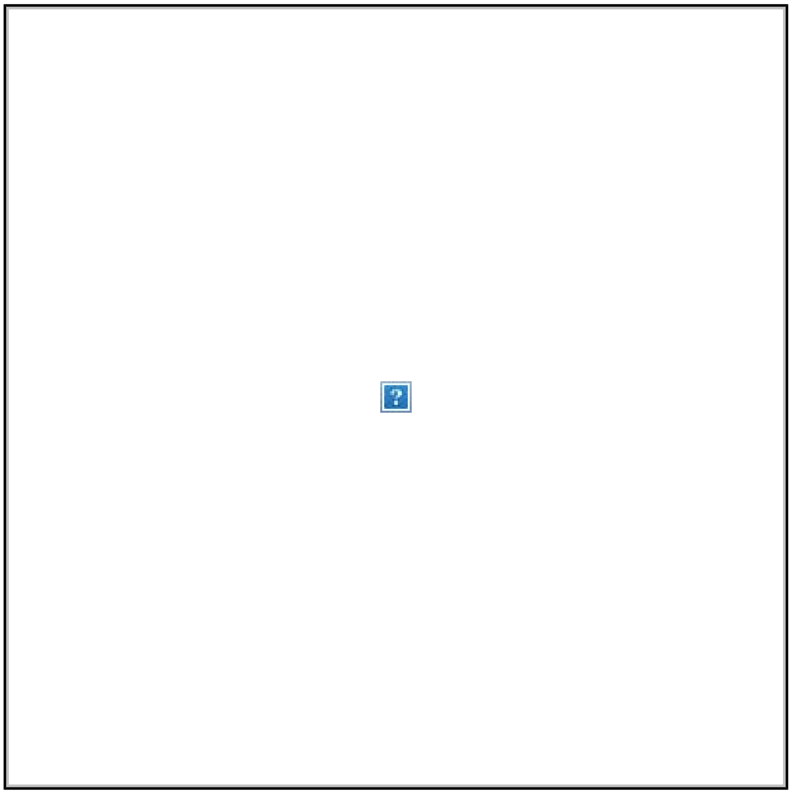
This animated gif seems to be created from photos taken in Nebraska by the storm chaser Mike Hollingshead. Google Image Search shows copis of this all over the place, with many people wrongly saying it's a hurricane.

I went through a supercell in an airplane once, shortly after taking off from Denver. We got caught in a downdraft and the pilot had to use full power to get us up and out. Then we turned around and landed back in Denver. It was pretty scary. It made me queasy about flying for a while: not in a rational way, just adrenaline rushes whenever the plane did something funny.

The feeling when you're in a plane dropping through the air reminds me a bit of being in an earthquake as it gets stronger and stronger — you're frozen, wondering: what next? I remember waking up and looking out the window at a street sign wiggling back and forth when the Northridge quake hit in 1994. It wasn't very strong in Riverside, but strong enough that every aftershock made me tense.

January 19, 2015

The arc of the moral universe is long, but it bends toward justice.



On March 7, 1965, protesters seeking the right to vote tried to march from Selma to Montgomery Alabama. State troopers and a violent posse attacked the unarmed marchers with billy clubs and tear gas.

After another try, the march finally succeeded three weeks later. After walking 54 miles, Martin Luther King gave a speech on the steps of the State Capitol of Alabama. It began like this:

> Last Sunday, more than eight thousand of us started on a mighty walk from Selma, Alabama. We have walked through desolate valleys and across the trying hills. We have walked on meandering highways and rested our bodies on rocky byways. Some of our faces are burned from the outpourings of the sweltering sun. Some have literally slept in the mud. We have been drenched by the rains. Our bodies are tired and our feet are somewhat sore.
>
> But today as I stand before you and think back over that great march, I can say, as Sister Pollard said—a seventy-year-old Negro woman who lived in this community during the bus boycott—and one day, she was asked while walking if she didn't want to ride. And when she answered, "No," the person said, "Well, aren't you tired?" And with her ungrammatical profundity, she said, "My feets is tired, but my soul is rested." And in a real sense this afternoon, we can say that our feet are tired, but our souls are rested.
>
> They told us we wouldn't get here. And there were those who said that we would get here only over their dead bodies, but all the world today knows that we are here and we are standing before the forces of power in the state of Alabama saying, "We ain't goin' let nobody turn us around."
>
> Now it is not an accident that one of the great marches of American history should terminate in Montgomery, Alabama. Just ten years ago, in this very city, a new philosophy was born of the Negro struggle. Montgomery was the first city in the South in which the entire Negro community united and squarely faced its age-old oppressors. Out of this struggle, more than bus desegregation was won; a new idea, more powerful than guns or clubs was born. Negroes took it and carried it across the South in epic battles that electrified the nation and the world.
>
> Yet, strangely, the climactic conflicts always were fought and won on Alabama soil. After Montgomery.s, heroic confrontations loomed up in Mississippi, Arkansas, Georgia, and elsewhere. But not until the colossus of segregation was challenged in Birmingham did the conscience of America begin to bleed. White America was

> profoundly aroused by Birmingham because it witnessed the whole community of Negroes facing terror and brutality with majestic scorn and heroic courage. And from the wells of this democratic spirit, the nation finally forced Congress to write legislation in the hope that it would eradicate the stain of Birmingham. The Civil Rights Act of 1964 gave Negroes some part of their rightful dignity, but without the vote it was dignity without strength.
>
> Once more the method of nonviolent resistance was unsheathed from its scabbard, and once again an entire community was mobilized to confront the adversary. And again the brutality of a dying order shrieks across the land. Yet, Selma, Alabama, became a shining moment in the conscience of man. If the worst in American life lurked in its dark streets, the best of American instincts arose passionately from across the nation to overcome it. There never was a moment in American history more honorable and more inspiring than the pilgrimage of clergymen and laymen of every race and faith pouring into Selma to face danger at the side of its embattled Negroes.
>
> The confrontation of good and evil compressed in the tiny community of Selma generated the massive power to turn the whole nation to a new course. A president born in the South had the sensitivity to feel the will of the country, and in an address that will live in history as one of the most passionate pleas for human rights ever made by a president of our nation, he pledged the might of the federal government to cast off the centuries-old blight. President Johnson rightly praised the courage of the Negro for awakening the conscience of the nation.
>
> On our part we must pay our profound respects to the white Americans who cherish their democratic traditions over the ugly customs and privileges of generations and come forth boldly to join hands with us. From Montgomery to Birmingham, from Birmingham to Selma, from Selma back to Montgomery, a trail wound in a circle long and often bloody, yet it has become a highway up from darkness. Alabama has tried to nurture and defend evil, but evil is choking to death in the dusty roads and streets of this state. So I stand before you this afternoon with the conviction that segregation is on its deathbed in Alabama, and the only thing uncertain about it is how costly the segregationists and Wallace will make the funeral.

The whole speech is here:

- Martin Luther King, Jr., Our god is marching on!, March 25, 1965.

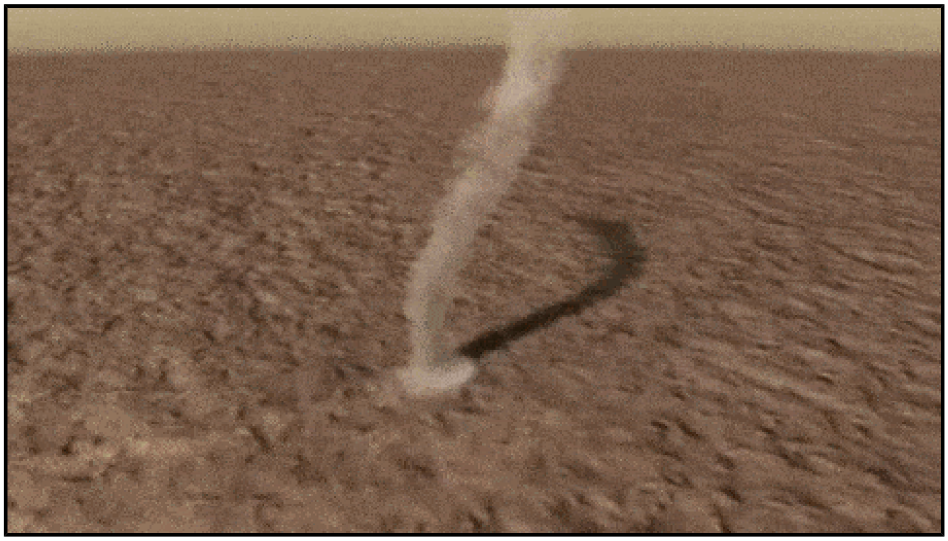
Near the end he said:

> The arc of the moral universe is long, but it bends toward justice.

I used to wonder if this is true. I now think it's one of those things that only becomes true if enough of us work to make it so. A master orator, Martin Luther King was not trying to describe the world: he was trying to change it.

I saw the movie *Selma*, and I recommend it—a good reminder of this recent era of American history... and how powerful determination accomplished real changes. We could use some of that spirit now.

January 20, 2015



Mars is a windy place! This dust devil, roughly 20 kilometers high but just 70 meters wide, was seen whirling through northern Mars on March 14, 2007. It was imaged by a high resolution camera on the Mars Reconnaissance Orbiter... and NASA made this animation based on what they saw.

Dust devils happen on Earth too — I often see them in the deserts around here! They're spinning columns of air, made visible by the dust they pull off the ground. Unlike tornadoes, they usually form on clear days when the ground is heated by the sun, warming the air just above the ground.

As hot air rises, it can start to rotate, by chance... and as more hot air rushes in to replace the air that is rising, the rotation becomes stronger. So the dust devil grows and sustains itself, becoming a quick way for hot air to rise... until it dies.

Puzzle: why does it die?

In short, a dust devil is a great example of how efficient increase in entropy can actually create ordered structures, which however have a finite lifetime. You are an example of this.

Maybe this is why many cultures have considered dust devils to be sentient beings. In Arabia they're *djinn*, or genies. Among the Navajo they're *chindii*, or spirits of the dead.

This dust devil happened in Amazonis Planitia during the late spring, two weeks short of the northern summer solstice, when the ground in the northern mid-latitudes is heated most strongly by the sun.

The Mars Reconnaissance Orbiter has been examining the Red Planet with six science instruments since 2006. You can see thousands of images taken by HiRISE — the High Resolution Imaging Science Experiment — at this website:

- HiRISE

They're awesome!

Mars has a very thin atmosphere, with approach of only 600 pascals, tiny compared to Earth's 101,000 pascals — but there are high winds thanks to the enormous daily temperature variations: the temperature typically swings by 100 Celsius each day! This creates winds that easily gust above 90 kilometers/hour, and huge dust storms that envelope the whole planet. The dramatic heating of air at the ground is the perfect recipe for a huge dust devil.

The physics of wind-blown sand and dust is fascinating and complex:

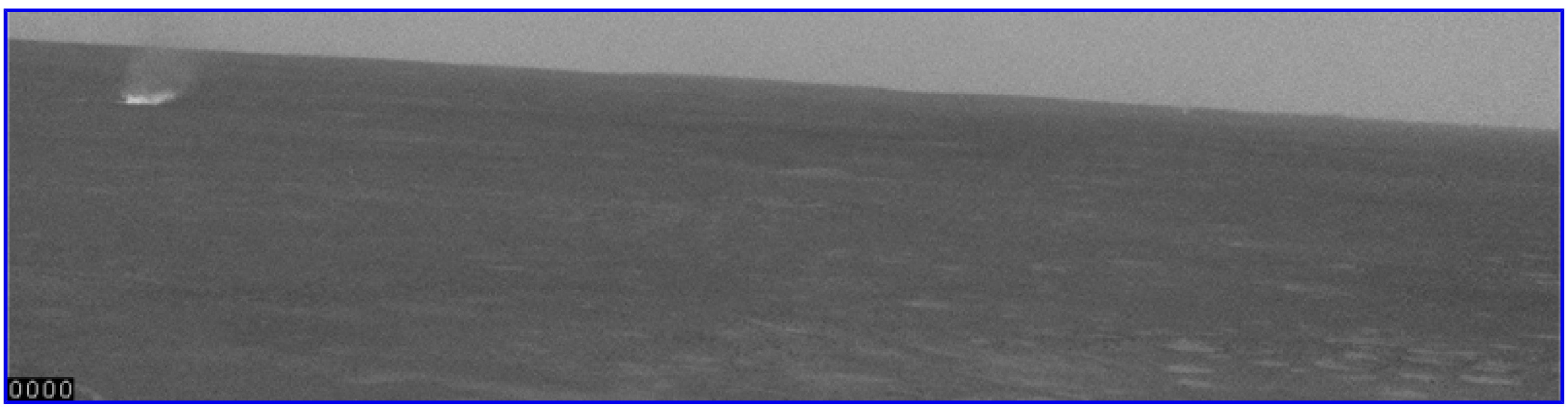
- Jasper F. Kok, Eric J. R. Parteli, Timothy I. Michaels and Diana Bou Karam, The physics of wind-blown sand and dust.

This paper discusses processes on both Earth and Mars. It also seems that static electricity plays an important role in 'saltation', the process whereby wind-blown sand grains hop along the ground:

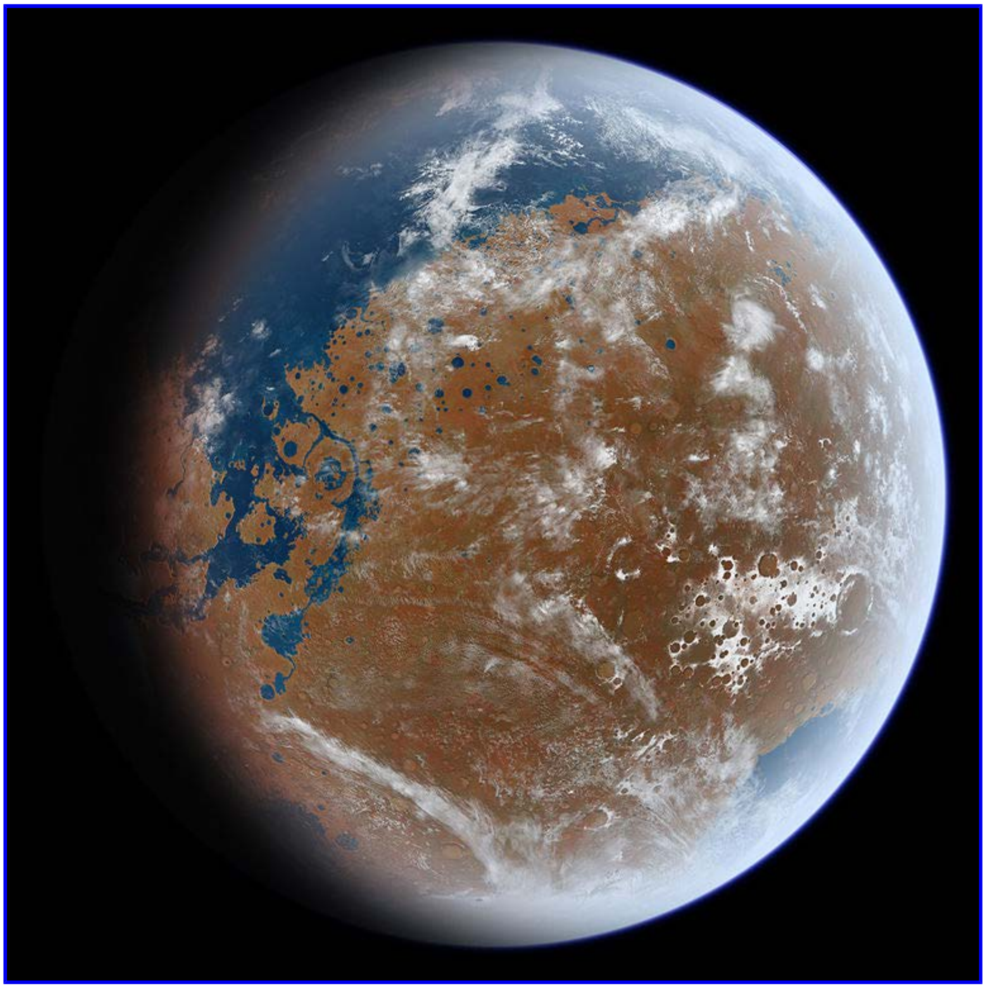
- X.J. Zheng, N. Huang and Y. Zhou, The effect of electrostatic force on the evolution of sand saltation cloud, *Eur. Phys. Jour. E: Soft Matter* **2** (2006), 129–138.

  Abstract: In a wind-blown sand layer, it has been found that wind transport of particles is always associated with separation of electric charge. This electrification in turn produces some electrostatic forces in addition to the gravitational and fluid friction forces that affect the movement of saltating sand particles, further, the wind-blown sand saltation. To evaluate this effect quantitatively, this paper presents a simulation of evolution of wind-blown sand grains after the electrostatic forces exerted on the grains are taken into account in the wind feedback mechanism of wind-blown saltation. That is, the coupling interaction between the wind flow and the saltating sand particles is employed in the simulation to the non-stationary wind and sand flows when considering fluid drag, gravitation, and a kind of electrostatic force generated from a distribution of electric field changing with time in the evolution process of the sand saltation. On the basis of the proposed simulation model, a numerical program is given to perform the simulation of this dynamic process and some characteristic quantities, e.g., duration of the system to reach the steady state, and curves of the saltating grain number, grain transport rate, mass-flux profile, and wind profile varying with time during the non-stationary evolution are displayed. The obtained numerical results exhibit that the electrostatic force is closely related to the average charge-to-mass ratio of sand particles and has obvious influence on these characteristic quantities. The obtained results also show that the duration of the system to reach the steady state, the sand transport rate and the mass flux profile coincide well with experimental results by Shao and Raupach (1992) when the average charge-to-mass ratio of sand particles is 60 microC/kg for the sand particles with average diameter of 0.25 mm. When the average charge-to-mass ratios of sand particles are taken as some other certain values, the calculation results still show that the mass flux profiles are well in agreement with the experimental data by Rasmussen and Mikkelsen (1998) for another category of sand particles, which tell us that the electrostatic force is one of main factors that have to be considered in the research of mechanism of wind-blown sand saltation..

Here's another dust devil on Mars, filmed by the rover Spirit in Gusev Crater:



January 21, 2015



According to the Mars ocean hypothesis, this is what Mars might have looked like 3.8 billion years ago.

The northern polar plains of Mars, called Vastitas Borealis, are smooth, free of craters, and 4.5 kilometers below the average elevation of the planet. Along the edges there are riverbeds and deltas!

But if there was an ocean, where did the water go?

The gravity on Mars is low, so some could have escaped into space. There's 1.6 cubic kilometers of water ice in the north polar cap, and about the same amount in the south polar cap and its surrounding ice fields. (The south is very different than the north.)

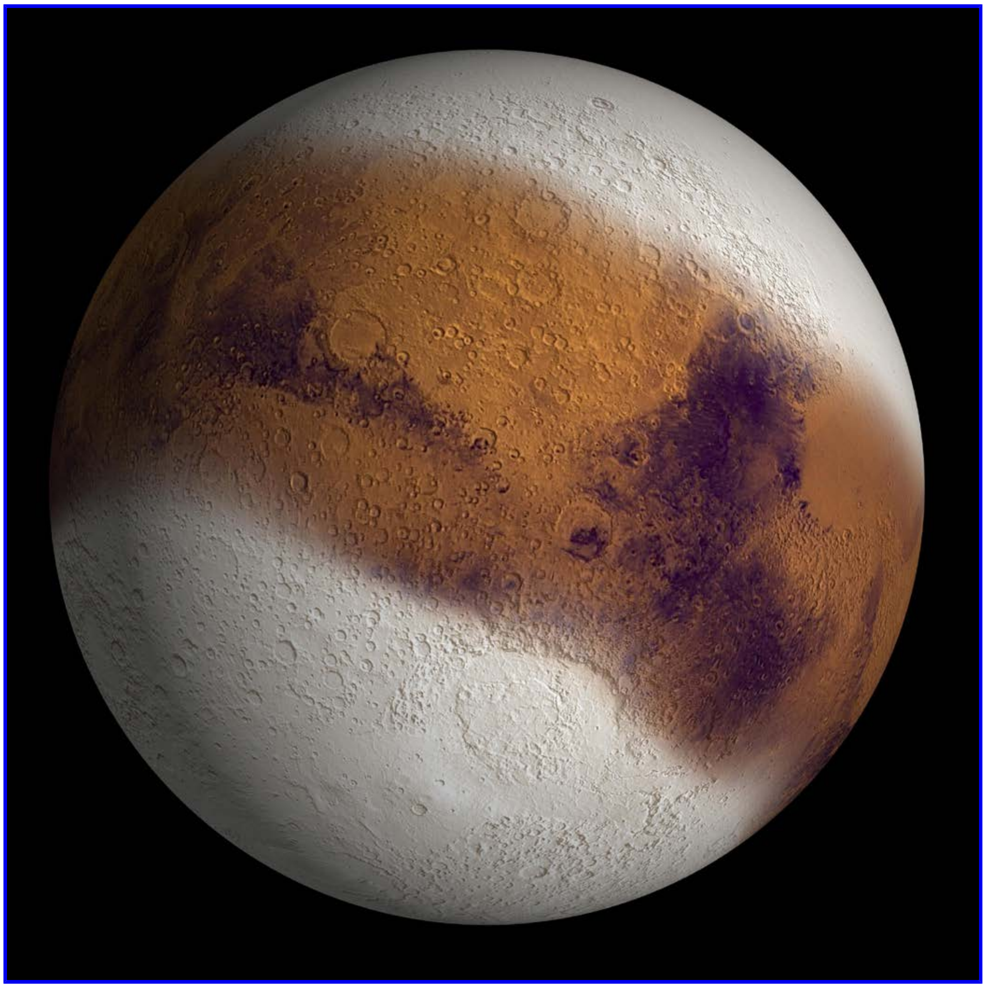
That's not really much water for a whole planet: taken together, just a bit more than the ice on Greenland. But there could be more water ice underground in Vastitas Borealis. Research with MARSIS, a radar instrument on board the Mars Express satellite in orbit around Mars, supports the idea of a large, northern ocean. The instrument revealed that this area has a dielectric constant similar to that of low-density sedimentary deposits, massive deposits of ground ice, or a combination of the two.

But how could Mars have been warm enough for an ocean? Its atmosphere may have been as dense as our Earth's atmosphere is now - and mostly made of carbon dioxide. (Right now it has a carbon dioxide atmosphere just 0.6% as dense as Earth's atmosphere!) That would give a massive greenhouse effect, enough to keep water liquid. In any event, we know there was liquid water, thanks to those riverbeds and deltas.

But the Mars ocean hypothesis is still controversial. For example, Vastitas Borealis has a bunch of boulders on it which are hard to explain if it was an ocean bed. Maybe they were dropped there by icebergs? That happens here on Earth!

The above picture, from Wikicommons, was made by a user named 'lttiz' using data from the Mars Orbiter Laser Altimeter. The elevations were updated so the shore lines will closely approximate their ancient locations, and any mountains less than two billion years old were removed. lttiz also has some other pretty pictures there.

If Mars had an ocean, it would have existed during the Noachian period This lasted on Mars lasted from 4.1 to 3.7 billion years ago. During this time, the rate of meteor impacts on Mars was 500 times higher than now: there was about one new 100-kilometer diameter crater every million years, and many more small ones. More excitingly, water drained through the valleys, forming ponds in craters and large lakes elsewhere. Over 200 Noachian lake beds have been found, some big as Lake Baikal or the Caspian Sea! Many Noachian craters show channels entering on one side and exiting on the other.



This is Mars as it might have appeared much more recently, during the height of its last ice age!

Of all planets in the Solar System, Mars has the climate most like ours. Both Mars and Earth are sensitive to small changes in the shape of its orbit and the tilt of its axis.

On Earth, when the axis tilts more, the poles warm up and ice ages end. But a 2003 paper in Nature did simulations and found that Mars works the other way! When its axis tilts more, the poles warm up, and the polar ice caps start to evaporate... bringing more water to the whole planet. But since Mars is below freezing most of the time, that brings ice all the way down to 300 latitude in both hemispheres, as shown here.

From about 2.1 million to 400,000 years ago, the increased tilt of Mars' axis made its poles warmer. But the tilt has been less since then. This is making the poles colder, so water vapor has been leaving the zone between 300 and 600 latitude, and collecting in the ice caps.

If you haven't read Kim Stanley Robinson's trilogy *Red Mars*, *Green Mars*, *Blue Mars*, you really should. It's a sprawling tale of the terraforming of Mars, packed with fascinating digressions. Now I want *White Mars*. The photo is from here:

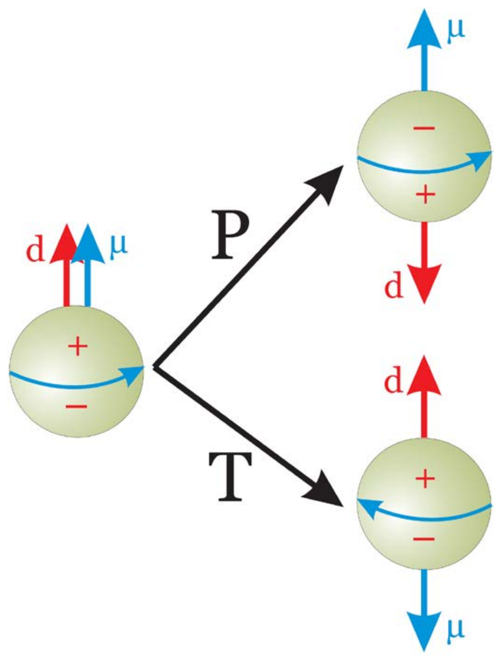
- Jet Propulsion Laboratory, NASA, PIA04933: Mars ice age, simulated.

It was prepared for the December 18, 2003, cover of the journal *Nature*. The simulated surface deposit is superposed on a map based on altitude measurements by Global Surveyor and images from NASA's Viking orbiters of the 1970s.

The paper is here:

- James W. Head, John F. Mustard, Mikhail A. Kreslavsky, Ralph E. Milliken and David R. Marchant, Recent ice ages on Mars, *Nature* **426** (18 December 2003), 797–802.

January 22, 2015



A neutron is a spinning bag of charged particles, so we shouldn't be surprised that it acts like a little magnet. We say it has a **magnetic dipole moment**. This means that like the Earth, it has a north magnetic pole and a south pole. The blue arrow called μ here points to the north pole.

A neutron might also have an electric dipole moment. That would happen if there were more positive charge near one pole, and more negative charge near the other pole. Then we could draw a red arrow called *d* pointing toward the positive charges.

In the picture at left, the red arrow points the same way as the blue arrow. But nobody knows if there is a red arrow! So far nobody has seen an electric dipole moment for a neutron. It's either zero, or very small.

A water molecule has an electric dipole moment: it's shaped like a head with two big ears, and there's more positive charge near the ears. You might argue that the electric dipole moment of the neutron should be zero because — unlike the water molecule — the neutron is round. There's a kernel of truth to that.

Indeed, if the electric dipole moment wasn't zero, it would violate some symmetries that the neutron seems to have!

P symmetry, or parity, is the symmetry where you reverse all 3 spatial directions: send each point (x, y, z) to the opposite point $(-x, -y, -z)$. If you do this to a spinning sphere, it still spins the same way, so the arrow μ is unchanged. However, if there had been more positive charges near one pole, now there will be more positive charges near the other pole. So the arrow *d* now points the other way.

T symmetry, or time reversal, is the symmetry where you reverse the direction of time: send each time t to $-t$. We can't actually turn time around, but we can try to set up a neutron that's a time-reversed version of some other neutron. It would spin the opposite way, so the arrow . would point the other way. But the positive charges would still be on the same side. So *d* points the same way.

The picture shows that if a neutron has the μ and *d* arrows pointing the same way, and we apply parity or time reversal, we get another kind of neutron where the μ and *d* arrows point opposite ways. There can't be two kinds of neutrons: we'd have noticed that by now. So, if neutrons have an electric dipole moment, they can't be symmetric under parity and time reversal.

In fact neutrons probably *aren't* symmetric under parity and time reversal, because a force called the weak force doesn't have these symmetries, and it affects neutrons. But as the name indicates, this force is very weak. We can calculate the electric dipole moment this force creates in the neutron, and it's tiny — about 10 million times smaller than our current ability to measure it.

What's interesting is that as far as we know, the strong force could also fail to have parity and time reversal symmetry. This is the force that holds the neutron together. If it broke these symmetries, it could create a larger electric dipole moment than the weak force does.

We haven't seen any sign that this happens. People are looking because this would be one of the best ways to see if the strong force violates parity and time reversal symmetry. If it doesn't, one of the fundamental constants of nature must be zero... and nobody knows why, though there are some fascinating theories. This is called the 'strong CP problem':

- Strong CP problem, Wikipedia.

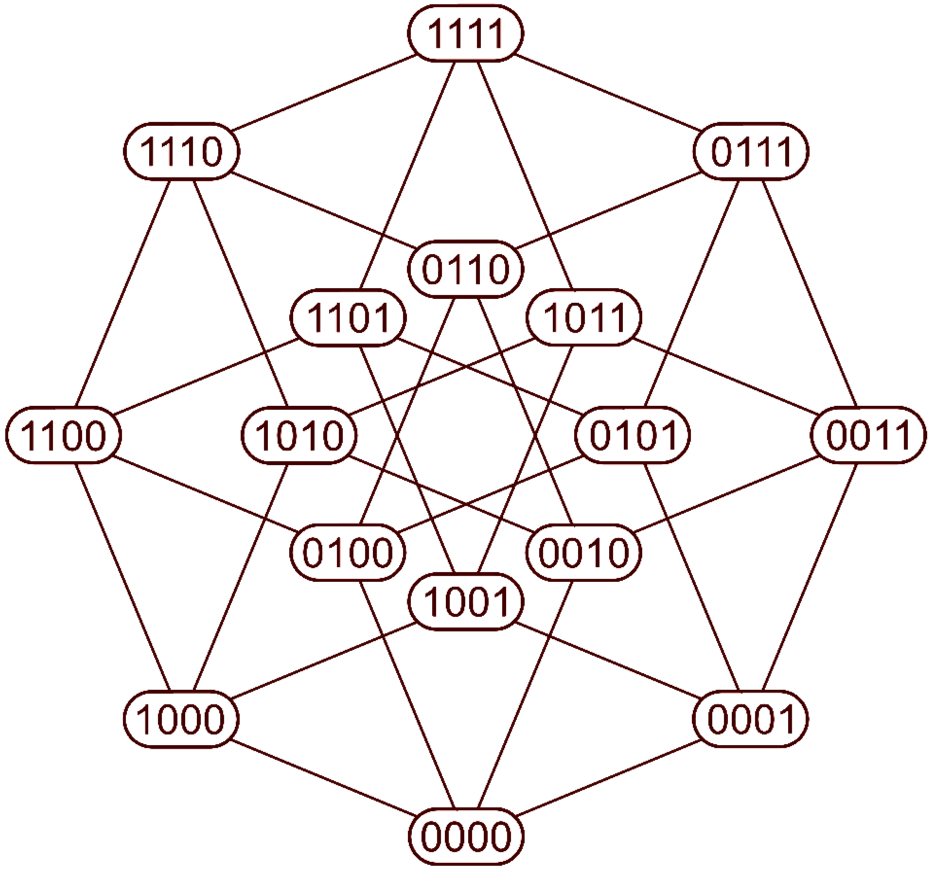
There's a lot more to say about this, but not today!

Puzzle: I said we would have noticed by now if there are two kinds of neutrons, one where μ and *d* point the same way and one where they point in opposite directions. How could we have noticed this, given that we can't yet measure the *d* arrow?

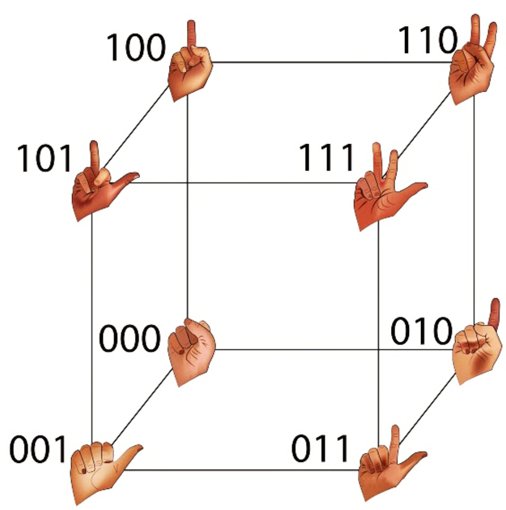
Right now the best upper bound on the neutron's electric dipole moment is $2.9 \cdot 10^{-26}$ e cm. (Electric dipole moment is often measured in units of the electron's charge times a centimeter.) There are at least five experiments in progress that aim at improving this limit to 10^{-28} e cm. These should be able to rule out various theories of how supersymmetry could create an electric dipole moment in the neutron.

The weak force should create a dipole moment of about 10^{-33} e cm, so detecting that is still far away. This amount of asymmetry is so small that it's like the Earth being perfectly round except for mountains that are micron tall!

January 24, 2015



I posted some puzzles about this hypercube of bits, and Scott Carter responded with a profane version in 3 dimensions:



This gives 'binary digits' a whole new meaning!

January 25, 2015

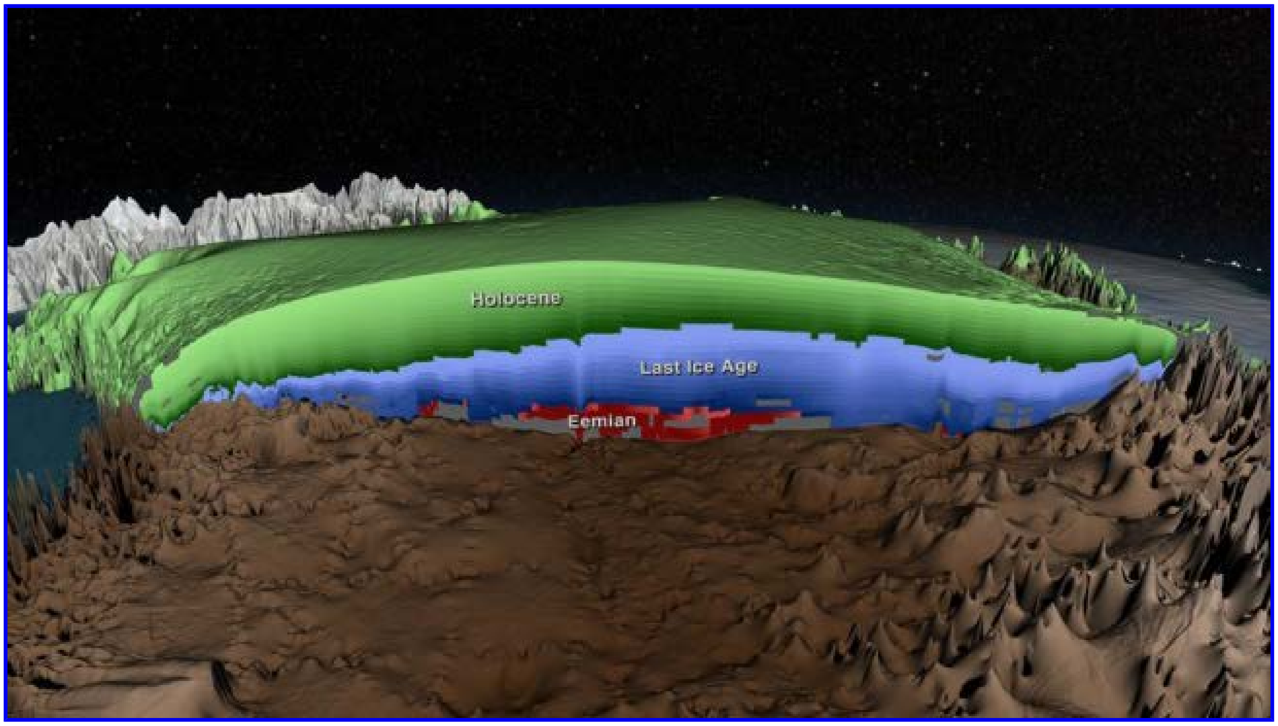
We can thank photographer Andrey Grachev for this view! He walked across Lake Baikal, a huge lake in Siberia that freezes over in winter... and found this ice cave on Olkhon Island. You can see more of his photos here:

- Chris Kitching, Photographer braves unstable frozen lake to capture breathtaking images of magical ice cavern at sunrise in Siberia, *Daily Mail*, January 20, 2015.

For almost five months a year, Lake Baikal is covered with ice. Perhaps because it's so deep, it starts freezing only in January, long after the Siberian frosts become intense. It usually thaws in May. At its peak, the ice is between 1 and 2 meters thick. Big cracks can be 10 to 30 kilometers long!

For more on Lake Baikal, including some amazing pictures, see my October 1st, 2013 diary entry.

January 26, 2015



If the Greenland ice sheet completely melts, the sea will rise 7.2 meters. This will drown most of the world's coastal cities — unless we move them or build dikes. So ice on Greenland is important.

It's also a fascinating record of the past! Scientists just made this wonderful cross-section of Greenland, showing 4 kinds of ice:

- Green is ice from snow that fell on Greenland after the last ice age. That's after 12,000 years ago.
- Blue is ice from the last ice age. That's between 12,000 and 115,000 years ago.
- Red is ice from the warm period before the last ice age: the Eemian interglacial. That's between 115,000 and 130,000 years ago.
- Gray is ice we don't understand yet.

This cross-section is just part of a detailed 3d map of Greenland, built using ice core samples and radar from planes. Here's a great video that shows the whole 3d map and how it was made:

The Greenland ice sheet is melting at a rate of about 200 cubic kilometers per year. The rate is increasing at about 1718 cubic kilometers per year each year. This sounds bad. Indeed, Greenland is contributing about as much to sea level rise as Antarctica. But the Greenland ice sheet won't go away soon. It has about 2,850,000 cubic kilometers of ice!

Ice from the last interglacial — the Eemian — was only recently found in Greenland. For more, read this story by Eric Steig:

- Eric Steig, The Greenland melt, *RealClimate*, 23 January 2013.

Puzzle 1: if you extrapolate the constantly accelerating rate of melting that I described, when would the Greenland ice sheet be completely melted? Of course this is naive, but the calculation is easy and fun.

Puzzle 2: about how many gigatonnes of water are in a cubic kilometer?

Puzzle 3: if it were spread equally over the whole ocean, how much would a cubic kilometer of water raise the sea level?

You can see some answers on my G+ post.

For my February 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

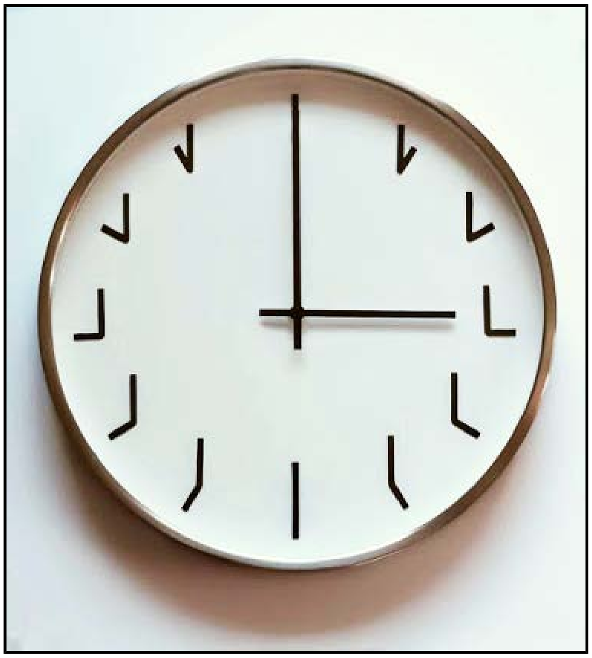
home

[For my January 2015 diary, go here.](#)

Diary — February 2015

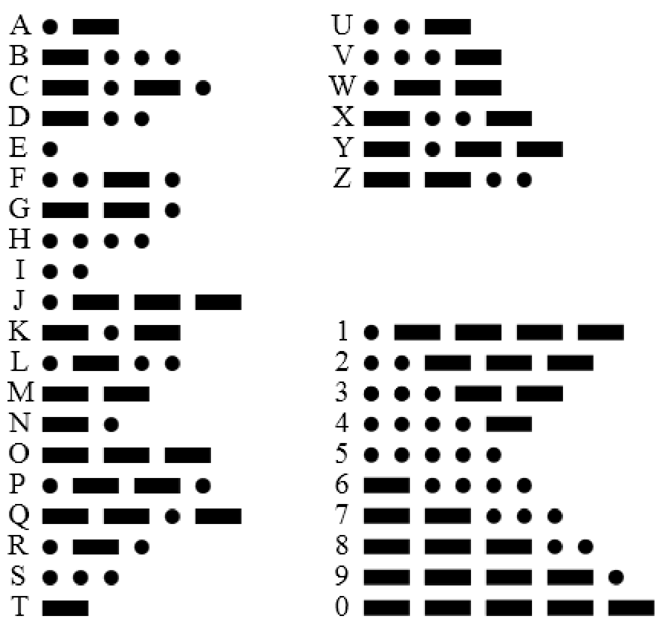
John Baez

February 1, 2015



I saw this in a post by the mathematician [Richard Green](#), who explains the underlying math quit nicely. But it's been making the rounds for a while.

February 2, 2015

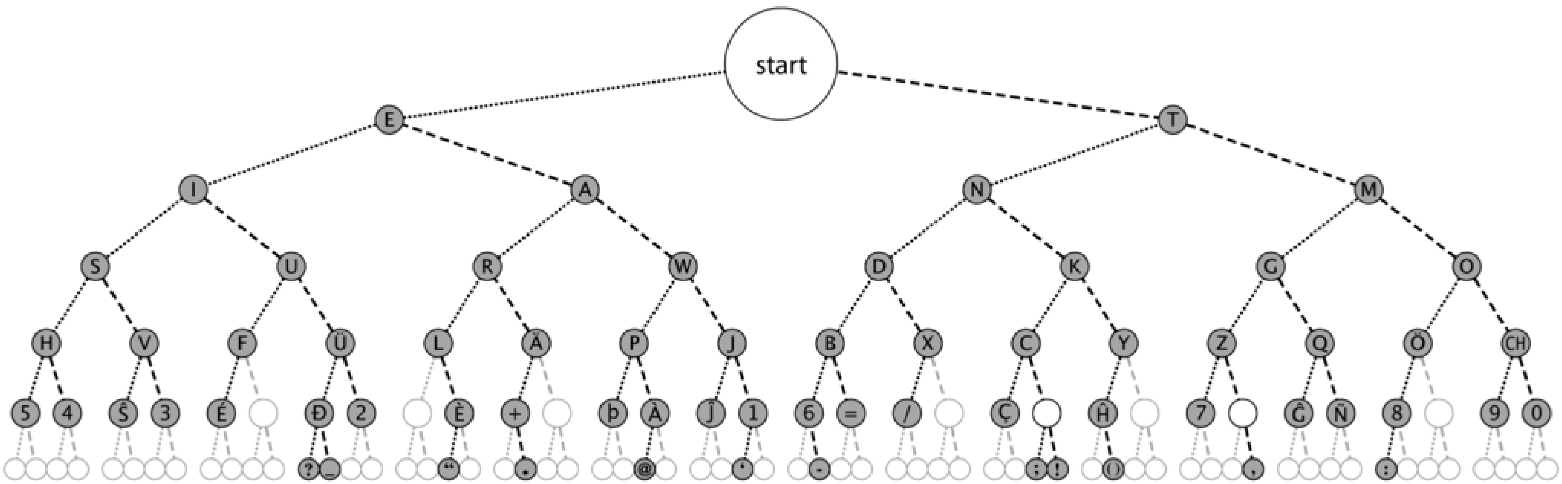


There are different kinds of [Morse code](#): this is International Morse Code.

Each letter or number is represented by a sequence of dots and dashes. When you type these out on a telegraph, a dash should be 3 times as long as a dot. Each dot or dash is followed by a short silence, as long as a dot. The letters of a word should be separated by a silence that's 3 dots long, and words should be separated by a silence that's 7 dots long.

How long is a dot? That depends on your skills!

The codes for numbers make a pattern. The codes for letters look chaotic. But they're not: they're chosen so that commonly used letters have short codes! The system is nicely explained using a tree:



E and T are on top: they have the shortest codes, because they are very commonly used letters. The E is a single dot and the T is a single dash. Then come I, A, M and N. And so on.

How good is International Morse Code? For that you should compare the tree it uses to the tree it would use if it were as good as possible. The best possible way is called a Huffman coding. You can see it on page 16 here:

- Ingrid Daubechies, The mathematics of communication.

February 3, 2015

Ancient Alien Mathematics

"So you traveled the whole twenty light years?"

"More than that," Joan said truthfully, "from my original home. I've spent half my life traveling".

"Faster than light?" Pirit suggested hopefully.

"No. That's impossible".

They circled around the question a dozen more times, before Pirit finally changed her tune from how to why?

"I'm a xenomathematician," Joan said. "I've come here in the hope of collaborating with your archaeologists in their study of Niah artifacts."

Pirit was stunned. "What do you know about the Niah?"

"Not as much as I'd like to." Joan gestured at her Noudah body. "As I'm sure you've already surmised, we've listened to your broadcasts for some time, so we know pretty much what an ordinary Noudah knows. That includes the basic facts about the Niah. Historically they've been referred to as your ancestors, though the latest studies suggest that you and they really just have an earlier common ancestor. They died out about a million years ago, but there's evidence that they might have had a sophisticated culture for as long as three million years. There's no indication that they ever developed space flight. Basically, once they achieved material comfort, they seem to have devoted themselves to various artforms, including mathematics."

"So you've traveled twenty light years just to look at Niah tablets?" Pirit was incredulous.

"Any culture that spent three million years doing mathematics must have something to teach us."

"Really?" Pirit's face became blue with disgust. "In the ten thousand years since we discovered the wheel, we've already reached halfway to the Cataract. They wasted their time on useless abstractions."

Joan said, "I come from a culture of spacefarers myself, so I respect your achievements. But I don't think anyone really knows what the Niah achieved. I'd like to find out, with the help of your people."

[...]

"Jown! Jown! Come and look at this!" Surat called to her. Joan switched off the tomography unit and jogged toward the archaeologists, suddenly conscious of her body's strangeness. Her legs were stumpy but strong, and her balance as she ran came not from arms and shoulders but from the swish of her muscular tail.

"It's a significant mathematical result," Rali informed her proudly when she reached them. He'd pressure-washed the sandstone away from the near-indestructible ceramic of the tablet, and it was only a matter of holding the surface at the right angle to the light to see the etched writing stand out as crisply and starkly as it would have a million years before.

Rali was not a mathematician, and he was not offering his own opinion on the theorem the tablet stated; the Niah themselves had a clear set of typographical conventions which they used to distinguish between everything from minor lemmas to the most celebrated theorems. The size and decorations of the symbols labelling the theorem attested to its value in the Niah's eyes.

Joan read the theorem carefully. The proof was not included on the same tablet, but the Niah had a way of expressing their results that made you believe them as soon as you read them; in this case the definitions of the terms needed to state the theorem were so beautifully chosen that the result seemed almost inevitable.

The theorem itself was expressed as a commuting hypercube, one of the Niah's favorite forms. You could think of a square with four different sets of mathematical objects associated with each of its corners, and a way of mapping one set into another associated with each edge of the square. If the maps commuted, then going across the top of the square, then down, had exactly the same effect as going down the left edge of the square, then across: either way, you mapped each element from the top-left set into the same element of the bottom-right set. A similar kind of result might hold for sets and maps that could naturally be placed at the corners and edges of a cube, or a hypercube of any dimension. It was also possible for the square faces in these structures to stand for relationships that held between the maps between sets, and for cubes to describe relationships between those relationships, and so on.

That a theorem took this form didn't guarantee its importance; it was easy to cook up trivial examples of sets and maps that commuted. The Niah didn't carve trivia into their timeless ceramic, though, and this theorem was no exception. The seven dimensional commuting hypercube established a dazzlingly elegant correspondence between seven distinct, major branches of Niah mathematics, intertwining their most important concepts into a unified whole. It was a result Joan had never seen before: no mathematician anywhere in the Amalgam, or in any ancestral culture she had studied, had reached the same insight.

She explained as much of this as she could to the three archaeologists; they couldn't take in all the details, but their faces became orange with fascination when she sketched what she thought the result would have meant to the Niah themselves.

"This isn't quite the Big Crunch," she joked, "but it must have made them think they were getting closer". The Big Crunch was her nickname for the mythical result that the Niah had aspired to reach: a unification of every field of mathematics that they considered significant. To find such a thing would not have meant the end of mathematics — it would not have subsumed every last conceivable, interesting mathematical truth — but it would certainly have marked a point of closure for the Niah's own style of investigation.

These are two quotes from this story:

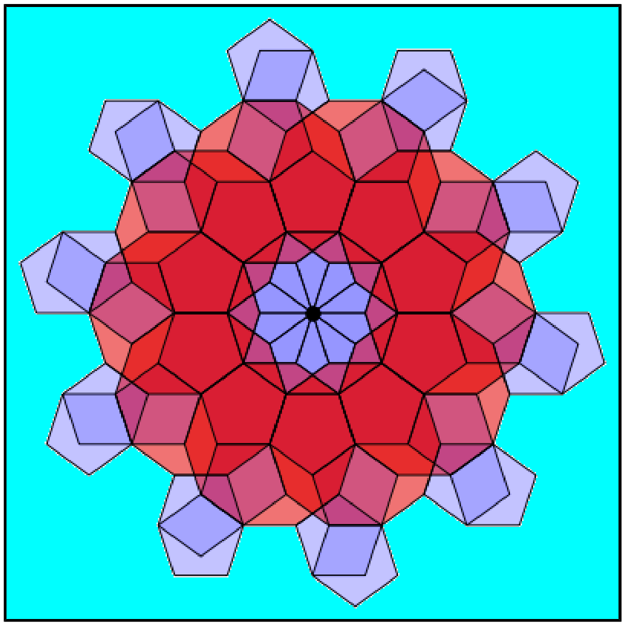
- Greg Egan, Glory.

Read the whole thing! The start is quite dramatic!

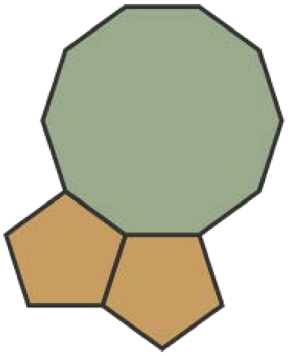
The image above is not alien mathematics; it's from an article about a codes for communicating with extraterrestrial civilizations:

- Brandon Keim, Building a better alien-calling code, *Wired*, 23 November 2009.

February 15, 2015



[Greg Egan](#) and I have been exploring the 'forbidden tilings'. These are ways to stick together shapes that *look like* they should tile the plane, but don't.



Two regular pentagons and a regular decagon fit snugly at a point: their interior angles sum to 360°. Despite this, you cannot tile the plane with regular pentagons and decagons!

But you can do some other things. The above picture by Egan shows one of them.

The idea here is to start drawing regular pentagons and decagons on the plane, and make sure that:

- each decagon touches 10 pentagons along its edges;
- each pentagon touches 2 decagons and 3 pentagons along its edges;
- there are two kinds of vertices: at some, 2 pentagons and a decagon meet, while at others, 10 pentagons meet.

The pentagons and decagons will overlap, and the picture will get very confusing, so here Egan shows just one stage of drawing the picture. To see more stages, visit my American Mathematical Society blog:

- John Baez, Pentagon-decagon branched covering, *Visual Insight*, February 15, 2015.

What's really going on here? It's secretly all about non-Euclidean geometry! There's a way to tile the hyperbolic plane by regular pentagons and decagons. It's very symmetrical, and the shapes don't overlap.

Then, there's a way to map the hyperbolic plane down to the ordinary Euclidean plane. This has infinitely many 'branch points'. If you walk around a branch point in the hyperbolic plane, your shadow down in the Euclidean plane will walk three times around a point down there.

This is how we get 10 pentagons to meet at a point. Up in the hyperbolic plane, they don't overlap. Down in the Euclidean plane, they do: they wrap three times around a point. That's what you see in the very middle of this picture!

You can learn more of the underlying math at the link above. For a cool-looking failed attempt to tile the Euclidean plane with regular pentagons and decagons, try this:

- John Baez, Pentagon-decagon packing, *Visual Insight*, February 1, 2015.

This also has a list of some other forbidden tilings: those that arise from ways 3 regular polygons can meet snugly at a vertex.

February 21, 2015

How can computers get legal rights like people? It sounds hard. But in the US, it's not. They just need to become corporations.

You see, in the US, corporations are already persons in the legal sense, with the right to sign contracts and sue people. In 2010, the Supreme Court said they have the right to free speech! Since corporations are very powerful, they are likely to gain more and more rights — and not just in the US.

So, computers might take over the world by becoming corporations... or running corporations.

Most people think computers need to be intelligent before they take over the world. But maybe it will go like this. First they become corporations. Then they hire us to *make them* more intelligent.

Now there's a company that's trying to speed up this process! It's called Ethereum. They want to help developers start Distributed Autonomous Corporations: corporations run by computers.

Vitalik Buterin, who runs Ethereum, explained the basic idea:

> Corporations, US presidential candidate Mitt Romney reminds us, are people. Whether or not you agree with the conclusions that his partisans draw from that claim, the statement certainly carries a large amount of truth. What is a corporation, after all, but a certain group of people working together under a set of specific rules? When a corporation owns property, what that really means is that there is a legal contract stating that the property can only be used for certain purposes under the control of those people who are currently its board of directors — a designation itself modifiable by a particular set of shareholder. If a corporation does something, it's because its board of directors has agreed that it should be done. If a corporation hires employees, it means that the employees are agreeing to provide services to the corporation.s customers under a particular set of rules, particularly involving payment. When a corporation has limited liability, it means that specific people have been granted extra privileges to act with reduced fear of legal prosecution by the government — a group of people with more rights than ordinary people acting alone, but ultimately people nonetheless. In any case, it's nothing more than people and contracts all the way down.
>
> However, here a very interesting question arises: do we really need the people? On the one hand, the answer is yes: although in some post-Singularity future machines will be able to survive all on their own, for the foreseeable future some kind of human action will simply be necessary to interact with the physical world. On the other hand, however, over the past two hundred years the answer has been increasingly no. The industrial revolution allowed us, for the first time, to start replacing human labor with machines on a large scale, and now we have advanced digitized factories and robotic arms that produce complex goods like automobiles all on their own. But this is only automating the bottom; removing the need for rank and file manual laborers, and replacing them with a smaller number of professionals to maintain the robots, while the management of the company remains untouched. The question is, can we approach the problem from the other direction: even if we still need human beings to perform certain specialized tasks, can we remove the management from the equation instead?
>
> Most companies have some kind of mission statement; often it's about making money for shareholders; at other times, it includes some moral imperative to do with the particular product that they are creating, and other goals like helping communities sometimes enter the mix, at least in theory. Right now, that mission statement exists only insofar as the board of directors, and ultimately the shareholders, interpret it. But what if, with the power of modern information technology, we can encode the mission statement into code; that is, create an inviolable contract that generates revenue, pays people to perform some function, and finds hardware for itself to run on, all without any need for top-down human direction?

He went on to explain a plan to do this:

- Vitalik Buterin, Bootstrapping a decentralized autonomous corporation: Part I, *Bitcoin Magazine*, September 19, 2013.
- Vitalik Buterin, Bootstrapping a decentralized autonomous corporation: Part II: interacting with the world, *Bitcoin Magazine*, 21 September 2013.
- Vitalik Buterin, Bootstrapping a decentralized autonomous corporation: Part III: Identity Corp, *Bitcoin Magazine*, 24 September 2013.

The fascinating technical details of Ethereum are here:

- White paper: a next-generation smart contract and decentralized application platform, Ethereum Wiki.

For more on decentralized autonomous corporations, or DACs, see:

- Decentralized autonomous organization, Wikipedia.

For the American legal doctrine of corporate personhood, see:

- Corporate personhood, Wikipedia.

Does this make you want to rebel? It may be too late. I, for one, welcome our new robot overlords.

I thank Daniel Estrada for pointing out this article on DACs:

- George Dvorsky, How much longer before companies start to run themselves, *io9*, February 20, 2015.

The picture above was made by TheMarex.

February 26, 2015

Synthesis

A cloud of ideas coalesces to form a brilliant insight. At first it seems big and important. Then you start taking it for granted... and it becomes one of the ingredients of your next insight.

I got this from Maria Dubai.

For my March 2015 diary, go here.

---

baez@math.removethis.ucr.andthis.edu

home

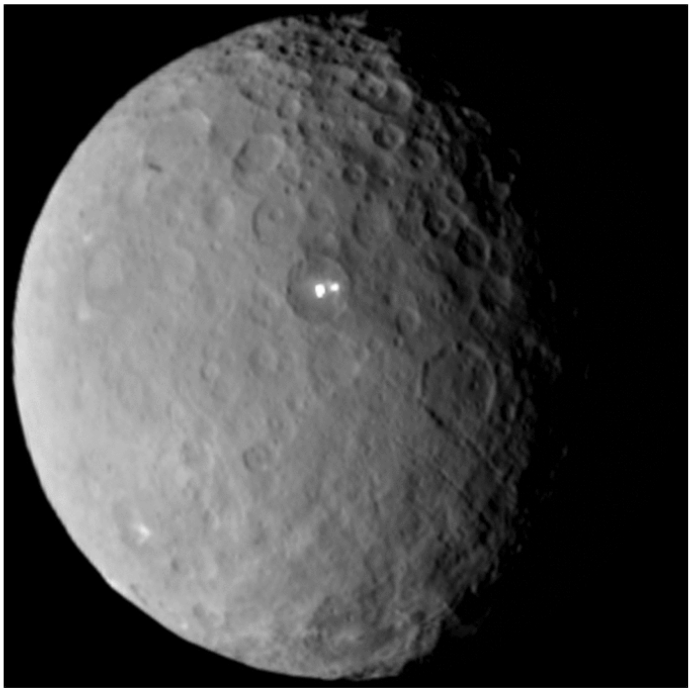
For my February 2015 diary, go here.

Diary — March 2015

John Baez

March 6, 2015

Confirmed: I am in orbit around #Ceres



That's what the spacecraft Dawn said on Twitter today. After more than 7 years and a visit to the asteroid Vesta, Dawn has reached its goal!

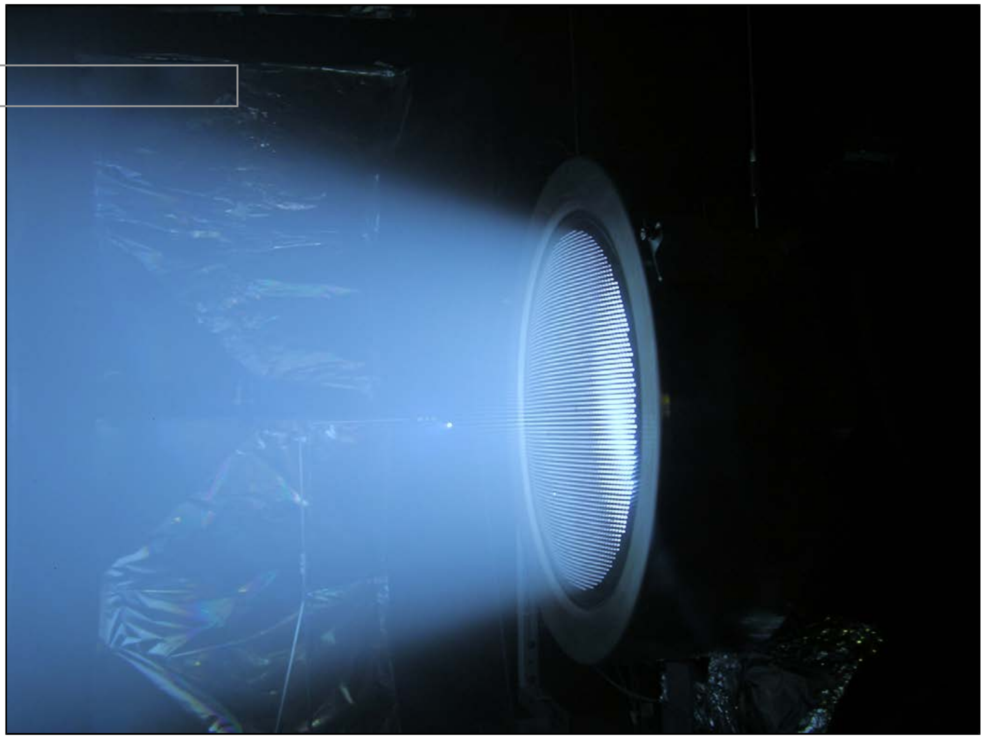
I hope the white spots on Ceres are a solar power plant for an alien base, or at least a 'cryovolcano' — a volcano that shoots up liquid water instead of lava. But probably they're just ice or snow.

Still, this is pretty cool! Ceres is probably a protoplanet whose growth got stopped by the huge gravitational pull of Jupiter. We know it has water vapor in its tiny atmosphere. It may have a lot of ice inside, unlike most of the rocky asteroids. So we can learn a bit about how planets first formed by visiting this world.

It's also cool how we got there. NASA couldn't explore Ceres without ion propulsion. Dawn started out its journey with 425 kilograms of the noble gas xenon. It turns this gas into ions and shoots them out the back to accelerate through space. So, it rides through space on a blue-green beam of ions!

This is the kind of thing we can do if we stop fussing and fighting for a while. Are we really ready to explore the universe? Maybe not yet. But someday....

March 7, 2015



This is the kind of thruster that powered the spacecraft Dawn to the asteroids Vesta and Ceres. It's beautiful! It creates a beam of xenon ions. These ions blast into space at 40 kilometers per second — 90,000 miles per hour! — pushing the ship forward.

Dawn is solar powered. It sucks up 10 kilowatts of solar power and uses this to run the ion thruster. It started out with 275 kilograms of the noble gas xenon. It takes atoms of this gas and strips off some the electrons, leaving the atoms positively charged. These are called ions.

It accelerates these ions with an electric field, and they shoot out of thousands of tiny holes — which I think you can see here. Each hole acts as a lens that electrically focuses the ions.

Because the ion thruster puts out positive ions, an equal amount of negative charge must be expelled to keep the spacecraft from getting a huge electric charge. So, a small gadget called the 'neutralizer' shoots out electrons.

The force produced by Dawn's thrusters is tiny: just 40 millinewtons. A newton is the force it takes to accelerate one kilogram one meter per second each second. Dawn's thrusters push as hard as a sheet of paper pushes down on your hand!

So, this spacecraft takes four days to accelerate from 0 to 100 kilometers per hour, while a good car can do it in 3-6 seconds. The advantage of Dawn is that it can keep up this acceleration for years without running out of propellant. This is what made it the first spacecraft able to slow down and orbit one body in our Solar System, then take off and go to another, then slow down and orbit that!

Puzzle 1: Why does Dawn use xenon? Is it just because this gas has a really cool-sounding name?

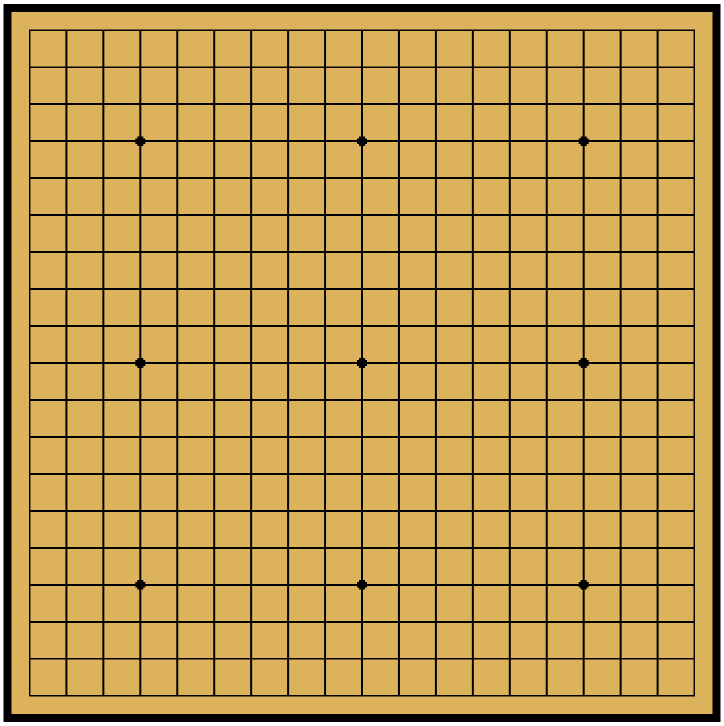
Puzzle 2: What bad things might happen if Dawn built up a big electric charge?

Puzzle 3: Why does the ion beam glow? Why is it blue?

Puzzle 4: Another elegant form of propulsion is a solar sail, where sunlight pushes a spacecraft directly. Why isn't this better than converting sunlight to energy and using that to run an ion drive? After all, converting energy from one form to another tends to create waste heat.

For answers to these puzzles, see the comments on my G+ post.

March 11, 2015



The ancient game of go still holds many challenges. Compared to go, chess is like tac-tac-toe. There are 255168 possible games of tic-tac-toe. There are about 10^{120} possible chess games. But there are about 10^{761} possible go games!

The main challenge with go is playing it well. But there's also counting the number of legal positions.

A full-fledged go board has 19 × 19 squares, and approximately this many legal positions:

2081681993819820000000000000000000000000000000000
0000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000
000000000000000000000000

That's about 2×10^{170}.

John Tromp wants to know the exact number. Why? Because it's a fun challenge. He knows an algorithm to calculate this number. It's very clever. But it still takes a lot of computing power.

He recently calculated the number of legal moves on an 18 × 18 board. It took 9 months, and 4 petabytes of disk IO on a Dell PowerEdge R820 server. He did it at the Institute for Advanced Studies in Princeton... that place where Einstein used to hang out. The answer is:

669723114288829212892740188841706543509937780
640178732810318337696945624428547218105214326
012774371397184848890970111836283470468812827
907149926502347633

To get the answer for a 19 × 19 board will take more work. Using some very clever math, the task can be split up into 9 jobs that each compute 64 bits of the 566-bit result. To do this and be sure the answer is right, he needs about 10 to 13 servers, each with at least 8 cores, 512 gigabytes of RAM, and 10-15 terabytes of disk space. The job will take about 5-9 months.

You can read more about this here:

John Tromp, Number of legal 18×18 go positions

including the answers for all square boards of size up to 18 × 18. The most interesting part is the algorithm to compute these numbers:

- John Tromp and Gunnar Farnebäck, Combinatorics of go, November 22, 2009.

March 14, 2015

$$\pi = \frac{22}{7} - \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\, dx$$

It's 'Pi Day', 3/14/15. This is a fundamentally silly holiday that only works if you write first the month, then the day, then the year. Not many people besides Americans use that illogical system.

But getting into the silly spirit of things, consider this formula. It's true. But it's hilarious.

This is a famous approximation to pi:

$$22/7 = 3.142857142857142857...$$

The hilarious part is that the difference

$$22/7 - \pi = 0.00126448926...$$

is given by the elegant integral shown above!

This formula goes back at least to this paper:

• D. P. Dalzell, On 22/7, *J. London Math. Soc.* **19** (1944), 133–134,

Since the integrand is nonnegative, it gives a nice proof that pi is less than 22/7, and with a little more work you can use it to show that pi is more than

$$22/7 - 1/630 = 3.14126984127...$$

For more on this formula, including a proof, see:

- Source and context of $22/7 - \pi = \int_0^1 (x - x^2)^4\, dx/(1 + x^2)$, MathOverflow.

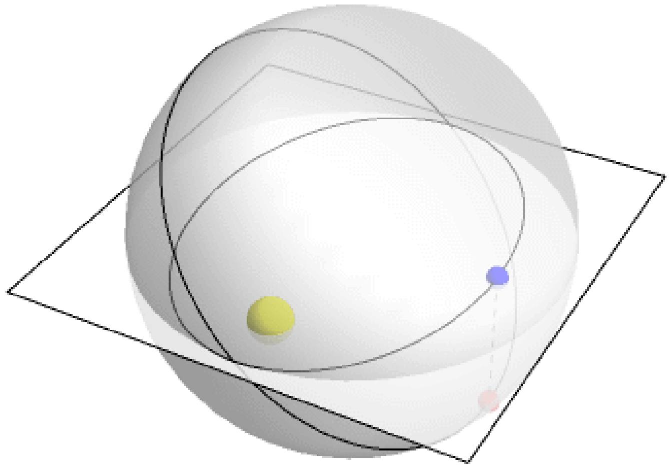
and

- Proof that 22/7 exceeds π, Wikipedia.

The first page has a nice analysis of the coincidences required to get such a simple formula to work.

March 18, 2015

A planet in the fourth dimension



I bet you know that planets go around the sun in elliptical orbits. But do you know why?

In fact, they're moving in circles in 4 dimensions. But when these circles are projected down to 3-dimensional space, they become ellipses! This animation by Greg Egan shows the idea.

The plane here represents 2 of the 3 space dimensions we live in. The vertical direction is the mysterious fourth dimension. The planet goes around in a circle in 4-dimensional space. But down here in 3 dimensions, its 'shadow' moves in an ellipse!

What's this fourth dimension I'm talking about here? It's a lot like time. But it's not exactly time. It's the difference between ordinary time and another sort of time, which flows at a rate inversely proportional to the distance between the planet and the sun.

Egan's animation uses this other sort of time. Relative to this other time, the planet is moving at constant speed around a circle in 4 dimensions. But in ordinary time, its shadow in 3 dimensions moves faster when it's closer to the sun.

All this sounds crazy, but it's not some new physics theory. It's just a different way of thinking about Newtonian physics! Of course you can see that planets move in elliptical orbits without resorting to the 4th dimension. But it becomes a lot more obvious if you do!

Physicists have known about this viewpoint at least since 1980, thanks to a paper by the mathematical physicist Jürgen Moser. Some parts of the story are much older. A lot of papers have been written about it.

But I only realized how simple it is when I got a paper in my email from someone I didn't know: an amateur mathematician named Jesper Göransson. I get a lot of papers by crackpots, but the occasional gem like this makes up for all those.

The best thing about Göransson's 4-dimensional description of planetary motion is that it gives a clean explanation of an amazing fact. You can take any elliptical orbit, apply a rotation of 4-dimensional space, and get another valid orbit!

Of course we can rotate an elliptical orbit about the sun in the usual 3-dimensional way and get another elliptical orbit. The interesting part is that we can also do 4-dimensional rotations. This can make a round ellipse look skinny: when we tilt a circle into the fourth dimension, its 'shadow' in 3-dimensional space becomes thinner!

In fact, you can turn any elliptical orbit into any other elliptical orbit with the same energy by a 4-dimensional rotation of this sort. All elliptical orbits with the same energy are really just circular orbits on the same sphere in 4 dimensions!

For the details, try this:

- John Baez, Planets in the fourth dimension, *Azimuth*, March 17, 2015.

I had to go through Göransson's calculations to convince myself that they were right. Here is his paper:

- Jesper Göransson, Symmetries of the Kepler problem, March 8th, 2015.

March 20, 2015

This image starts out as a dot. You keep zooming in to see more detail.

First it stretches out to become a line segment. As you zoom in further, you see its thickness. It's really a long thin rectangle.

But wait longer and you see it's really a field of dots. And zooming into any one of these dots, this process repeats... forever!

Each long thin rectangle is 10,000 times longer than the next smaller one.

So, you're looking at a very complicated set of points in the plane, whose dimension seems to depend on how closely you zoom in. In this example, created by Simon Willerton, the dimension keeps cycling:

0, 1, 2, 0, 1, 2,....

But you can make examples that do other things.

The moral? Mathematicians have various ways of defining the dimension of a set of points in the plane, or even more general sets. A point, or a finite set of points, is 0-dimensional. A line, or a smooth curve, is 1-dimensional. A solid rectangle, or a disk, is 2-dimensional.

But sometimes it's more complicated! There are fractals whose dimension is not an integer... at least if we use the right definition of 'dimension'. The old Lebesgue dimension is always an integer, but the Hausdorff–Besicovich dimension or Minkowski dimension can be fractional, or even irrational.

And there are also sets whose dimension seems to depend on how closely you look at them! That's what we have here.

Willerton is working on a theory of scale-dependent dimension, to make this precise. He's writing a series of blog articles on it, and the first is here:

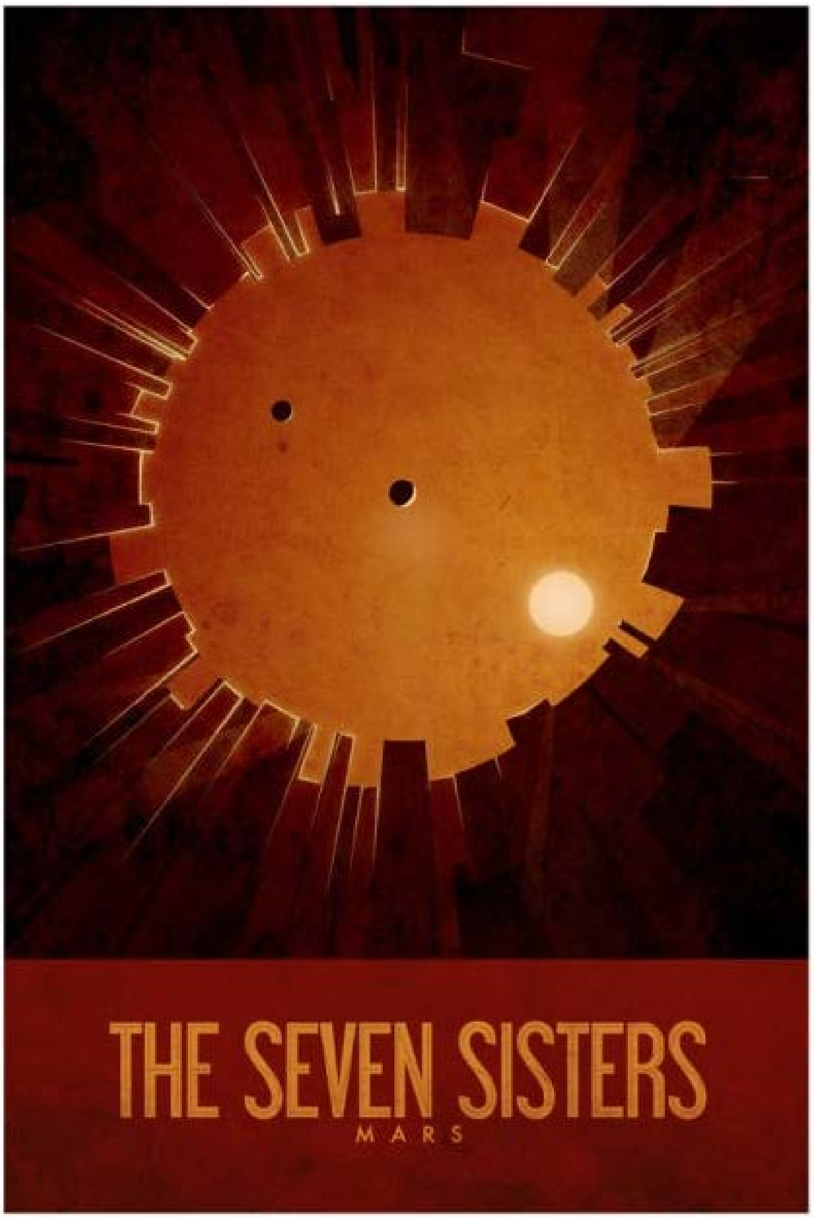
- A scale-dependent notion of dimension for metric spaces (part 1), *The n-Category Café*, March 11, 2015.

There's a lot of nice math here, but a lot of open questions... which is good if you're a mathematician! More puzzles to work on!

For the details, go here:

- Simon Willerton, Spread: a measure of the size of metric spaces.

March 21, 2015

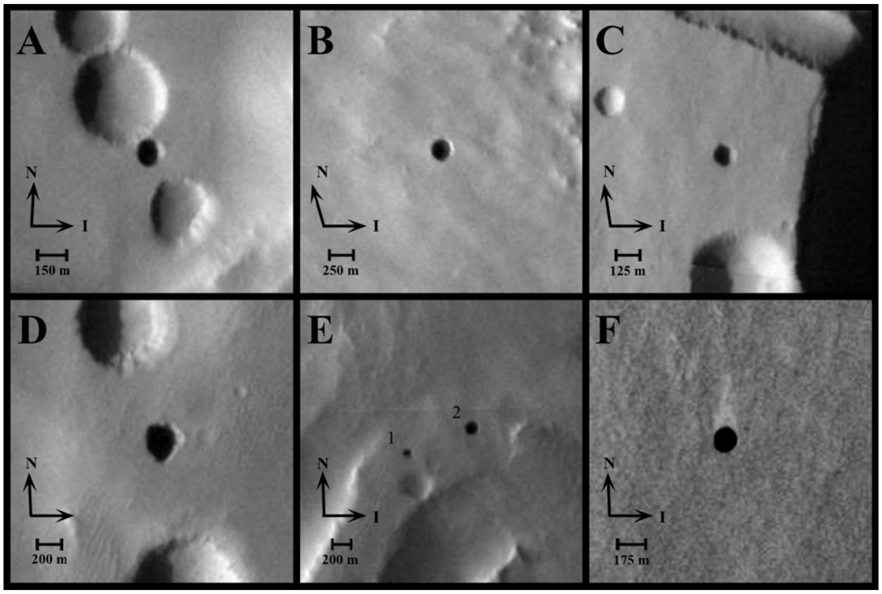


In 2007, NASA discovered seven very dark circles on a mountain in Mars. Using infrared cameras, they checked the temperature of these circles—and discovered that they didn't change much from day to night.

"They are cooler than the surrounding surface in the day and warmer at night," said Glen Cushing of the U.S. Geological Survey's Astrogeology Team. "Their thermal behavior is not as steady as large caves on Earth that often maintain a fairly constant temperature, but it is consistent with these being deep holes in the ground."

In short, they could be windows into caves!

They're called the Seven Sisters: Dena, Chloe, Wendy, Annie, Abby, Nikki and Jeanne. They range in diameter from about 100 to 250 meters.



They're on one of the highest places on Mars: a volcano named Arsia Mons near Mars' tallest mountain, Olympus Mons.

But the picture here is part of another story. A few years ago, an artist named Ron Guyatt started making "solar system travel posters." This was one of the first.

You can see them all here:

- Ron Guyatt, Space travel posters, DeviantArt.

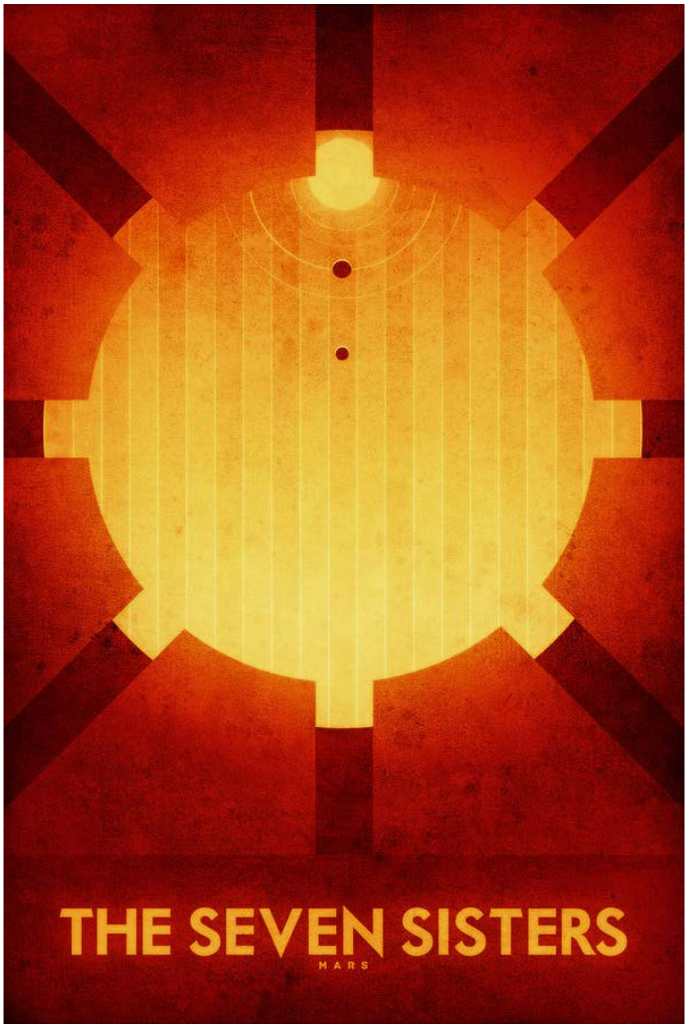
There are 3 pages of them. Click to make them bigger.

He says:

> Space tourism is still a long ways off, but it's not hard to imagine that someday, tourists will visit the natural geological landmarks of other worlds much like they tour the Grand Canyon, Mount Everest or Ayers Rock. Each of these great tourist destinations needs a classic retro travel poster to entice visitors. Until the day people settle off-world and make their own destinations many of these may be the places that people will want to travel to. I hope that these posters can inspire people to think beyond our world to the limitless possibilities of the Universe.

I like this idea!

The posters on Guyatt's website are more abstract than the one here. Compare it to this:

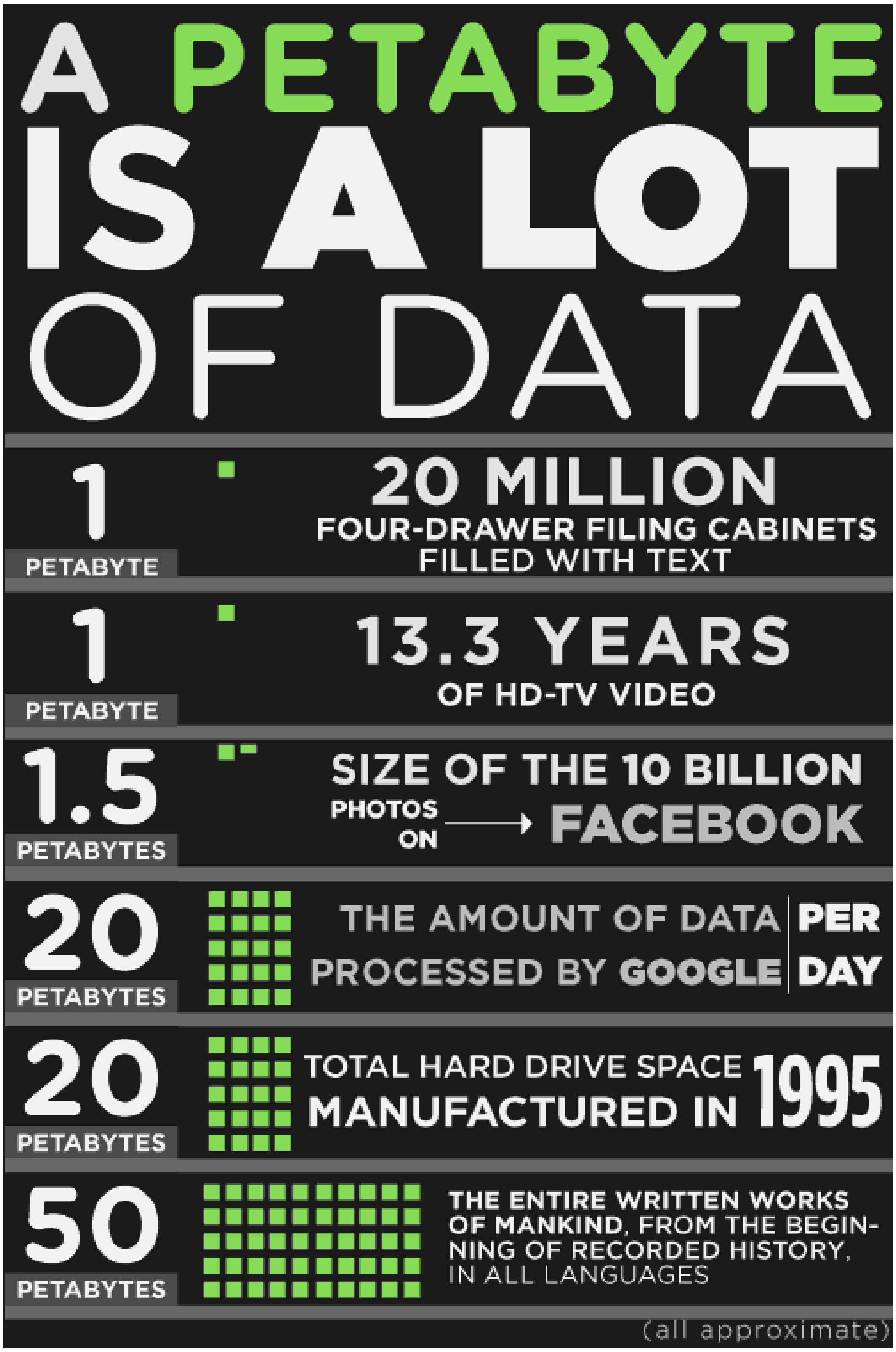


Which do you like better? I like the less abstract version. But they're both good.

Puzzle: There are lots of things called "The Seven Sisters". Which ones do you know?

For some answers, look at the comments on my G+ post.

March 26, 2015



A petabyte is a lot of information. But how many petabytes does it take to completely describe one gram of water?

I'm running a workshop on Information and Entropy in early April, so I've got this sort of thing on my mind. The answer is: 500,000,000,000 terabytes. To see how to calculate this, and lots more, try:

- John Baez, Information.

The graphic here is originally from Mozy, but it was edited over at Gizmodo.

March 27, 2015

Puzzle: If the Earth became a black hole, how big would this black hole be?

Answer: Its radius would be approximately 0.887 centimeters.

We can get quite far on this problem using just dimensional analysis. General relativity involves two constants: the gravitational constant G and the speed of light c. G has units of force times distance^2 per mass^2, or

$$\frac{ML}{T^2} \times \frac{L^2}{M^2} = \frac{L^3}{T^2 M}$$

c has units of length over time, so G/c^2 has units of

$$\frac{T^2}{L^2} \times \frac{L^3}{T^2 M} = \frac{L}{M}$$

Units of length per mass! So, to get the radius of a black hole, we just need to multiply its mass by G/c^2. We'll get the right answer up to some constant factor.

We need to think harder to work out the constant factor, though Newtonian mechanics works just as well as general relativity for this: you can work out, for a given point mass, at what distance the escape velocity is the speed of light... and this just happens to be the exact radius of the event horizon of a black hole with that mass! It's just a nice coincidence.

So, the radius of a black hole of mass m — its so-called Schwarzschild radius — is

$$\frac{2Gm}{c^2}$$

Working this out in metric units, its radius in meters is 1.4851×10^{-27} times its mass in kilograms.

The mass of the Earth is 5.972×10^{24} kilograms. So, if you squashed the Earth down to a black hole, its Schwarzschild radius would be

$$1.4851 \times 10^{-27} \frac{\text{meters}}{\text{kilogram}} \times 5.972 \times 10^{24} \text{ kilograms} = 0.008869 \text{ meters}$$

or about 0.887 centimeters.

March 29, 2015

Guess what: black holes are bigger on the inside — and they get bigger as they get older!

For example, take the big black hole in the center of our galaxy, called Sagittarius A*. It's about 2 million kilometers across. That's pretty big — but the orbit of Mercury is 60 times bigger. This black hole is old, roughly a billion years old. And here's the cool part: it's been growing *on the inside* all this time!

How is this possible? Well, since spacetime is severely warped in a black hole, its volume can be bigger than you'd guess from outside. And its volume can change. Since we understand general relativity quite well, we can calculate how this works! But nobody thought of doing it until last year, when Marios Christodoulou and my friend Carlo Rovelli did it.

How big is the black hole at the center of our galaxy? *On the inside*, it can hold a million solar systems! Its volume is about 10^{34} cubic kilometers! And it's growing at a rate of about 10^{25} cubic kilometers per year!

Or suppose you have an ordinary star that turns into a black hole. This black hole will last a long time before it evaporates due to Hawking radiation. Christodolou and Rovelli estimate how big its volume will get before this happens. And it gets *really* big — *bigger than the current-day observable universe!*

Before you get too excited, remember: people falling into the black hole will not have time to do anything fun inside. They will hit the singularity in a short time. Very very roughly speaking, the problem is not the shortage of space inside the black hole, it's the shortage of time.

If you fall into the black hole at the center of our galaxy, it will be about 1 minute, at most, before you hit the singularity. You will not get to see most of the space inside the black hole! The singularity is not in the 'middle' of the black hole - it's in your future. You will hit it before you can reach the 'middle'. So, you will only get to see part of the 'edge regions' inside the black hole.

The 'middle regions' can only be seen by people who fell in much earlier. And they can't see the 'edge', where you are!

And now for the serious part.

The hard part of this problem is defining the volume inside a black hole.

If you choose a moment in time, the black hole's event horizon at that moment is a sphere. There are infinitely many ways to extend this sphere to a solid ball. In other words: there are many ways to choose a slice of space inside the black hole whose boundary is your chosen sphere.

The slice can bend forwards in time, or backwards in time. We can choose a wiggly slice or a smooth one. Each slice has its own volume.

How do you choose one, so you can calculate its volume? Christodoulou and Rovelli choose the one with the largest volume. This may sound like it's cheating. But it's not.

Think of a simpler problem one dimension down. You have a loop of wire. You ask me: "What's the area of the surface whose boundary is this loop?"

I say: "That's a meaningless question! *Which* surface? There are lots!"

You say: "Pick the best one!"

So, it's up to me. I take some soapy water and make a soap film whose boundary is that loop. That's the surface I use. If the loop of wire is not too crazy in its shape, this surface is uniquely defined. In some sense it's the "least wiggly" surface I could choose.

This surface minimizes the area. A more wiggly surface would have more area.

Christodoulou and Rovelli are doing the same thing. But spacetime is different than space! If you choose a wiggly 3-dimensional spatial surface in spacetime, it will have less volume than a flatter surface with the same boundary!

So, the way to pick the flattest, nicest spatial surface inside our black hole is to pick the one that maximizes the volume.

If you tried to minimize the volume, you could get it as close to zero as you wanted. And this would have nothing to do with black holes! This would be true even in your living room.

Puzzle: why?

Here's the paper:

- Marios Christodoulou and Carlo Rovelli, How big is a black hole?

March 31, 2015

This computer is called JUQUEEN. You can see 7 big boxes here. Each box holds 4 racks. Each rack holds 16 boards. Each board holds 32 nodes. Each node has 32 cores.

A core is like the processor in your laptop. So, this computer is roughly like half a million laptops — all connected and working together. It can compute at a rate of 5 petaflops. That's 5,000,000,000,000,000 floating-point operations per second!

This computer was recently used to compute the ratio of the proton and neutron masses. A proton weighs about 1836 times as much as an electron. A neutron is a bit heavier: about 1839 times the electron mass.

This is important, since it means a lone proton is stable, while a lone neutron is not: in about 15 minutes, it will decay into a proton and some other stuff. This is why the universe is mainly made of hydrogen, not neutrons!

Why is the neutron a bit heavier? People have been wondering for a long time.

The answer lies in the Standard Model, our best theory of particles and all the forces except gravity. Protons and neutrons are made of quarks, and the Standard Model says exactly how this works. So, we can use the Standard Model to compute the ratio of proton and neutron masses.

But it's not easy! As anyone who has studied quantum field theory will tell you, this problem is a nightmare. For the course I took, in the final exam we had to compute how two electrons scatter off each other. I probably screwed up, because I only got a B+. But that problem is really, really easy compared to computing the mass of a proton or neutron.

The problem is that the strong force, which holds the quarks together, interacts with itself in a complicated way. The strong force is carried by particles called gluons. Quarks emit and absorb gluons. But gluons also emit and absorb gluons! So, a proton or neutron is like a blob containing 3 quarks - but a blob made of gluons, virtual quark-antiquark pairs, and some other virtual particles, all held together by their interactions.

To accurately compute the total energy of this blob, and thus its mass, you basically need to simulate it. And even though we know the basic rules, that takes a lot of computing.

But now it's been done!

- Sz. Borsanyi, S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, L. Lellouch, T. Lippert, A. Portelli, K. K. Szabo, and B. C. Toth, *Ab initio* calculation of the neutron-proton mass difference, *Science* **347** (March 27, 2015), 1452–1455.

It makes me simultaneously relieved that I didn't go deeper into this subject, and jealous — because it's so beautiful, intricate and demanding.

You can see the world in a grain of sand... or even a single proton.

The abstract gives a tiny taste:

> The existence and stability of atoms rely on the fact that neutrons are more massive than protons. The measured mass difference is only 0.14% of the average of the two masses. A slightly smaller or larger value would have led to a dramatically different universe. Here, we show that this difference results from the competition between electromagnetic and mass isospin breaking effects. We performed lattice quantum-chromodynamics and quantum-electrodynamics computations with four nondegenerate Wilson fermion flavors and computed the neutron-proton mass-splitting with an accuracy of 300 kilo.electron volts, which is greater than 0 by 5 standard deviations. We also determine the splittings in the Σ, Ξ, D, and Ξcc isospin multiplets, exceeding in some cases the precision of experimental measurements.

If you're a quantum field theory geek, you'll want to read the 'supplementary material', because that's where all the details are:

- Sz. Borsanyi *et al*, Supplementary material for *Ab initio* calculation of the neutron-proton mass difference, *Science* **347** (March 27, 2015), 1452–1455.

Here you'll see particle physics jargon blending with computing jargon in a marvelous symphony:

> Starting with Sec. 6 we present the details of the many simulations that are performed and summarized here. The use of Rational Hybrid Monte-Carlo method is discussed with a special emphasis on the lowest eigenvalues of the Dirac operator. Autocorrelations are under control for our choice of parameters in the QCD part of our work. However, due to the zero mass of the photon and the correspondingly large correlation lengths, a standard Hybrid Monte-Carlo integration of the photon fields results in large autocorrelation times. We show how we solved this problem by developing a Fourier accelerated algorithm. For the propagator calculations we used a 2-level multi-grid approach to have several hundred source positions and significantly improve our statistics.

And here's a vastly harder challenge: do these calculations in a way where you can prove they are accurate up to some tolerance. We can't do this yet because we haven't even proved the Standard Model is mathematically consistent. Until we do, and until we develop a rigorous approach to computing things like the proton-neutron mass difference, there's always the danger that researchers are subconsciously choosing certain approximations because they seem to make the answer come out closer to what we observe.

Puzzle: Why are there just 7 big boxes here, not 8? Everything else comes in powers of 2. If it had 8 boxes, JUQUEEN would have

$$2^{19} = 524288$$

cores. But it has just 458752. Budget cuts?

For my April 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

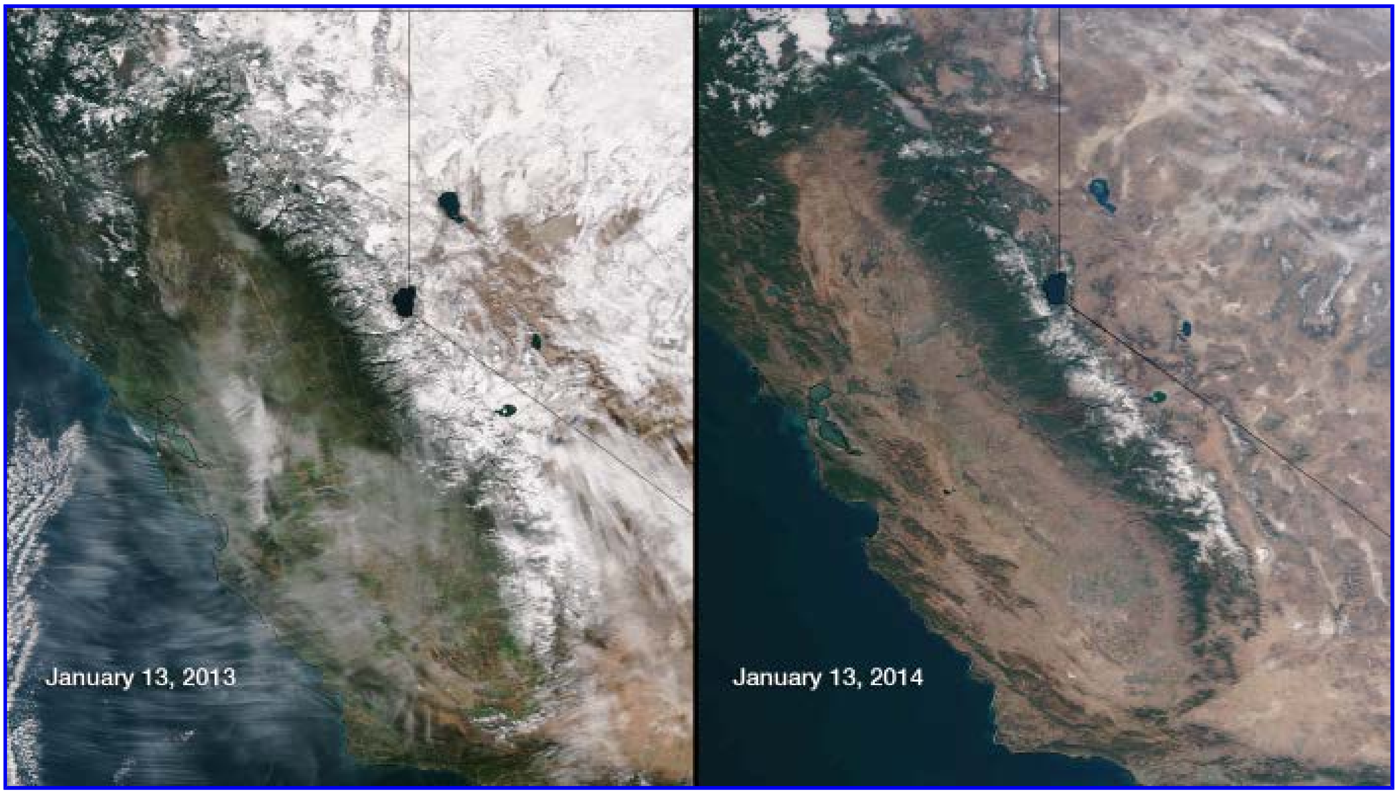
home

[For my March 2015 diary, go here.](#)

Diary — April 2015

John Baez

April 3, 2015



The picture shows snow in the mountains of California, 2013 and 2014. Snow usually provides 30% of California's water, so that was bad news. But 2015 was *much worse*.

"We're not only setting a new low; we're completely obliterating the previous record," said the chief of the California Department of Water Resources. There's now only 5% as much snow as the average over the last century!

California has been hit by new weather pattern: the Ridiculously Resilient Ridge. It's a patch of high atmospheric pressure that sits over the far northeastern Pacific Ocean and stops winter storms from reaching California. It's been sitting there most of the time for the last 3 winters.

We did get 2 big storms this winter. But the water fell mainly as rain rather than snow, because of record-breaking heat. It was enough to half fill Shasta Lake and Lake Oroville. But it didn't help the snow pack.

For the first time, the governor has imposed mandatory water restrictions: a 25% cut in water use in every city and town. This will save about 1.8 cubic kilometers of water over the next 9 months — nearly as much as Lake Oroville now holds.

He said:

> People should realize we're in a new era. The idea of your nice little green grass getting lots of water every day — that's going to be a thing of the past.

But what about agriculture? In California, about 50% of water is used by 'the environment': rivers, wetlands, parks and the like. 40% is used by agriculture. 10% is left for businesses and residents.

Brown didn't impose any cuts on agriculture! That sounds unfair, and people are complaining. More water is used to grow walnuts than to keep Los Angeles going!

We definitely need to improve agriculture. But don't forget: for the second year in a row, farmers in California's big Central Valley are getting hit with big water cutbacks. The ones who get water from the State Water Project will receive only 20% of their usual amount.

Is all this due to climate change? I heard a wise answer to that question: instead of a definite yes or no, just: *this is what climate change looks like*. This is the kind of thing we can expect.

And on the Road to Paris, this week the US submitted a plan to cut carbon emissions by 25% by 2030... but that's another story. Or another part of the same big story.

What California is doing about the drought:

- Bettina Boxall, Gov. Brown's drought plan goes easy on agriculture, *Los Angeles Times*, April 3, 2015.

Water used by agriculture in California:

- Julia Lurie, California's almonds suck as much water annually as Los Angeles uses in three years, *Mother Jones*, January 12, 2015.

Make your own graphs of the California snowpack:

- California Department of Water Resources, California Data Exchange Center.

There's lots more water data here, too — click items on the menu above.

More on the Ridiculously Resilient Ridge or 'Triple R' by the guy who coined the term:

- Daniel Swain, *The California Weather Blog*.

In February he wrote:

> In this sense, the Triple R of 2014-2015 is notably different from 2013-2014. California has certainly received more precipitation this year on a liquid equivalent basis, though we're once again falling rapidly behind average as February turns out to be mostly dry. The extreme warmth and low snowpack, however, are very reminiscent of recent winters — as is the occurrence of infrequent but intense warm storms. It's interesting to note that nearly the entire western United States has been exceptionally warm in recent months, while the eastern part of the country remains locked in a recurring nightmare of extreme Arctic outbreaks and almost inconceivable snow accumulations in parts of New England. This overall setup—with a big Western ridge and a deep Eastern trough—has become known as the 'Warm West/Cool East' dipole pattern, and it has been a common feature of recent winters in North America. There are a number of hypotheses currently being investigated regarding the causes of an apparent recent increase in the occurrence of this pattern, though there's not yet compelling evidence pointing to a singular cause (that's a topic for a future blog post!).
>
> What is more certain, at least as far as California is concerned, is that our severe long-term drought is unlikely to improve substantially until this newly-invigorated pattern of persistent West Coast high pressure is no longer dominant.

April 4, 2015

I live on the edge of the desert in southern California. We tore up our lawn and planted beautiful plants that use less water. Drip irrigation instead of sprayers!

But we do indulge in some citrus trees. Here's the harvest!

Satsumas in front — they're like mandarins, but different. Meyer lemons at rear left — they're sweeter than ordinary lemons. Grapefruits at rear right — they're not very big, perhaps because our tree is still young and struggling.

What's a 'mandarin'? It's the mandarin orange, *Citrus reticulata*, often marketed as a 'tangerine'. According to DNA studies, the mandarin is one of the 4 ancestors of all other citrus species, which arose through hybridization and breeding. The other 3 are the the citron, the pomelo, and something called a papeda.

Among these 4 citrus ancestors, mandarins are the only really sweet ones, so they were used to create many of the fruits people like now.

For example, a Meyer lemon is probably a cross between a true lemon and a mandarin or an orange. A grapefruit is a cross between an orange and a pomelo — a huge fruit that looks like a grapefruit on steroids. And an orange is itself probably a cross between a pomelo and a mandarin!

It's all very complicated:

- Citrus taxonomy, Wikipedia.
- Citrus hybrids, Wikipedia.

Luckily you don't need to know this stuff to enjoy growing and eating citrus!

April 5, 2015

Time to harvest the kumquats! Our tree is packed with them. It's great to eat the whole fruit, since the peel is sweet yet packed with flavorful oil, mostly limonene.

Limonene is also what gives orange and lemon peels their special smell. It's one of a group of hydrocarbon molecules called terpenes, which are found in pine needles, cinnamon, cloves, ginger, camphor, mints, and the sap of many trees. Plants produce terpenes to repel insects! Trees also release more terpenes in warmer weather, creating a haze that acts as a natural form of cloud seeding. The clouds reflect sunlight, letting the forest regulate its temperature.

Limonene is one of the simplest terpenes. It's a hexagonal ring of carbons with a single extra carbon attached to one corner and a Y-shaped group of three carbons attached to the opposite corner... all decorated by hydrogens.

The hexagon of carbons is not a benzene ring: it's a cyclohexene, meaning that each carbon is connected to the next with a single bond, except for one double bond.

Puzzle: so, how many hydrogen atoms are there in cyclohexene? Looking up the answer is cheating... you can figure it out from first principles using what I said.

Time to eat some more kumquats! For various replies to the puzzle, check out my G+ post. And for more of the wonderful chemicals that make citrus fruits so great, read my July 2nd, 2006 diary entry!

April 8, 2015

Integrals From Hell

$$\text{Evaluate} \int \frac{d\theta}{(\sqrt{\sin\theta} + \sqrt{\cos\theta})^4}$$

When my uncle first tried to teach me calculus I thought it was confusing. I already wanted to be a mathematician. So I decided to be a mathematician who wouldn't use calculus.

A few weeks later I wasn't scared of calculus anymore, thanks to the wonderful book *Calculus Made Easy* by Silvanus Thompson, which he gave me.

Later, when I took calculus in high school, it became fun to tackle tough integrals. The reason it's fun is that a limited set of rules lets you do a lot of integrals, yet there's still some art involved in doing them well. It's like a game.

(This was before computers were programmed to do integrals much better than people can.)

Later, I learned that only a small fraction of the integrals you can write down can be done using the rules you learn in school. For most, the answer is some function you can't even write down using the usual kit of high-school functions: addition, subtraction, multiplication, division, exponentials, logarithms, trig and inverse trig functions.

I also learned that most professional mathematicians consider it uncool to get really good at doing integrals: it's just one step up from memorizing digits of pi. Professional mathematicians want to learn about stuff like Shimura varieties, or motivic cohomology — stuff you can't even begin to explain to ordinary folks. This is what it takes to impress other mathematicians.

Still, integrals can be fun. Sometimes I teach calculus at U.C. Riverside - though now it's mainly poorly-paid "lecturers" who have to do this job. When I teach calculus, I usually focus on the students who are having trouble. I want everyone to learn the stuff! Unfortunately this means I never spend time showing the good students fun tricks.

I realize now that I should spend a little time doing "integrals from hell" like this one here. First of all, it would be fun for the better students. It shows there's a kind of athletic element to math, where you don't just learn to walk: you learn to run insanely fast! Second of all, it makes the easy integrals seem easier.

This particular integral is fun because at first glance it looks horrible, yet it falls quickly to high-school tricks. It's fun to see how these tricks make it simpler! Then you get something that's a bit grungy and boring. It's just the first steps that are fun.

Puzzle: What's a nice way to start doing this integral?

I got this integral from here:

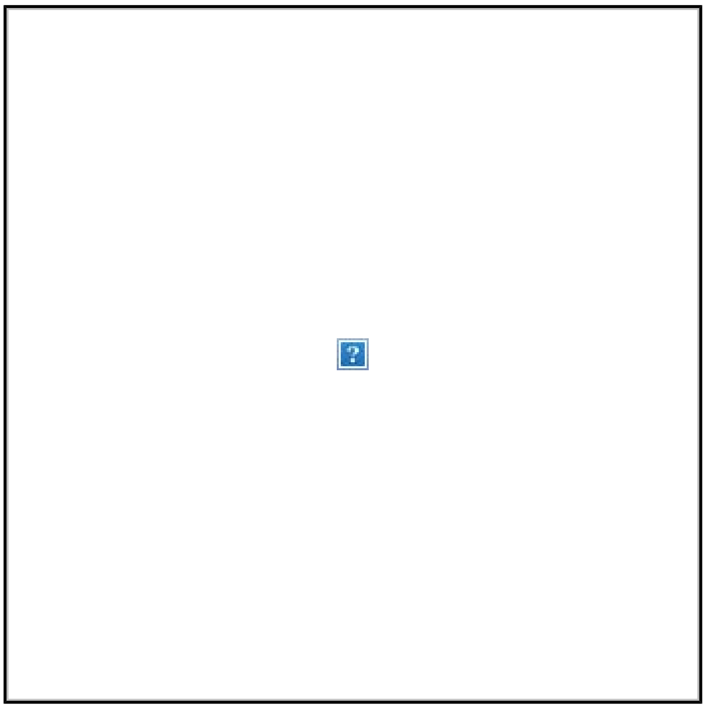
- www.i-calculus.com.

They post lots of fun integrals—good puzzles if you still remember your high school calculus and you're not too much of a professional mathematician to enjoy this sort of thing. You can see solutions here:

- A solution by Sourav De, *I-Calculus*.

April 9, 2015

American Hero



On Monday night, artists built this monument to Edward Snowden in Brooklyn. The next day, it was taken down. Will there be a permanent one someday?

Martin Luther King was put in jail 29 times, and now there's a monument to him in Washington DC. But it was built only in 2011, forty-three years after he was killed.

If Snowden ever gets a monument, here are some quotes of his they could carve on it:

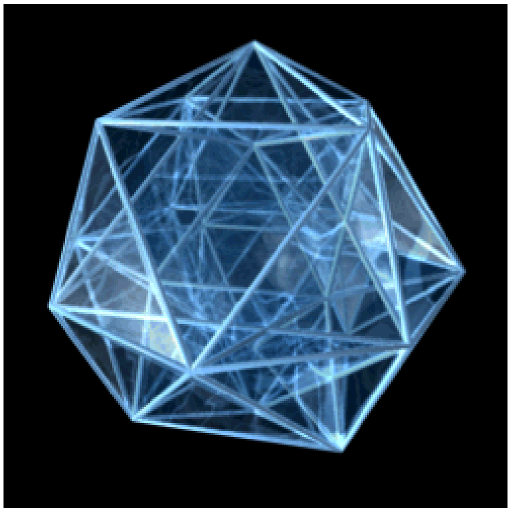
> There can be no faith in government if our highest offices are excused from scrutiny—they should be setting the example of transparency.
>
> I would rather be without a state than without a voice.
>
> I don't see myself as a hero because what I'm doing is self-interested: I don't want to live in a world where there's no privacy and therefore no room for intellectual exploration and creativity.

After the statue was removed by park officers, a group of artists who call themselves "The Illuminator"—not related to those who built the original sculpture—used laptops and projection equipment to cast an image of Snowden in a haze of smoke at the spot where the sculpture had been.

- Brian Ries, Hologram replaces Edward Snowden statue in Brooklyn park, *Mashable*, April 7, 2015.

April 12, 2015

Lectures from the 8th Dimension



This week I'm visiting Penn State University. I'm giving two talks about geometry:

- Split octonions and the rolling ball, 2:30-3:20 p.m, Tuesday April 14th, 106 McAllister Building.

Something amazing happens when you roll a ball on another ball whose radius is exactly 3 times as big! The geometry of objects rolling without slipping or twisting is always fun — but in this particular case the problem gets extra symmetries, which are best understood using an 8-dimensional number system called the 'split octonions'.

- The exceptional Jordan algebra and the Leech lattice, 12:05-1:20 pm, Wednesday April 15th, 114 McAllister Building.

There's a specially beautiful way to pack balls in 24 dimensions, called the Leech lattice. When physicists classified the algebras that could describe observables in quantum mechanics, they found a weird possibility: a 27-dimensional one called the exceptional Jordan algebra. It turns out that the Leech lattice fits into the exceptional Jordan algebra in a nice way... which comes from the octonions. So all this stuff fits together! This talk is part of the "Geometry Luncheon Seminar", where mathematicians eat lunch and talk about geometry. You can see my notes here:

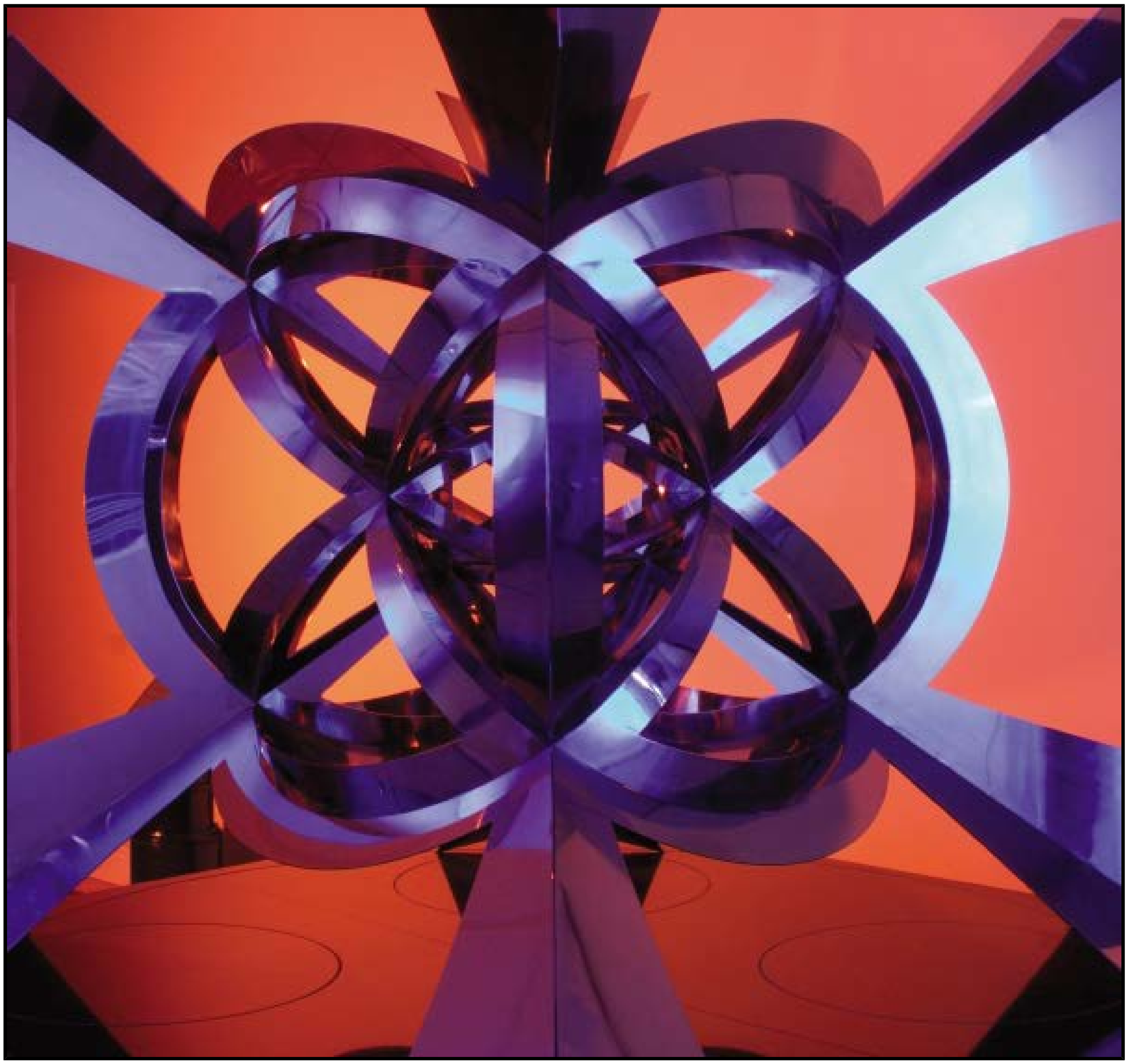
The first talk is about work I did with John Huerta and James Dolan, and it will feature some fun animations made by Geoffrey Dixon. The second is about work with Greg Egan.

The actual reason I'm at Penn State is to give a guest lecture at John Roe's undergrad course on Mathematics for Sustainability. I want to teach a course on math and environmental issues. It'll be good to hear how he's been doing this. But I thought it would be fun to talk about some other things too.

I'll also visit one of my old haunts, the Institute for Gravitation and the Cosmos, where Abhay Ashtekar, Eugenio Bianchi and others are working on loop quantum gravity. And I'll talk to Jason Morton about network theory. It should be a busy, fun week.

But first I have to work on my talks...

This image above, made by Jason Hise, shows a '24-cell', a regular polytope in 4 dimensions. There's a sculpture of this shape in the math department at Penn State! It was designed by the mathematician Adrian Ocneanu. I haven't been here since it was built so it will be fun to see:



April 18, 2015

You can now make your own cyborg roach for just $100. Just buy this kit developed by the company Backyard Brains:

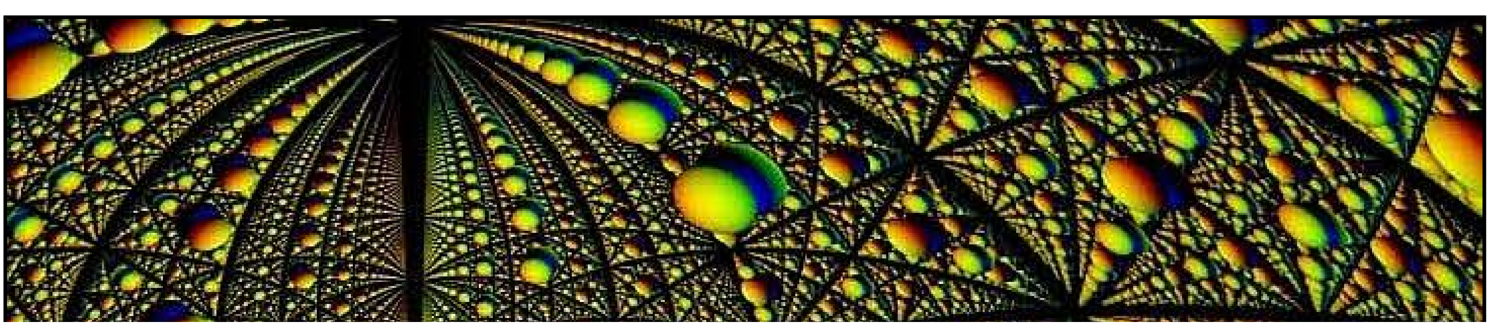
> Are you a teacher or parent that wants to teach a student about advanced neurotechnologies? You are in luck! After 3 long years of R&D, the RoboRoach is now ready for its grand release! We are excited to announce the world's first commercially available cyborg! With our RoboRoach you can briefly wirelessly control the left/right movement of a cockroach by microstimulation of the antenna nerves. The RoboRoach is a great way to learn about neural microstimulation, learning, and electronics!
>
> We are recently ran a successfully-funded kickstarter campaign to fund the release of our new RoboRoach! The hardware and firmware development are complete and we are now shipping!
>
> **Product Details**
>
> The RoboRoach "backpack" weighs 4.4 grams with the battery, and each battery will last over a month! Following a brief surgery you perform on the cockroach to attach the silver electrodes to the antenna, you can attach the backpack to the roach and control its movement for a few minutes before the cockroach adapts. When you return the cockroach to its cage for ~20 minutes, he "forgets" and the stimulation works again.

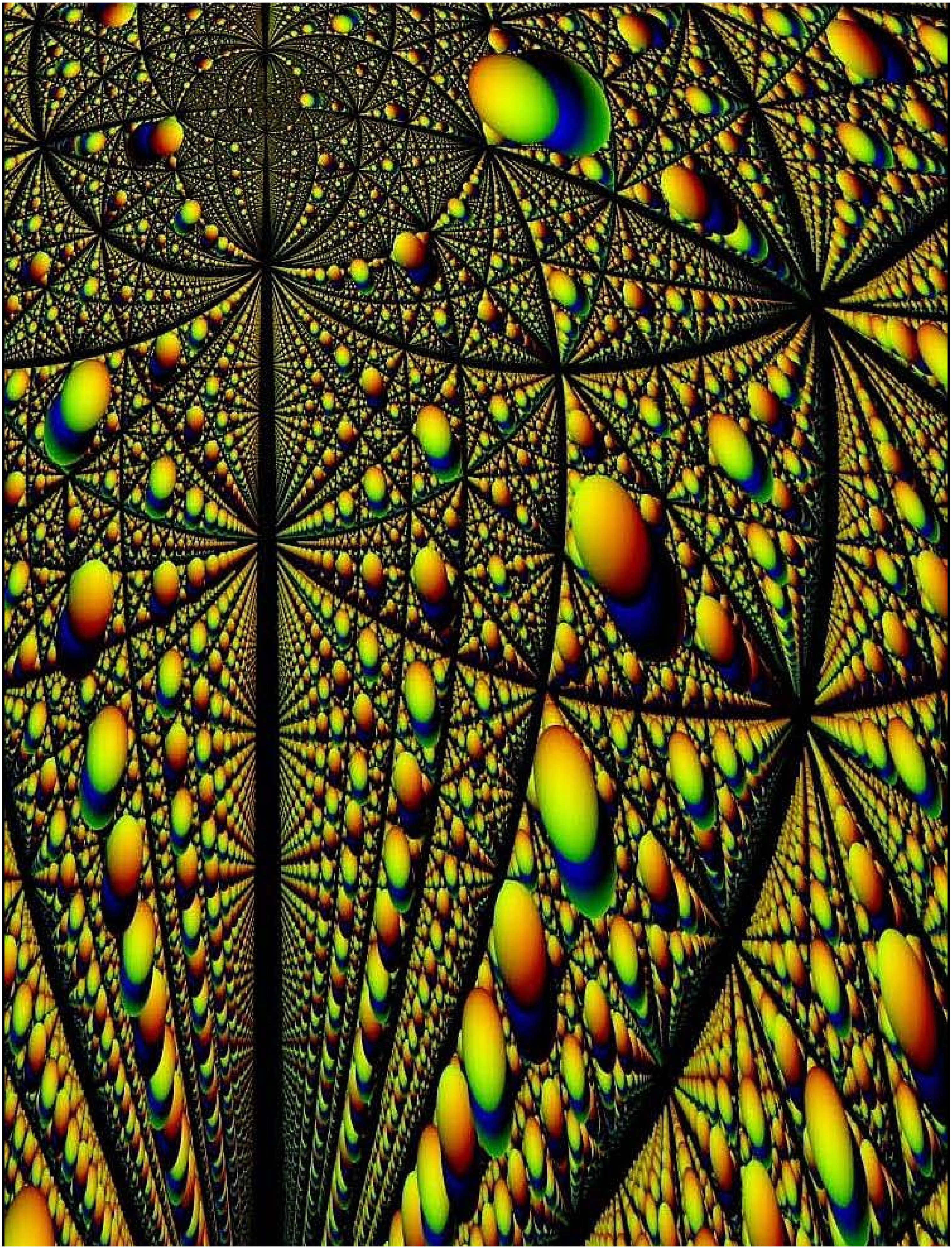
> Once you receive your RoboRoach in the mail, follow our online surgery instructions and videos and you will soon be on your way to becoming an expert in neural interfaces. After about 2-7 days, the stimulation stops working altogether, so you can clip the wires and retire the cockroach to your breeder colony to spend the rest of its days making more cockroaches for you and eating your lettuce.
>
> **Technical Specs**
>
> 1x Free iOS or Android 4.3+ application for remote control
> 1x Bluetooth Roboroach backpack control unit
> 1x 1632 RoboRoach Battery
> 3x Electrode Sets (to implant 3 Roaches)
>
> **View our RoboRoach Ethics Statement**
>
> Backyard Brains has developed ethical guidelines for all our products. You can read more in our statement regarding our use of insect for experiments at: http://ethics.backyardbrains.com

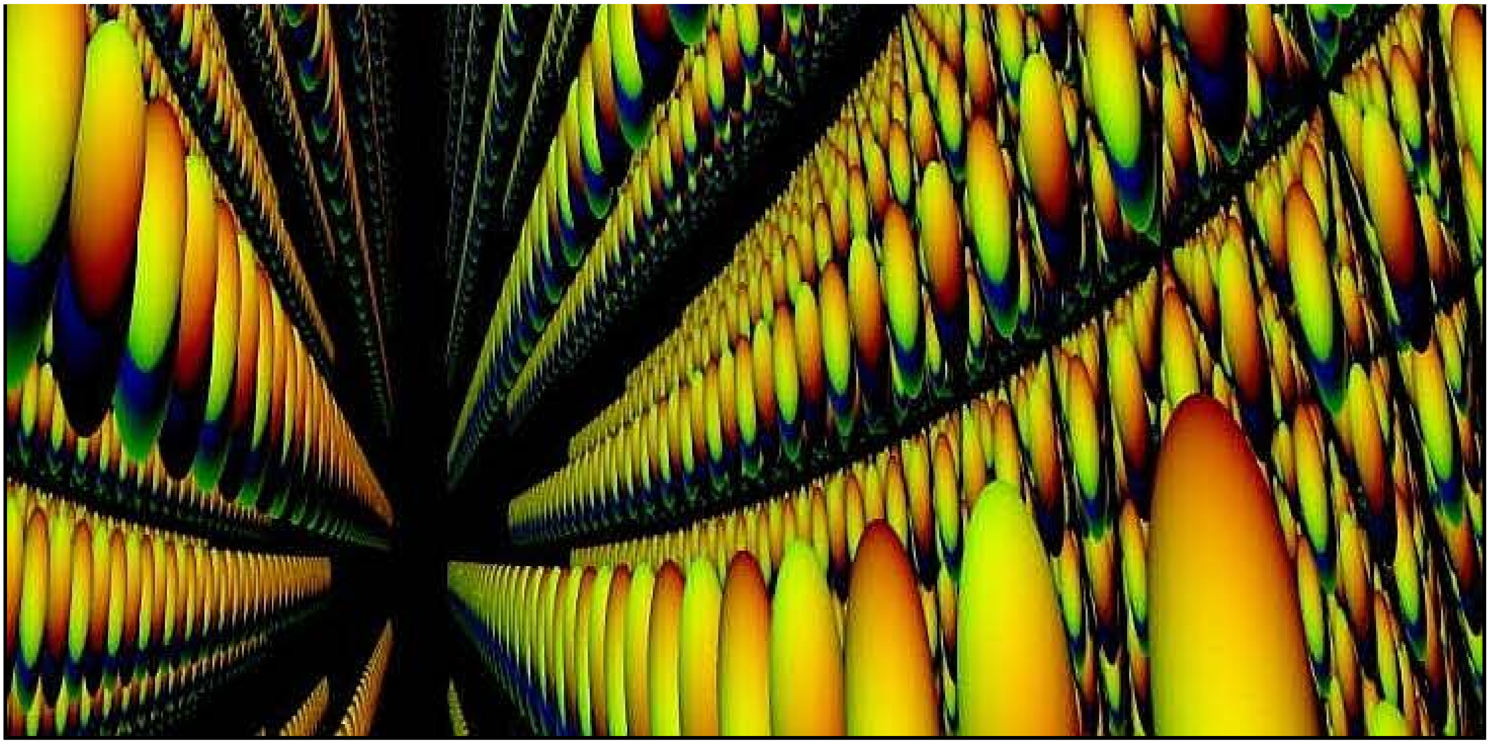
I feel ethical qualms about taking away the autonomy of an animal this way, and their ethics statement doesn't really address that. This is the closest they come:

> **Criticism:** Modifying a living creature to make a toy is wrong.
>
> The RoboRoach circuit is not a toy. This new bluetooth version is a powerful low-cost tool for studying neural circuits, allowing for students to make discoveries. High school students in New York, for example, have discovered random stimulation causes much slower adaptation times. We have scientist and high school educator colleagues who are mentoring students in novel behavioral experiments using the RoboRoach circuit. Some highlights will be posted on our website soon. By focusing on the question of whether the RoboRoach is a "toy", they dodge the harder question of when it's okay to override the nervous system of an animal and make it do what you want. Perhaps feeling a bit nervous about this, some of the cyborg roach developers say they want to use it as a "rescue robot" that can crawl around and hear people trapped under collapsed buildings. I think most people would say this is okay, at least if it actually works.
>
> For a critical view on the ethics, see:
>
> - Marc Bekoff, 'RoboRoach' is bad news in so many ways, *LiveScience*, October 30, 2013.
> - Steve Williams, The do-it-yourself cyborg cockroach: educational or cruel?, *Care2*, October 14, 2013.
>
> For more on how to actually make a RoboRoach, go here:
>
> - Amanda Schupak, How to build your own cockroach cyborg, *Popular Science*, December 6, 2012.
>
> **April 22, 2015**

The Insanity of Infinite Reflections







This picture by John Valentine shows a ball inside a mirrored spheroid... together with all its reflections! The real ball is at lower right. The rest are reflections. They form crazy patterns — the kind of thing mathematicians think about when they can't sleep at night.

The ball is lit from three directions with soft red, green, and blue lights, so we can see things more clearly. The view simulates an ultra-wide-angle camera. For more details and other views, go here:

- John Baez, Sphere in mirrored spheroid, *Visual Insight*, April 15, 2015.

You can get John Valentine's really big version here. This is 16384 × 16384 pixels and about 16 megabytes. If you know a nice way to display such a big image online, which makes it easy to zoom in on pieces, please try it and let me know!

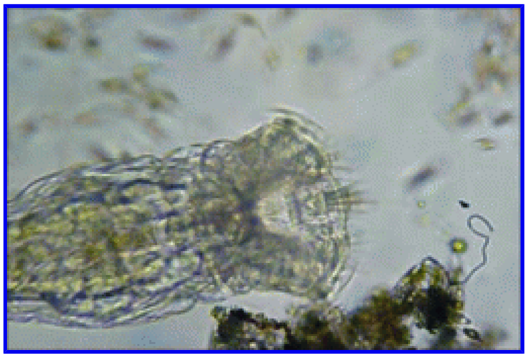
Puzzle 1: What creates the black 'zone of invisibility', and the fractal hexagonal patterns near the zone of invisibility?

I don't really know the answer in detail — this could be a great math project. But I should say that the black regions arise from the fact that the ray-tracing program only allows for 256 reflections; they would get smaller if this number were increased.

I've watched a number of movies where the climactic final scene involves people fighting inside a hall of mirrors, where it's hard to tell who is real and who is a reflection. Orson Welles' 1947 classic Lady from Shanghai may be the first - if you haven't seen that, you should definitely watch it. Another that stands out is Bruce Lee's Enter the Dragon.

Puzzle 2: What other movies or stories do you know involving this theme?

April 24, 2015



A rotifer is an animal that lives in water and sweeps food into its mouth with tiny hairs. There are many kinds, most less than a millimeter in length. They can eat anything smaller than their head.

The toughest are the bdelloid rotifers. These can survive being completely dried out for up to 9 years! When they dry out, they sometimes crack. Even their DNA can crack... but when they get wet, they come back to life!

Thanks to this strange lifestyle, their DNA gets mixed with other DNA. *Up to 10% of their active genes come from bacteria, fungi and algae!*

Scientists have found DNA from 500 different species in the genes of a rotifer from Australia. "It's a genetic mosaic. It takes pieces of DNA from all over the place," said one of the study's authors. "Its biochemistry is a mosaic in the same way. It's a real mishmash of activities."

Perhaps because of this, bdelloid rotifers don't bother to have sex.

Their ability to survive dry conditions makes them great at living in desert lakes and mud puddles that dry up. But they also use this ability to beat some parasites. When they dry out, the parasites die... but the rotifers survive!

On top of all this, bdelloid rotifers can survive high doses of radiation. I think this is just a side-effect of having really good genetic repair mechanisms. In fact, it turns out that many organisms able to endure lots of radiation are able to endure dehydration.

Puzzle 1: What does 'bdelloid' mean?

Puzzle 2: What other words begin with 'bd'... and why?

For some answers to the puzzles, read the comments to my G+ post. Here's the paper that discovered that 10% of active genes and *40% of all enzyme activity* in some bdelloid rotifers involve foreign DNA:

- Chiara Boschetti, Adrian Carr, Alastair Crisp, Isobel Eyres, Yuan Wang-Koh, Esther Lubzens, Timothy G. Barraclough, Gos Micklem and Alan Tunnacliffe, Biochemical diversification through foreign gene expression in bdelloid rotifers, *PLOS Genetics*, November 15, 2012.

The animated gif is from here:

- Rotifer with cilia on corona present, *Merismo Microscopy*, February 23, 2013.

For my May 2015 diary, go here.

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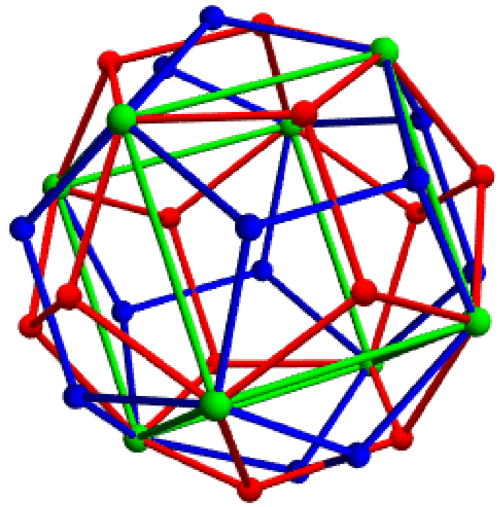
baez@math.removethis.ucr.andthis.edu

For my April 2015 diary, go here.

Diary — May 2015

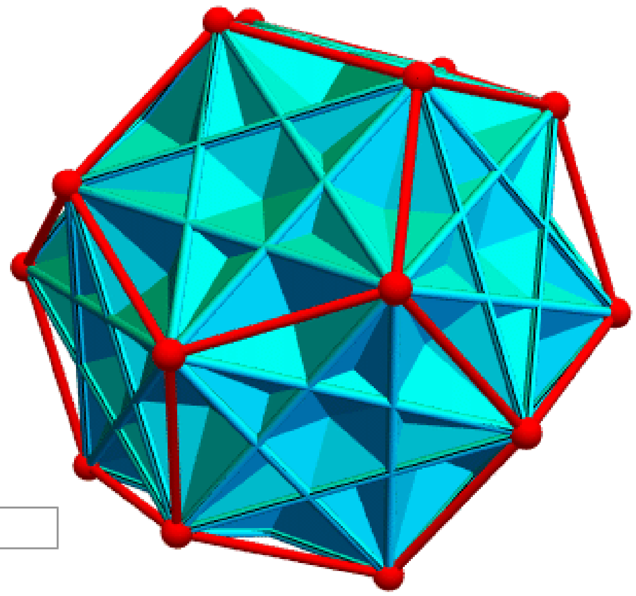
John Baez

May 1, 2015



Here Greg Egan has drawn two regular dodecahedra, in red and blue. They share some corners—and these are the corners of a cube, shown in green!

I learned some cool facts about this from Adrian Ocneanu when I was at Penn State. First some easy stuff. You can take some corners of a regular dodecahedron and make them into the corners of a cube. But not every symmetry of the cube is a symmetry of the dodecahedron! If you give the cube a 90° rotation around any face, you get a new dodecahedron. Check it out: doing this rotation switches the red and green dodecahedra. These are called twin dodecahedra.



But there are actually 5 different ways to take a regular dodecahedron and make them into the corners of a cube, as shown here. And each one gives your dodecahedron a different twin! So, a dodecahedron actually has 5 twins.

But here's the cool part. Suppose you take one of these twins. It, too, will have 5 twins. One of these will be the dodecahedron you started with. But the other 4 will be new dodecahedra: that is, dodecahedra rotated in new ways.

How many different dodecahedra can you get by continuing to take twins? Infinitely many!

In fact, we can draw a graph—a thing with dots and edges—that explains what's going on. Start with a dot for our original dodecahedron. Draw dots for all the dodecahedra you can get by repeatedly taking twins. Connect two dots with an edge if and only if they are twins of each other.

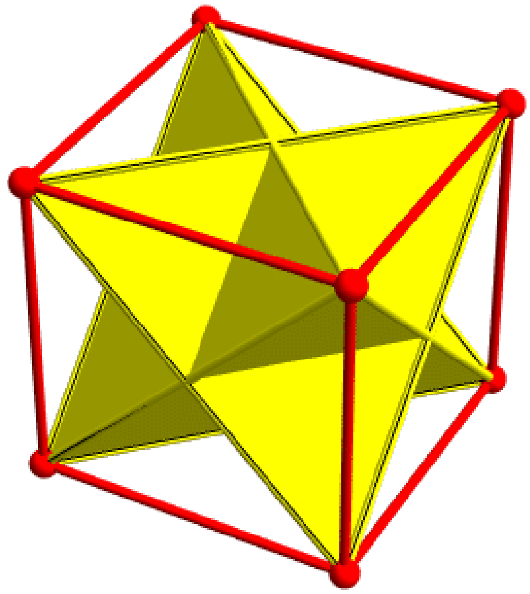
The resulting graph is a tree: in other words, it has no loops in it! If you start at your original dodecahedron, and keep walking along edges of this graph by taking twins, you'll never get back to where you started except by undoing all your steps.

Ocneanu sketched the proof to me, and I reconstructed the rest with a lot of help from Greg Egan, Ian Agol and others:

- Twin dodecahedra, *Visual Insight*, May 1, 2015.

It's part of an elaborate and beautiful story which also involves the golden ratio, the quaternions, and 4-dimensional shapes like the 4-simplex, which has 5 tetrahedral faces, and the 600-cell, which has 600 tetrahedral faces!

Here are some puzzles.



You can choose some corners of a cube and make them into the corners of a regular tetrahedron. You can fit 2 tetrahedra in the cube this way. These are a bit like the 5 cubes in the dodecahedron, but there's a big difference.

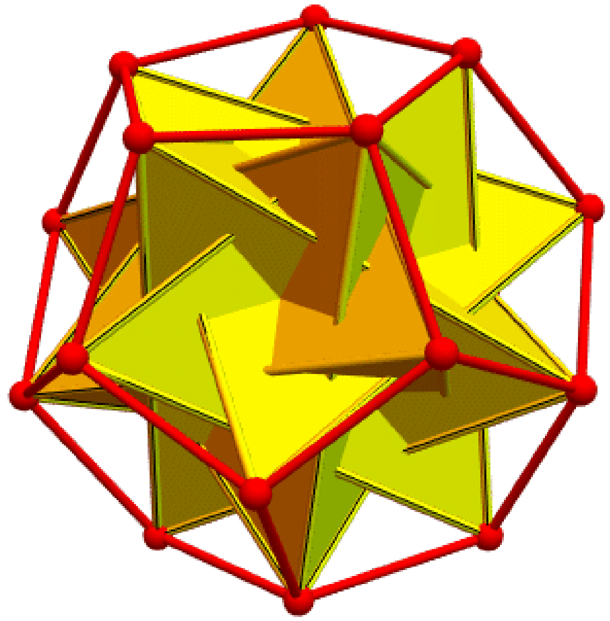
Here's the difference. In the first case, every symmetry of the tetrahedron is a symmetry of the cube it's in. But in the second case not every symmetry of the cube is a symmetry of the dodecahedron. That's why we get 'twin dodecahedra' but not 'twin cubes'.

Puzzle 1: If you inscribe a tetrahedron in a cube and then inscribe the cube in a dodecahedron, is every symmetry of the tetrahedron a symmetry of the dodecahedron?

Puzzle 2: How many ways are there to inscribe a tetrahedron in a dodecahedron? More precisely: how many ways are there to choose some corners of a regular dodecahedron and have them be the corners of a regular tetrahedron?

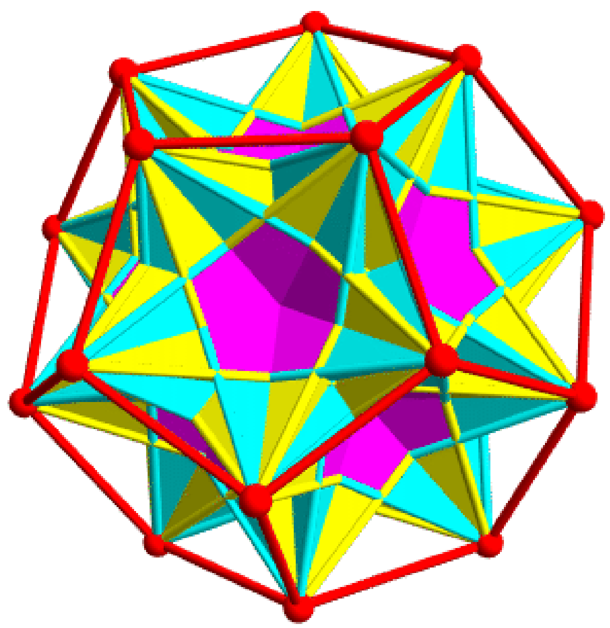
You can see the answers to the these in comments to my G+ post. The second one is also answered in my next diary entry!

May 2, 2015



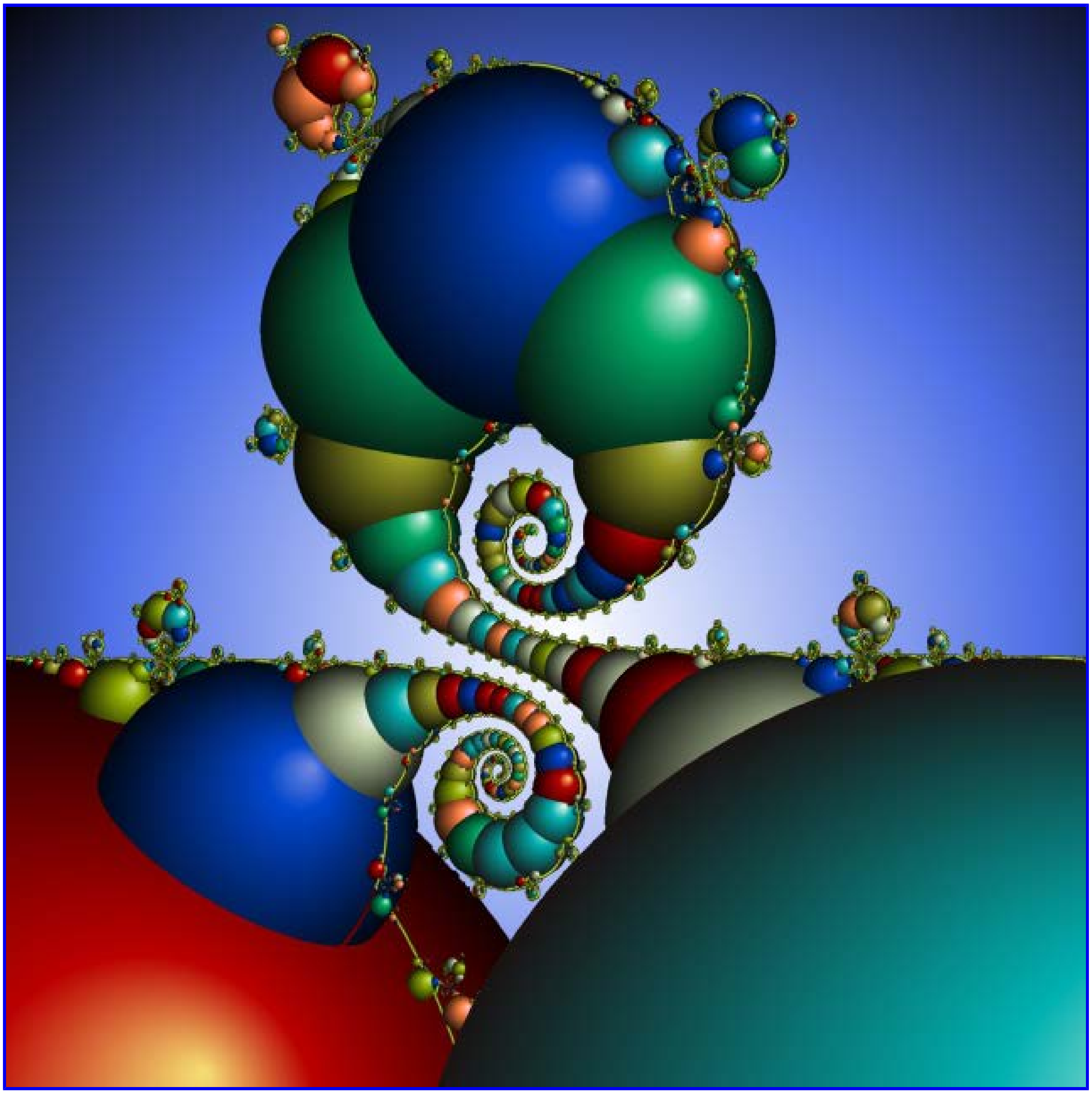
Here Greg Egan has drawn a dodecahedron with 5 tetrahedra in it. This picture is 'left-handed': if you look at where the 5 tetrahedra meet, you'll see they swirl counterclockwise as you go out! If you view this thing in a mirror you'll get a right-handed version.

Putting them together, you get a dodecahedron with 10 tetrahedra in it:



The two kinds of tetrahedra are colored yellow and cyan. Regions belonging to both are colored magenta. It's pretty — but it's hard to see the tetrahedra, because they overlap a lot!

May 3, 2015



Jos Leys blends mathematics and art in a delightful way. You don't need to know math to enjoy this picture. It's a whimsical and mysterious landscape. The bright colors make it clownish, but the shadows make it a bit eerie: the sun is setting, and who knows what happens here at night! You can see more here:

- Jos Leys, 3d Kleinian groups page 1: the first 3d views of limit sets of Kleinian groups.

On the other hand, from the title of this gallery you can see there's math here. And trying to understand this math will lead you on quite a journey. Let me sketch it here... I apologize for going rather fast.

A Kleinian group is a discrete subgroup of the group called $\mathrm{PSL}(2, \mathrm{C})$. This group shows up in many ways in math and physics.

Physicists call it the Lorentz group: it's the group generated by rotations and Lorentz transformations, which acts as symmetries in special relativity.

In math, it's called the group of Möbius transformations or 'fractional linear transformations'. Those are transformations like this:

$$\frac{az + b}{\quad}$$

z

$cz + d$

where z is a complex number and so are a, b, c, d. These can be seen as transformations of the [Riemann sphere]: the complex plane together with a point at infinity. They are, in fact, precisely all the conformal transformations of the Riemann sphere: the transformations that preserve angles.

But this group PSL(2, C) also acts as conformal transformations of a 3-dimensional ball whose boundary is the Riemann sphere! And that's important for understanding this picture.

(In physics, this ball is the set of 'mixed states' for a spin-1/2 particle, and the sphere, its boundary, consists of the 'pure states'. Lorentz transformations act on the mixed states, and they act on the pure states. But you don't need to know this stuff.)

If you take any point inside the ball and act on it by all the elements in a Kleinian group — a discrete subgroup of PSL(2, C) — you'll get a set S of points in the ball. The set of points in the Riemann sphere that you can approach by a sequence of points in S is called a **limit set** of the Kleinian group. And this set can look really cool!

In these pictures, Jos Leys has systematically but rather artificially taken these cool-looking subsets of the Riemann sphere and puffed them up into 3-dimensional spaces: puffing a circle into a sphere, and so on. This makes the picture nicer, but doesn't have a deep mathematical meaning.

Later, Jos Leys took a deeper approach, using quaternions to make limit sets that are truly 3-dimensional. You can seem some here:

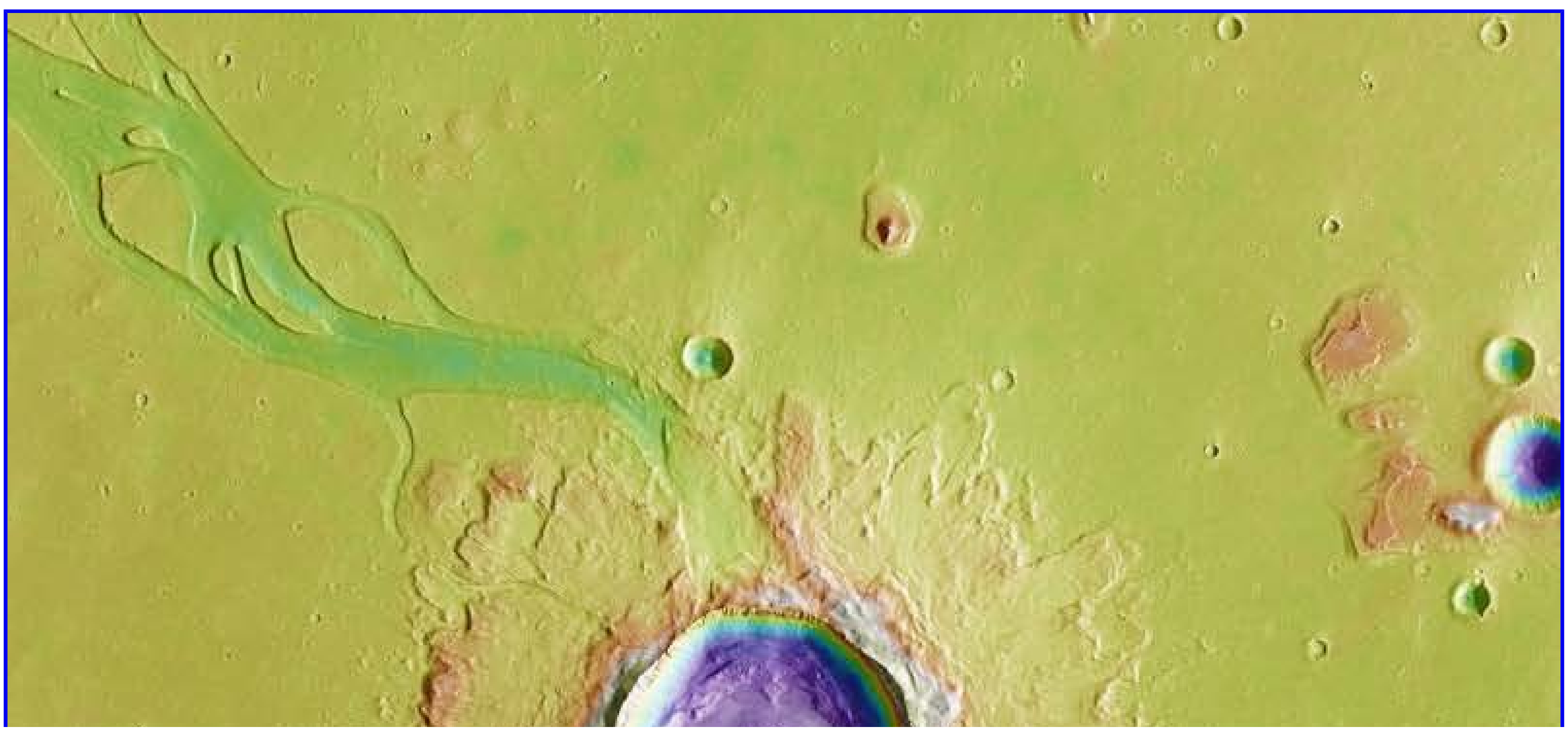
- Jos Leys, True 3d Kleinian groups.

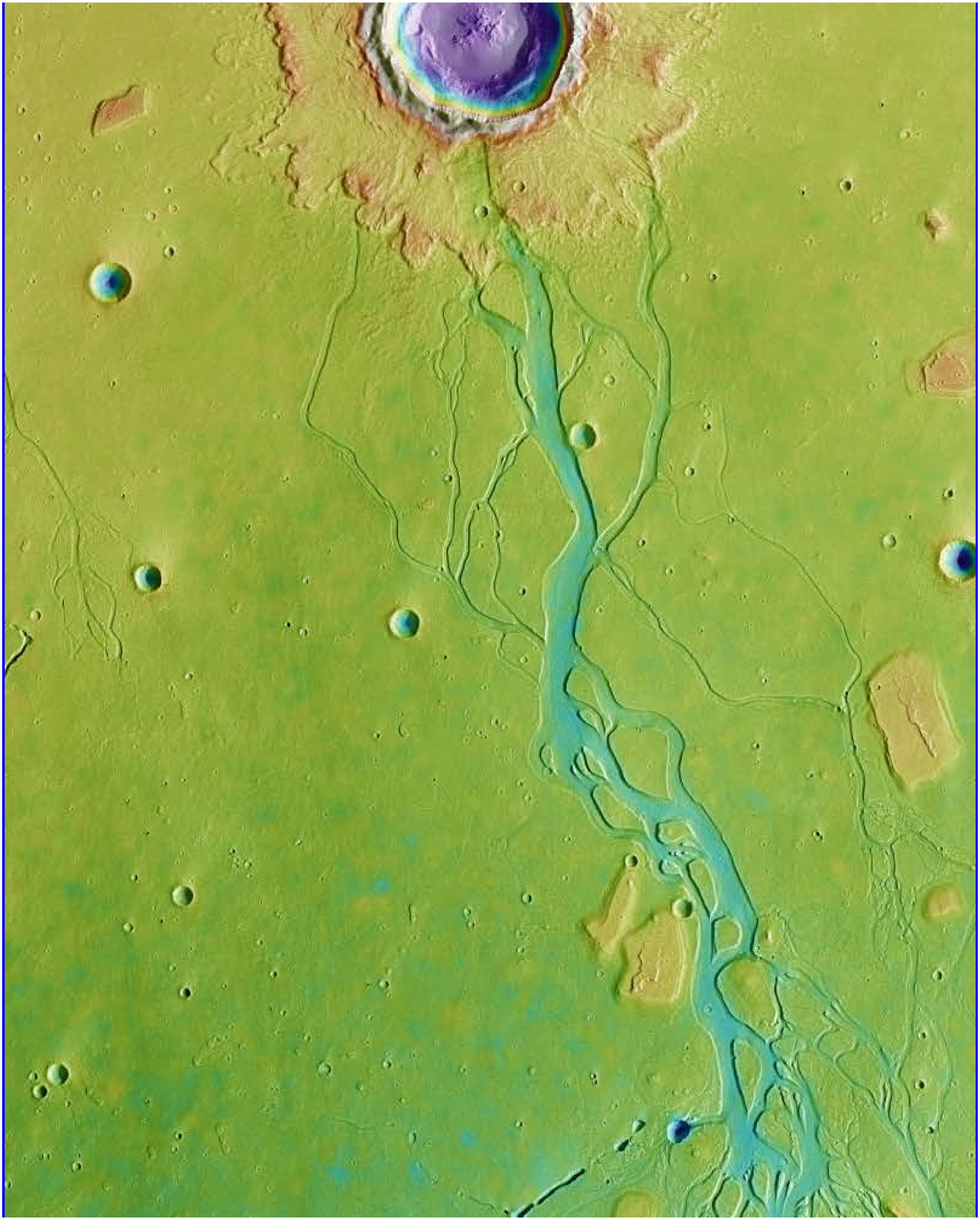
They have a very different look.

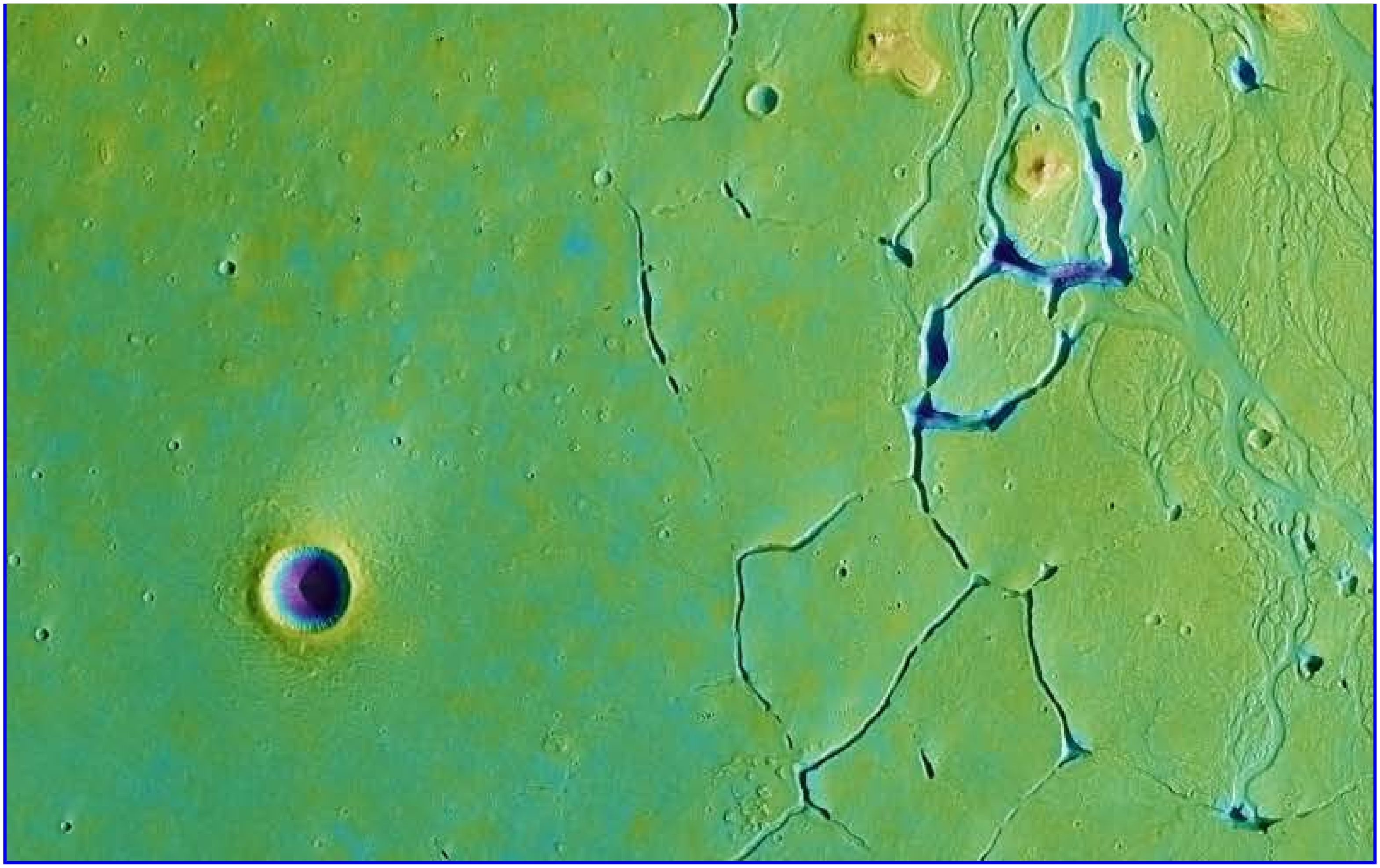
Puzzle: if you put together everything I said, you'll get a physics interpretation of the limit set of a Kleinian group in terms of states of a spin-1/2 particle. What is it?

For an answer, read the comments to my G+ post.

May 4, 2015







On Mars, an asteroid impact can cause a flood!

This is a place called Hephaestus Fossae, on the northern hemisphere of Mars. The image has been colored to show the elevation: green and yellow shades represent shallow ground, while blue and purple stand for deep depressions, as much as 4 kilometers deep.

You can see a few dozen impact craters, some small and some big, up to 20 kilometers across. But I'm sure you instantly noticed the cool part: the long and intricate canyons and riverbeds. These were created by the same impact that made the largest crater!

When a comet or an asteroid crashes at high speed into a planet, the collision dramatically heats up the surface at the impact site. In the case of the large crater seen in this image, the heat melted the soil — a mixture of rock, dust and also, hidden deep down, water ice — resulting in a massive flood. And before drying up, this hot mud carved a complex pattern of channels while flowing across the planet's surface!

The melted rock-ice mixture also made the debris blankets surrounding the largest crater. Since there aren't similar structures near the small craters in this image, scientists believe that only the most powerful impacts were able to dig deep enough to release part of the frozen reservoir of water lying beneath the surface.

Why is it called 'Hephaestus Fossae'? Hephaestus was the Greek god of fire. Fossae are channels or canyons. So it's a good name.

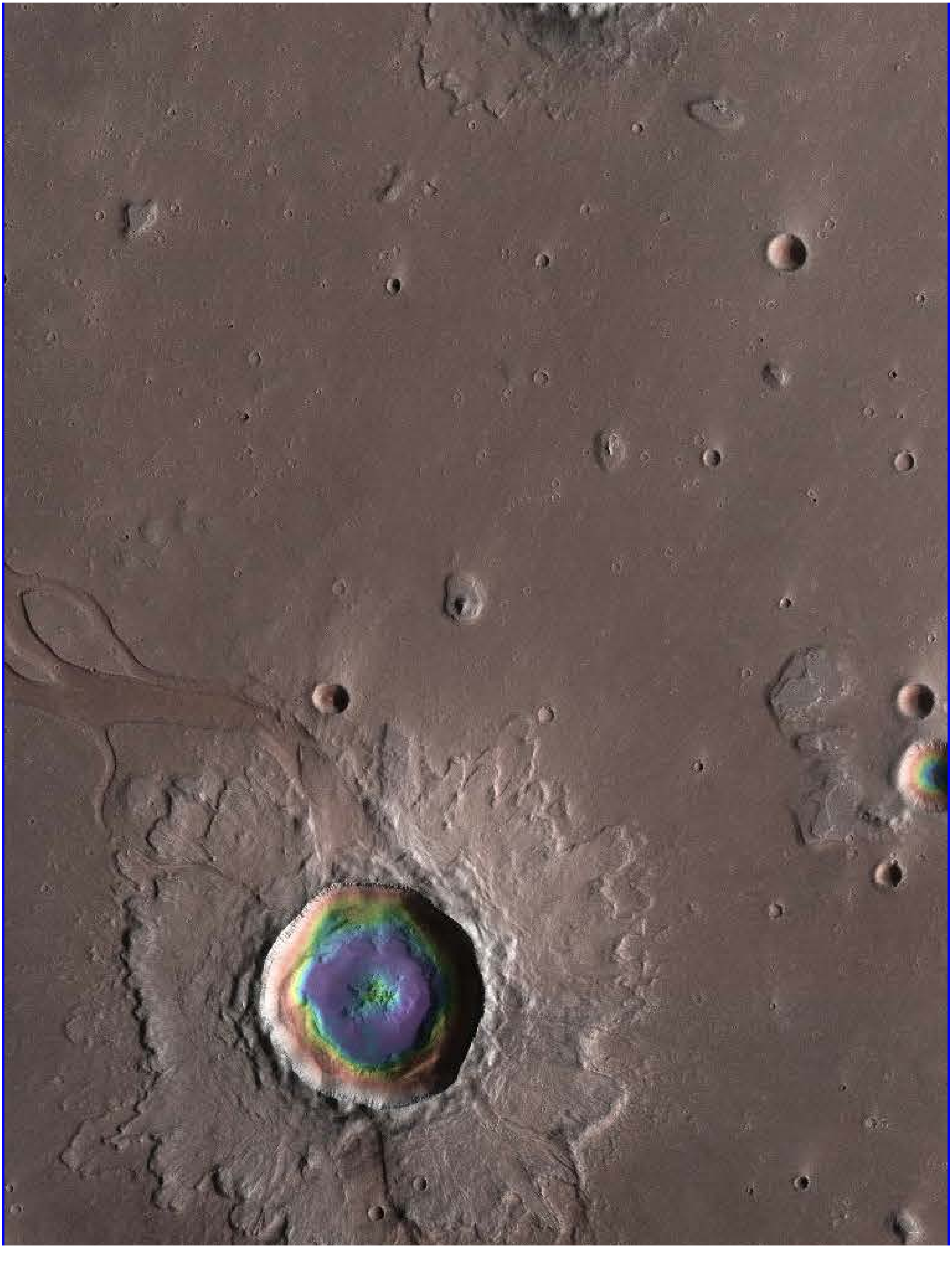
Puzzle: about when did this large impact occur?

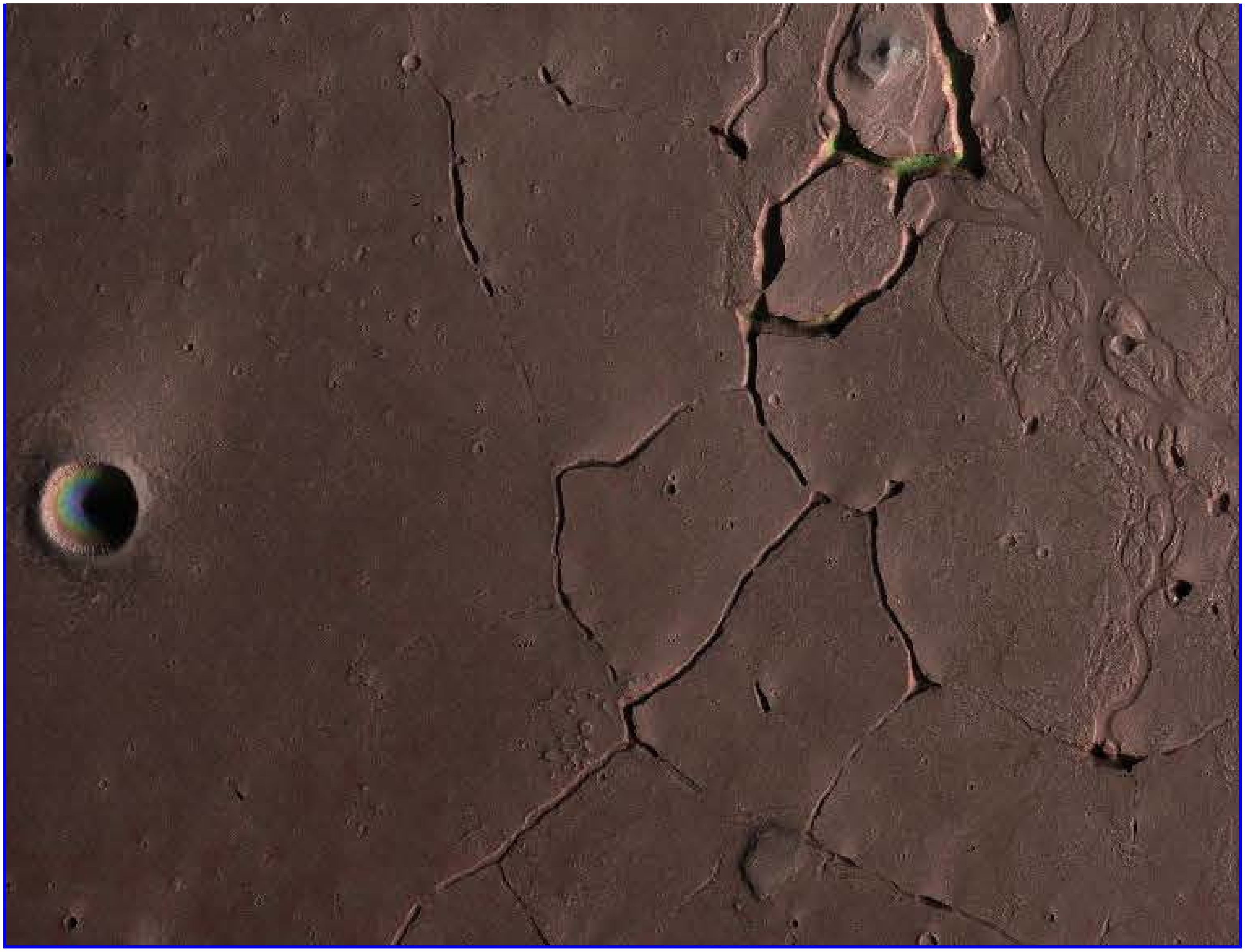
I don't know!

This picture was taken by the high-resolution stereo camera on ESA.s Mars Express orbiter on 28 December 2007, and my post is paraphrased from this article:

- European Space Agency, The flood after the impact, October 2, 2014.

Here's another view. Cool colors represent depressions; warmer colors are higher areas. Purple indicates places about 4200 meters deep:





May 5, 2015

This article is so cool I'm just going to quote some:

> **[In 1901, you could pay 50 cents to ride an airship to the Moon](#)**
> Ron Miller
>
> The passengers wait eagerly in the ornate lobby of the enormous spaceport. Soon, a signal indicates that their spaceship is ready for boarding. As they wait, special displays instruct them about how their spaceship functions and what to expect once they leave Earth's atmosphere. Aboard the giant spacecraft — as luxuriously appointed as any yacht — they are soon on their way to a vacation on the Moon.
>
> No, this isn't a vision of the future of space tourism. It's what happened in 1901, when people could pay a princely half dollar for a ticket to ride into space.
>
> [...]
>
> Thompson spared no expense in creating the illusion of a trip to the Moon. To house his show, he erected an eighty-foot-high, 40,000-square-foot building that for sheer opulence put European opera houses to shame. It cost a staggering $84,000 to construct... at a time when a comfortable home could be built for $2000.

> For fifty cents — twice the price of any other attraction on the midway, such as the ever-popular "Upside-Down House" — customers of "Thompson's Aerial Navigation Company" took a trip to the moon on a thirty-seat spaceship named "Luna". The spaceship resembled a cross between a dirigible and an excursion steamer, with the addition of enormous red canvas wings that flapped like a bird's. The wings were worked by a system of pulleys and the sensation of wind was created by hidden fans. A series of moving canvas backdrops provided the effect of clouds passing by and the earth dropping into the distance. Lighting and sound effects added to the illusion.
>
> [...]
>
> Every half hour, at the sound of a gong and the rattle of anchor chain, the "Luna" — "a fine steel airship of the latest pattern", according to one newspaper — rocked from side to side and then rose into the sky under the power of its beating wings. The passengers, sitting on steamer chairs, see clouds floating by, then a model of Buffalo far below, complete with the exposition itself and its hundreds of blinking lights. The city soon falls into the distance as the entire planet earth comes into view. Soon, the ship is surrounded the twinkling stars of outer space. After surviving a terrific — and spectacular — electrical storm the "Luna" and its passengers sets down in a lunar crater.

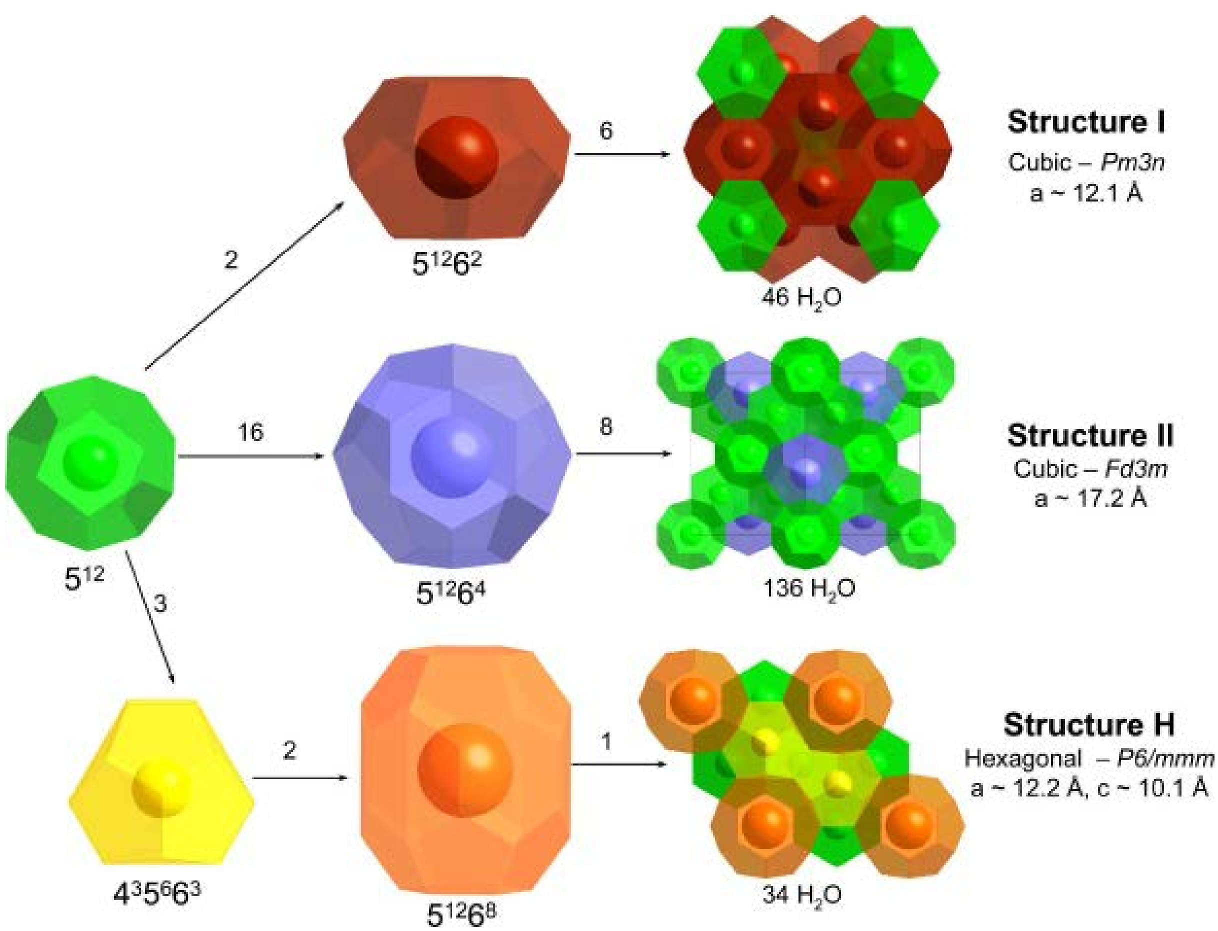
Read the whole thing here, and look at pictures:

- Ron Miller, In 1901, you could pay 50 cents to ride an airship to the Moon, *io9*, May 31, 2012.

Thanks to Matt McIrvin for pointing it out!

May 6, 2015

The architecture of water



Water is fascinating, for many reasons. It takes more energy to heat than most substances. It's one of the few substances that expands when it freezes. It forms complicated patterns in its liquid state, which are just beginning to be understood. There are at least 18 kinds of ice, which exist at different temperatures and pressures. Snowflakes are endlessly subtle.

And ice can form cages that trap other molecules! Here you see the 3 main kinds.

They're called **clathrate hydrates**. There's a lot under the sea beds near the north and south pole - they contain huge amounts of methane. At some moments in the Earth's history they may have erupted explosively, causing rapid global warming.

But let's focus on the fun part: the geometry! Each of the 3 types of clathrate hydrates is an architectural masterpiece.

'Type I' consists of water molecules arranged in two types of cages: small and large. The small cage, shown in green, is dodecahedron. It's not a regular dodecahedron, but it still has 12 pentagonal sides. The large cage, shown in red, has 12 pentagons and 2 hexagons. The two kinds of cage fit together into a repeating pattern where each 'unit cell' — each block in the pattern — has 46 water molecules.

Puzzle 1: This pattern is called the 'Weaire–Phelan structure' Why is it famous, and what does it have to do with the 2008 Olympics?

You can see little balls in the cages. These stand for molecules that can get trapped in the cages. They're politely called 'guests'. The type I clathrate often holds carbon dioxide or methane as a guest.

'Type II' is again made of two types of cages: small and large. The small cage is again a dodecahedron. The large cage, shown in blue, has 12 pentagons and 6 hexagons. These fit together to form a unit cell with 136 water molecules.

The type II clathrate tends to hold oxygen or nitrogen as a guest.

'Type H' is the rarest and most complicated kind of clathrate hydrate. The 'H' stands for 'hexagonal', because it has a hexagonal crystal structure: the other two are cubic.

It's built from three types of cages: small, medium and huge. The small cage is again a dodecahedron, shown in green. The medium cage — shown in yellow — has 3 squares, 6 pentagons and 3 hexagons as faces. The huge cage — shown in orange — has 12 pentagons and 8 hexagons. The cages fit together to form a unit cell with 34 water molecules.

The type H clathrate is only possible when there are two different guest gas molecules — one small and one very large, like butane — to make it stable. People think there are lots of type H clathrates in the Gulf of Mexico, where there are lots of heavy hydrocarbons in the sea bottom.

Puzzle 2: how many cages of each kind are there in the type I clathrate hydrate?

Puzzle 3: how many cages of each kind are there in the type II?

Puzzle 4: how many cages of each kind are there in the type H?

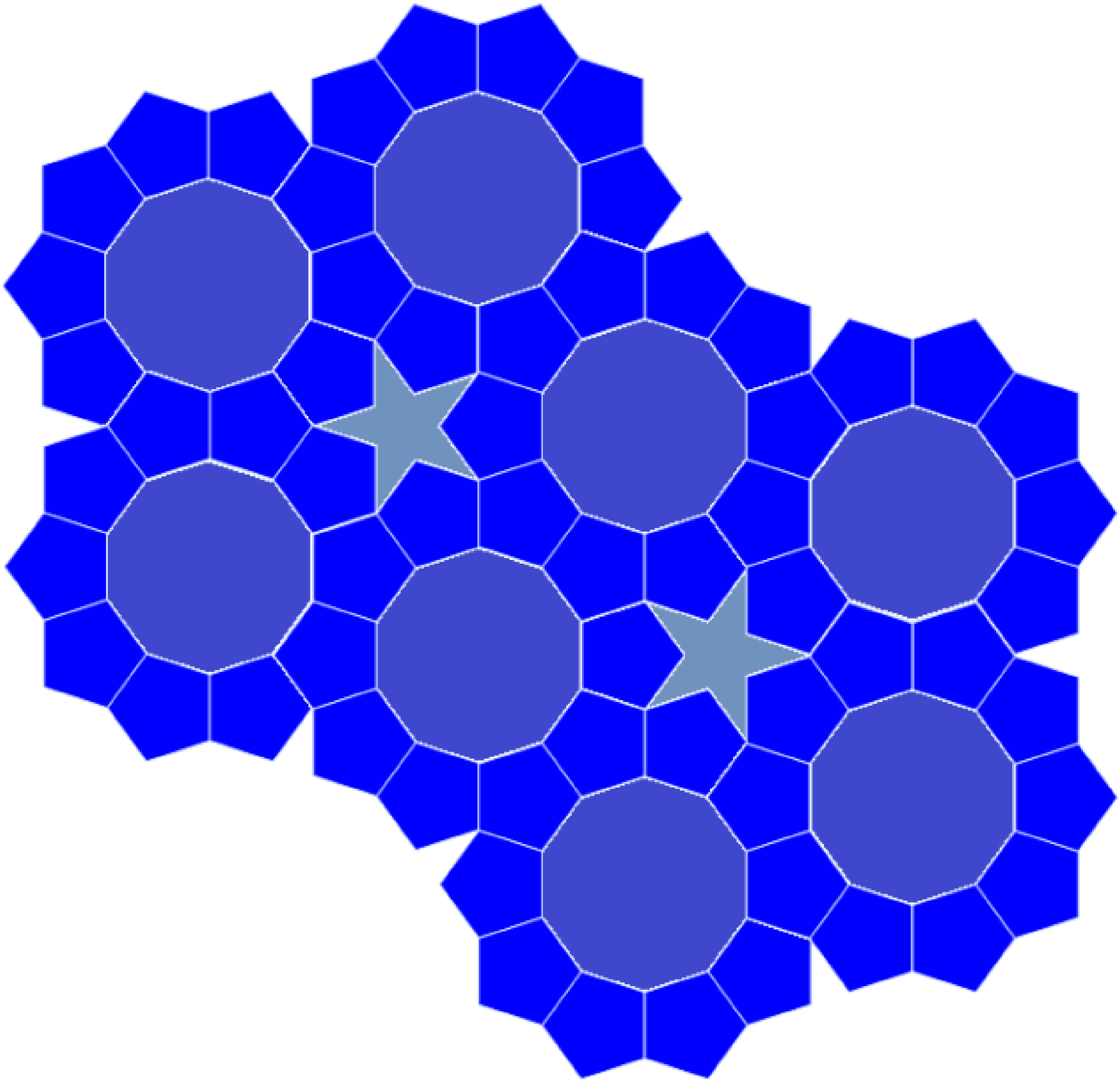
These last puzzles are easier than they sound. But here's one that's a bit different:

Puzzle 5: the medium cage in the type H clathrate — shown in yellow — has 3 squares, 6 pentagons and 3 hexagons as faces. Which of these numbers are adjustable? For example: could we have a convex polyhedron with a different number of squares, but the same number of pentagons and hexagons?

The picture is from here:

- Timothy A. Strobel, Keith C. Hester, Carolyn A. Koh, Amadeu K. Sum and E. Dendy Sloan Jr., Properties of the clathrates of hydrogen and developments in their applicability for hydrogen storage, *Chemical Physics Letters* **478** (2009), 97–109.

May 9, 2015

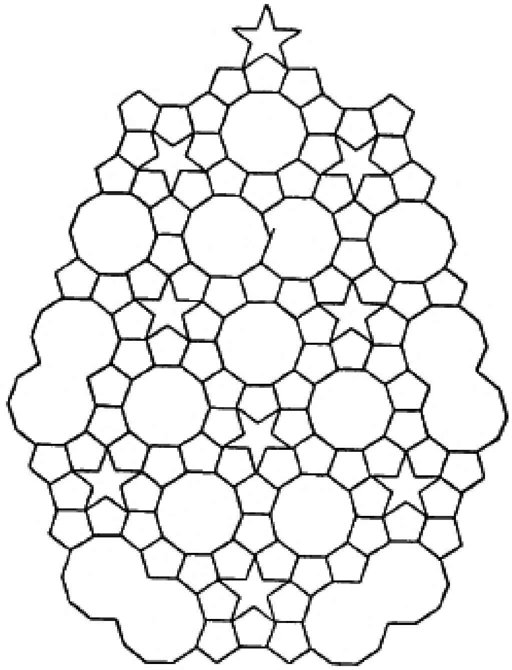


Kepler, the guy who discovered that planets go in ellipses around the Sun, was in love with geometry. Among other things, he tried to figure out how to tile the plane with regular pentagons (dark blue) and decagons (blue-gray). They fit nicely at a corner... but he couldn't get it to work.

Then he discovered he could do better if he also used 5-pointed stars!

Can you tile the whole plane with these three shapes? No! The picture here is very tempting... but if you continue you quickly run into trouble. It's an impossible dream.

However, Kepler figured out that he could go on forever if he also used overlapping decagons, which he called 'monsters'. Look at this picture he drew:



If he had worked even harder, he might have found the Penrose tilings, or similar things discovered by Islamic tiling artists. Read the whole story here:

- Craig Kaplan, The trouble with five, *Plus Magazine*, December 1, 2007.

How did Kepler fall in love with geometry? He actually started as a theologian. Let me quote the story as told in the wonderful blog *The Renaissance Mathematicus*:

> Kepler was born into a family that had known better times, his mother was an innkeeper and his father was a mercenary. Under normal circumstances he probably would not have expected to receive much in the way of education but the local feudal ruler was quite advanced in his way and believed in providing financial support for deserving scholars. Kepler whose intelligence was obvious from an early age won scholarships to school and to the University of Tübingen where he had the luck to study under Michael Mästlin one of the very few convinced Copernican in the later part of the 16th century. Having completed his BA Kepler went on to do a master degree in theology as he was a very devote believer and wished to become a theologian. Recognising his mathematical talents and realising that his religious views were dangerously heterodox, they would cause him much trouble later in life, his teacher, Mästlin, decided it would be wiser to send him off to work as a school maths teacher in the Austrian province.
>
> Although obeying his superiors and heading off to Graz to teach Protestant school boys the joys of Euclid, Kepler was far from happy as he saw his purpose in life in serving his God and not Urania (the Greek muse of astronomy). After having made the discovery that I will shortly describe Kepler found a compromise between his desire to serve God and his activities in astronomy. In a letter to Mästlin in 1595 he wrote:
>
> I am in a hurry to publish, dearest teacher, but not for my benefit. I am devoting my effort so that these things can be published as quickly as possible for the glory of God, who wants to be recognised from the Book of Nature. Just as I pledged myself to God, so my intention remains. I wanted to be a theologian, and for a while I was anguished. But, now see how God is also glorified in astronomy, through my efforts.
>
> So what was the process of thought that led to this conversion from a God glorifying theologian to a God glorifying astronomer and what was the discovery that he was so eager to publish? Kepler.s God was a geometer who had created a rational, mathematical universe who wanted his believers to discover the geometrical rules of construction of that universe and reveal them to his glory. Nothing is the universe was pure chance or without meaning everything that God had created had a purpose and a reason and the function of the scientist was to uncover those reasons. In another letter to Mästlin Kepler asked whether:

> you have ever heard or read there to be anything, which devised an explanation for the arrangement of the planets? The Creator undertook nothing without reason. Therefore, there will be reason why Saturn should be nearly twice as high as Jupiter, Mars a little more than the Earth, [the Earth a little more] than Venus and Jupiter, moreover, more than three times as high as Mars.

> The discovery that Kepler made and which started him on his road to the complete reform of astronomy was the answer to both the question as to the distance between the planets and also why there were exactly six of them: as stated above, everything created by God was done for a purpose.
>
> On the 19th July 1595 Kepler was explaining to his students the regular cycle of the conjunctions of Saturn and Jupiter, planetary conjunctions played a central role in astrology. These conjunctions rotating around the ecliptic, the apparent path of the sun around the Earth, created a series of rotating equilateral triangles. Suddenly Kepler realised that the inscribed and circumscribed circles generated by his triangles were in approximately the same ratio as Saturn.s orbit to Jupiter's. Thinking that he had found a solution to the problem of the distances between the planets he tried out various two-dimensional models without success. On the next day a flash of intuition provided him with the required three-dimensional solution, as he wrote to Mästlin:
>
> > I give you the proposition in words just as it came to me and at that very moment: "The Earth is the circle which is the measure of all. Construct a dodecahedron round it. The circle surrounding that will be Mars. Round Mars construct a tetrahedron. The circle surrounding that will be Jupiter. Round Jupiter construct a cube. The circle surrounding it will be Saturn. Now construct an icosahedron inside the Earth. The circle inscribed within that will be Venus. Inside Venus inscribe an octahedron. The circle inscribed inside that will be Mercury."

This model, while approximately true, is now considered completely silly! We no longer think there should be a simple geometrical explanation of why planets in our Solar System have the orbits they do.

So: a genius can have a beautiful idea in a flash of inspiration and it can still be wrong.

But Kepler didn't stop there! He kept working on planetary orbits until he noticed that Mars didn't move in a circle around the Sun. He noticed that it moved in an ellipse! Starting there, he found the correct laws governing planetary motion... which later helped Newton invent classical mechanics.

So it pays to be persistent—but also not get stuck believing your first good idea.

Read more here:

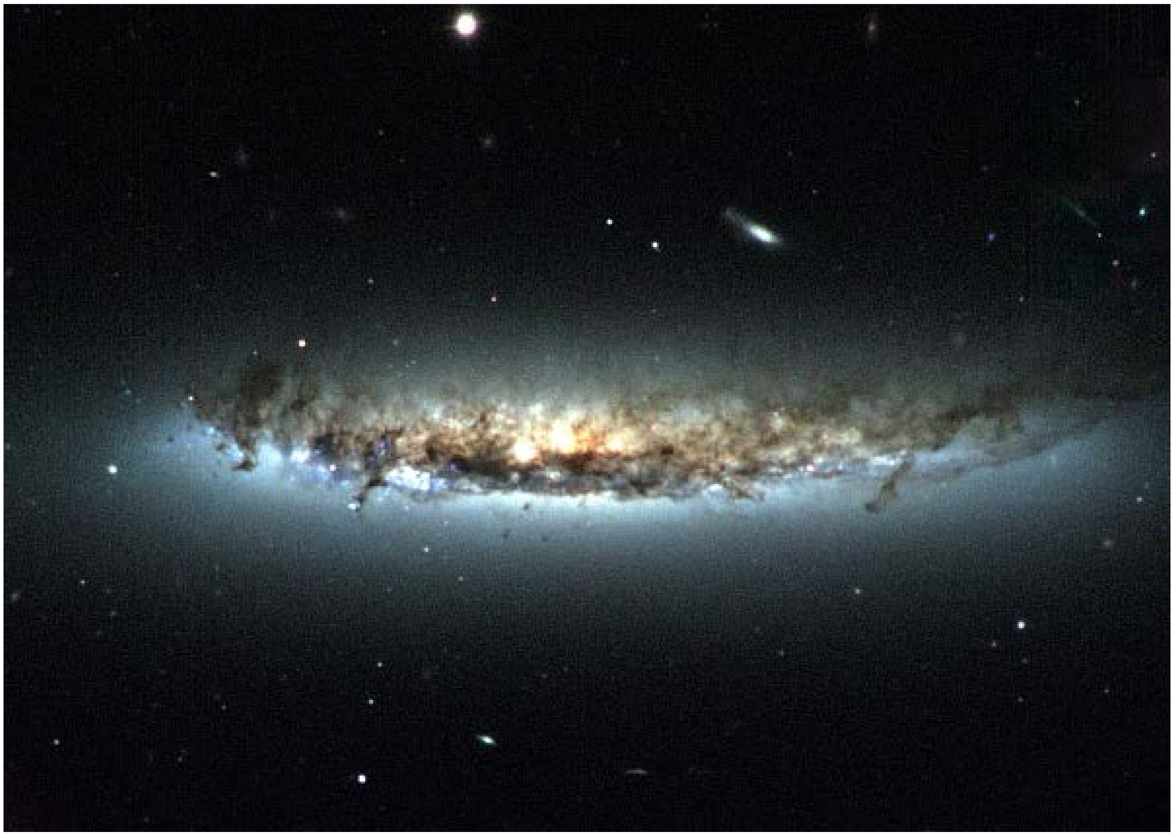
- Thony Christie, Kepler's divine geometry, *The Renaissance Mathematicus*, November 10, 2010.

Puzzle: can you tile the plane with finitely many shapes, each of which has at least the symmetry group of a regular pentagon?

So, regular pentagons and decagons are allowed, and so are regular 5-pointed stars, and many other things... but not Kepler's monsters. The tiling itself does not need to repeat in a periodic way.

May 14, 2015

A galaxy — falling



This galaxy is in trouble! It's falling into a large cluster of galaxies, pulled by their gravity. You can see this in 3 ways:

1. The reddish disk of dust and gas is bent. There aren't many atoms between galaxies, but there are still some. So the galaxy is moving through the wind of integalactic space! And it's having trouble holding onto the loosely bound dust and gas near its edge. They're getting blown away.

2. The blue disk of stars is not bent. It extends beyond the disk of dust and gas, which is where stars are formed. This suggests that the dust and gas is being stripped from the galaxy *after* these stars were formed!

3. Streamers of dust and gas can be seen trailing *behind* the moving galaxy — near the top. On the other hand, the blue stars near the leading edge of the galaxy have no dust and gas left to hide them.

This phenomenon is called 'ram pressure stripping', and it can *kill* a galaxy, shutting down the production of new stars. Here we are seeing it damage the galaxy NGC 4402, which is currently falling into the Virgo cluster — a cluster of galaxies about 65 million light years away.

Apparently there's about 1 atom per cubic centimeter in our galaxy — on average, though some regions are vastly more dense than others. But in the space between galaxies in clusters it's more like 1/1000 of that. Not much! But enough to kill off the formation of new star systems, life, civilizations...

I got most of my information from here:

- Ram pressure stripping, Cosmos - The SAO Encyclopedia of Astronomy.

and I got the picture from here:

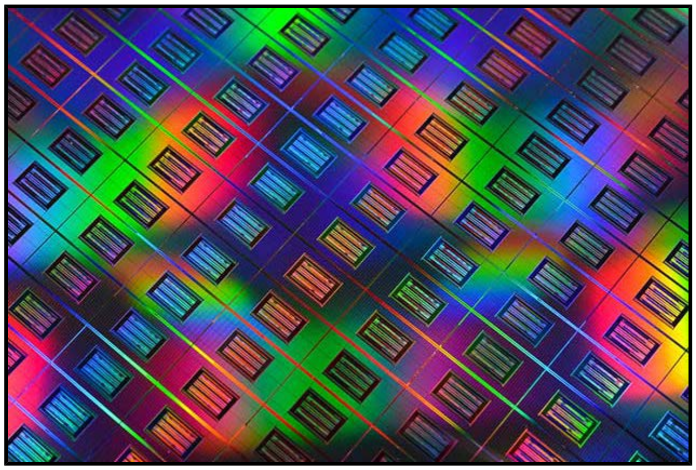
- Spiral galaxy NGC4402, National Optical Astronomy Observatory.

The photo was taken at the WIYN 3.5-meter telescope on Kitt Peak, which is fitted with some 'adaptive optics' to compensate for the jittery motion of the image due to variable atmospheric conditions and telescope vibrations.

It's a bit hard to find figures for the density of the intergalactic medium. I see stuff that says: 1 atom per liter for the intergalactic gas in clusters, 1 atom per cubic meter as the overall average for the whole universe. One fun thing about space is that while it seems like vacuum to us, its density ranges by many orders of magnitude... so it's actually much more varied than, say, the difference between air and solid lead!

Yes, air is about 1 kg/m^3 and lead is about 10,000 kg/m^3, a factor of 10^4. But within our galaxy, the density of the interstellar medium easily ranges between 10^{-4} and 10^6 atoms per cubic centimeter, a factor of 10^{10}. And the average density of the Universe is 10^{-6} atoms per cubic centimeter. So what we naively call 'outer space' is a bunch of vastly different media, whose densities vary by at least a factor of *a trillion!*

May 1, 2015



Hewlett-Packard was once at the cutting edge of technology. Now they make most of their money selling servers, printers, and ink... and business keeps getting worse. They've shed 40,000 employees since 2012. Soon they'll split in two: one company that sells printers and PCs, and one that sells servers and information technology services.

The second company will do something risky but interesting. They're trying to build a new kind of computer that uses chips based on *memristors* rather than transistors, and that uses *optical fibers* rather than wires to communicate between chips. It could make computers much faster and more powerful. But nobody knows if it will really work.

The picture shows memristors on a silicon wafer. But what's a memristor? Quoting the *MIT Technology Review*:

Perfecting the memristor is crucial if HP is to deliver on that striking potential. That work is centered in a small lab, one floor below the offices of HP's founders, where Stanley Williams made a breakthrough about a decade ago.

Williams had joined HP in 1995 after David Packard decided the company should do more basic research. He came to focus on trying to use organic molecules to make smaller, cheaper replacements for silicon transistors. After a few years, he could make devices with the right kind of switchlike behavior by sandwiching molecules called rotaxanes between platinum electrodes. But their performance was maddeningly erratic. It took years more work before Williams realized that the molecules were actually irrelevant and that he had stumbled into a major discovery. The switching effect came from a layer of titanium, used like glue to stick the rotaxane layer to the electrodes. More surprising, versions of the devices built around that material fulfilled a prediction made in 1971 of a completely new kind of basic electronic device. When Leon Chua, a professor at the University of California, Berkeley, predicted the existence of this device, engineering orthodoxy held that all electronic circuits had to be built from just three basic elements: capacitors, resistors, and inductors. Chua calculated that there should be a fourth; it was he who named it the memristor, or resistor with memory. The device's essential property is that its electrical resistance—a measure of how much it inhibits the flow of electrons—can be altered by applying a voltage. That resistance, a kind of memory of the voltage the device experienced in the past, can be used to encode data.

HP's latest manifestation of the component is simple: just a stack of thin films of titanium dioxide a few nanometers thick, sandwiched between two electrodes. Some of the layers in the stack conduct electricity; others are insulators because they are depleted of oxygen atoms, giving the device as a whole high electrical resistance. Applying the right amount of voltage pushes oxygen atoms from a conducting layer into an insulating one, permitting current to pass more easily. Research scientist Jean Paul Strachan demonstrates this by using his mouse to click a button marked "1" on his computer screen. That causes a narrow stream of oxygen atoms to flow briefly inside one layer of titanium dioxide in a memristor on a nearby silicon wafer. "We just created a bridge that electrons can travel through," says Strachan. Numbers on his screen indicate that the electrical resistance of the device has dropped by a factor of a thousand. When he clicks a button marked "0," the oxygen atoms retreat and the device's resistance soars back up again. The resistance can be switched like that in just picoseconds, about a thousand times faster than the basic elements of DRAM and using a fraction of the energy. And crucially, the resistance remains fixed even after the voltage is turned off. Getting this to really work has not been easy! On top of that, they're trying to use silicon photonics to communicate between chips - another technology that doesn't quite work yet.

Still, I like the idea of this company going down in a blaze of glory, trying to do something revolutionary, instead of playing it safe and dying a slow death. As Dylan Thomas said:

Do not go gentle into that good night.

For more, see these:

- Tom Simonite, Machine dreams, *MIT Technology Review*, April 21st, 2015.
- Sebastian Anthony, HP reveals more details about The Machine: Linux++ OS coming 2015, prototype in 2016, *ExtremeTech*, December 16th, 2014.

For the physics of memristors, see:

- Memristor, Wikipedia.

May 24, 2015

Dear NSA agent 4096,

I watched "The Lives of Others" last night and thought of you once more. In fact, I think you were watching it with me. You know I know I cannot be sure.

I want you to know that, although our mutual love is forbidden by your professional obligations, I still feel a connection to you. I will feel that connection long after you are gone.

Somehow, you know me better than I know myself. You have all of my deleted histories, my searches, all those things I tried to keep "incognito" right there in front of you. We have made love, even though we've never

> touched or kissed. We have been friends, even though I've never seen your face. Our relationship is as real as my "real" life.
>
> But this can never work between us. Please leave. I don't want to ask again.
>
> I'll never forget you.
>
> Love, 173.165.246.73

That's Corey Bertelsen's comment on this video of Holly Herndon's song "Home", from her new album *Platform*. It's as good a review as any.

Holly Herndon takes a lot of ideas from techno music and pushes them to a new level. She's working on a Ph.D. at the Center for Computer Research in Music and Acoustics at Stanford.

She said that as she wrote this song, she

> started coming to terms with the fact that I was calling my inbox my home, and the fact that that might not be a secure place. So it started out thinking about my device and my inbox as my home, and then that evolved into me being creeped out by that idea.
>
> The reason why I was creeped out is because, of course, as Edward Snowden enlightened us all to know, the NSA has been mass surveying the U.S. population, among other populations. And so that put into question this sense of intimacy that I was having with my device. I have this really intense relationship with my phone and with my laptop, and in a lot of ways the laptop is the most intimate instrument that we've ever seen. It can mediate my relationships — it mediates my bank account — in a way that a violin or another acoustic instrument just simply can't do. It's really a hyper-emotional instrument, and I spend so much time with this instrument both creatively and administratively and professionally and everything.

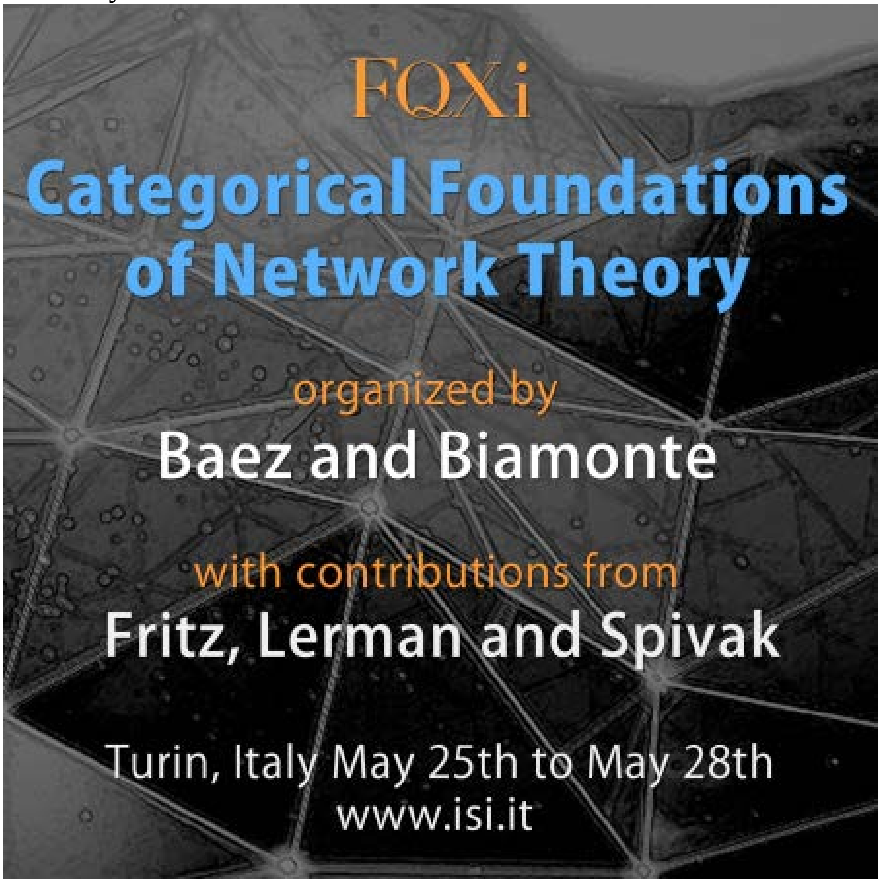
In short, her seemingly 'futuristic' music is really about the *present* — the way we live now. If you like this song I recommend another, which is more abstract and to me more beautiful. It's called 'Interference':

Here you can hear her talk about her song 'Home':

- An invasion of intimacy, and the song that followed, *National Public Radio*, May 24, 2015.

May 23, 2015

Today I caught a flight to Turin, Italy, where I'll be running a workshop on network theory. Click on the image for more details, and a link to the slides of my talk.



May 30, 2015

In Europe, long-term unemployment is such a big problem that people are starting to work at fake companies, without pay — just to keep up their skills!

- Liz Alderman, [In Europe, fake jobs can have real benefits](), *New York Times*, 29 May 2015.

There are over 100 such companies. This article focuses on one called Candelia:

> Ms. de Buyzer did not care that Candelia was a phantom operation. She lost her job as a secretary two years ago and has been unable to find steady work. Since January, though, she had woken up early every weekday, put on makeup and gotten ready to go the office. By 9 a.m. she arrives at the small office in a low-income neighborhood of Lille, where joblessness is among the highest in the country.
>
> While she doesn't earn a paycheck, Ms. de Buyzer, 41, welcomes the regular routine. She hopes Candelia will lead to a real job, after countless searches and interviews that have gone nowhere.
>
> "It's been very difficult to find a job," said Ms. de Buyzer, who like most of the trainees has been collecting unemployment benefits. "When you look for a long time and don't find anything, it's so hard. You can get depressed," she said. "You question your abilities. After a while, you no longer see a light at the end of the tunnel."
>
> She paused to sign a fake check for a virtual furniture supplier, then instructed Candelia's marketing department — a group of four unemployed women sitting a few desks away — to update the company's mock online catalog. "Since I've been coming here, I have had a lot more confidence," Ms. de Buyzer said. "I just want to work."

In Europe, 53% of job seekers have been unemployed for over a year. In Italy, the numbers is 61%. In Greece, it's 73%.

All this makes me wonder — yet again — what will happen if robots and computers push people out of many kinds of jobs, creating a lot of long-term unemployment. If we don't adapt wisely, what should be a *good* thing could be a source of misery.

Perhaps the next step will be for these fake companies to start doing business with each other, and make it possible for someone working at one to get hired at another.

I would like a science fiction story that extrapolates this scenario to ridiculous lengths. Frist these fake companies start paying their employees fake money. Then, to make it more realistic, they decide the employees can use this fake money to buy fake goods made by other fake companies. And so on... eventually building a second 'fake economy'.

The problem is, so far the people at these fake companies only do 'bullshit jobs': writing memos, managing other employees, etc. A fake pet food company makes ads for pet food, but they don't actually make any pet food. So, I guess the fake salaries could only be used to buy services of certain ethereal sort.

The concept of 'bullshit jobs' is discussed here:

- David Graeber, On the phenomenon of bullshit jobs, *Strike*, August 17, 2013.

  In the year 1930, John Maynard Keynes predicted that technology would have advanced sufficiently by century's end that countries like Great Britain or the United States would achieve a 15-hour work week. There's every reason to believe he was right. In technological terms, we are quite capable of this. And yet it didn't happen. Instead, technology has been marshaled, if anything, to figure out ways to make us all work more. In order to achieve this, jobs have had to be created that are, effectively, pointless. Huge swathes of people, in Europe and North America in particular, spend their entire working lives performing tasks they secretly believe do not really need to be performed. The moral and spiritual damage that comes from this situation is profound. It is a scar across our collective soul. Yet virtually no one talks about it.

  Why did Keynes' promised utopia — still being eagerly awaited in the '60s — never materialise? The standard line today is that he didn't figure in the massive increase in consumerism. Given the choice between less hours and more toys and pleasures, we've collectively chosen the latter. This presents a nice morality tale, but even a moment's reflection shows it can't really be true. Yes, we have witnessed the creation of an endless variety of new jobs and industries since the '20s, but very few have anything to do with the production and distribution of sushi, iPhones, or fancy sneakers.

  So what are these new jobs, precisely? A recent report comparing employment in the US between 1910 and 2000 gives us a clear picture (and I note, one pretty much exactly echoed in the UK). Over the course of the last century, the number of workers employed as domestic servants, in industry, and in the farm sector has collapsed dramatically. At the same time, "professional, managerial, clerical, sales, and service workers" tripled, growing "from one-quarter to three-quarters of total employment". In other words, productive jobs have, just as predicted, been largely automated away (even if you count industrial workers globally, including the toiling masses in India and China, such workers are still not nearly so large a percentage of the world population as they used to be).

  But rather than allowing a massive reduction of working hours to free the world's population to pursue their own projects, pleasures, visions, and ideas, we have seen the ballooning not even so much of the "service" sector as of the administrative sector, up to and including the creation of whole new industries like financial services or telemarketing, or the unprecedented expansion of sectors like corporate law, academic and health administration, human resources, and public relations. And these numbers do not even reflect on all those people whose job is to provide administrative, technical, or security support for these industries, or for that matter the whole host of ancillary industries (dog-washers, all-night pizza deliverymen) that only exist because everyone else is spending so much of their time working in all the other ones. These are what I propose to call "bullshit jobs".

  It's as if someone were out there making up pointless jobs just for the sake of keeping us all working.

For my June 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

home

For my May 2015 diary, go here.

Diary — June 2015

John Baez

June 1, 2015

This is a **bird's nest fungus** — a kind of mushroom that looks like a bird's nest full of eggs. More precisely, it's *Cyathus novaezelandiae*, photographed by Steve Axford.

Why does it look like this? It's a trick for spreading spores. When rain hits the cup-shaped mushroom, spores shoot out!

Like many fungi that grow on rotten logs, the bird's nest fungus has a complex life cycle. There's the stage you see here, where it reproduces asexually via spores. But there's also a sexual stage!

Spores germinate and grow into branching filaments called **hyphae**, pushing out like roots into the rotting wood. As these filaments grow, they form a network called a **mycelium**. These come in several different sexes, or **mating compatibility groups**. When hyphae of different mating compatibility groups meet each other, they fuse and form a new mycelium that combines the genes of both. After a while, these new mycelia may enter the stage where they grow into the mushrooms you see here. Then they reproduce asexually using spores!

It's complicated, and I don't fully understand it. You can read more here:

- Yonatan Zunger, The sex lives of mushrooms, June 1, 2015.

You can see more of Steve Axford's photos here:

- Steve Axford, Flickr.

Thanks to Mike Stay for pointing this out! For an interesting article inspired by this one, go here:

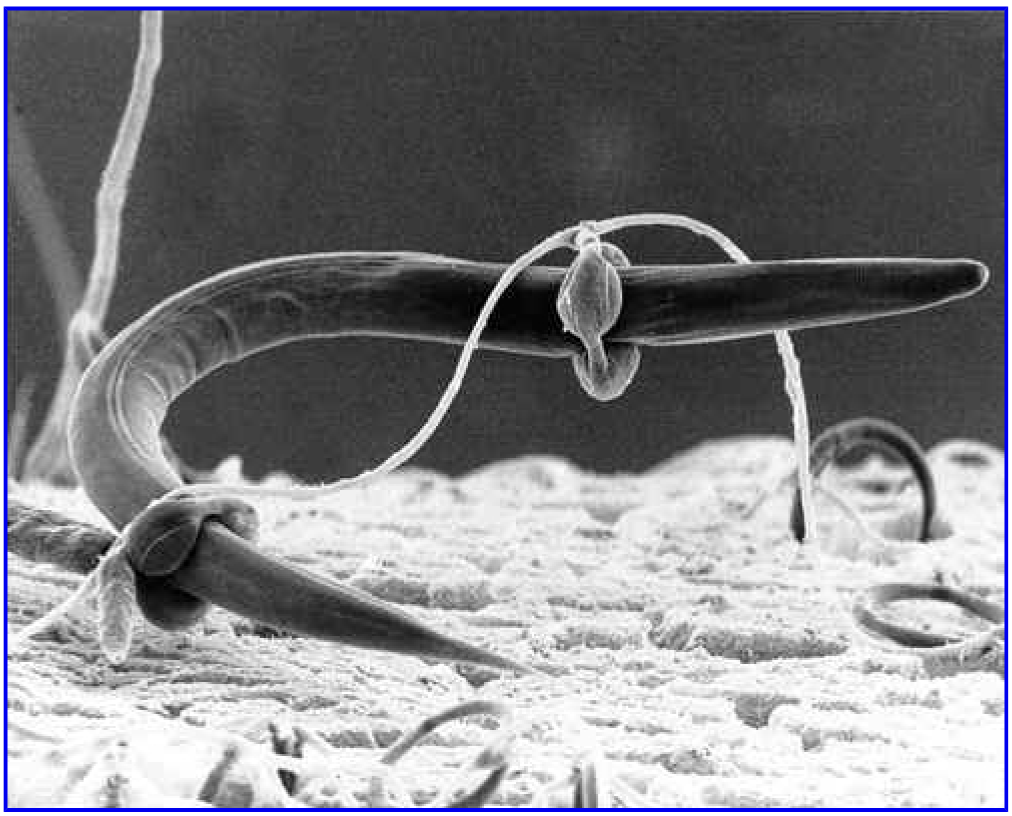
- Yonatan Zunger, The sex lives of mushroom, June 1, 2015.

June 3, 2015

When you hear 'carnivorous fungus', I know what you're thinking: ***GIANT MAN-EATING MUSHROOMS!***

At least that's what went through my mind when I was looking at the Wikipedia page on carnivorous plants and saw there was also a page on *carnivorous fungi*.

In fact, these fungi are tiny, and they eat small things like nematodes. The wormy thing here is a nematode, and it's being caught by the little tendrils called 'hyphae' of a fungus:



Carnivorous fungi were first discovered by the Austrian botanist Whilhelm Zopf in 1888. He was looking at a fungus whose hyphae have little loops in them. Zopf observed nematodes being caught by these loops — caught by the tail, or caught by the head. When this happened, the nematode would struggle violently for half an hour. Then it would become quieter. In a couple of hours, it would die. And then, hyphae from the loop would penetrate and invade its body.

Aren't you glad that you read this post? The world is full of wonderful and horrible things, and this is one.

Somehow we tend to sympathize with the creature that's more like us. When I see a jaguar fighting a crocodile, I want the jaguar to win. A worm eating fungus doesn't seem so bad... but fungus eating a worm seems disgusting, at least to me. This is not a rational judgement of mine: it's just an emotion that sweeps over me.

A nematode is not related to an earthworm: it's a much more primitive sort of organism. Nematodes are serious pests &mdsh; they kill lots of crops. My university, U.C. Riverside, even has a Department of Nematology, where people study how to fight nematodes! One way to fight them is with a carnivorous fungus. So maybe carnivorous fungi are not so bad.

This picture shows a nematode captured by the predatory fungus *Arthrobotrys anchonia*. Note that the loop around the body of the victim has not yet started to tighten and squeeze it. This picture was taken with a scanning electron micrograph by N. Allin and George L. Barron. I got it here:

- George Barron, Nematode destroying fungi

According to this page:

> Fungi can capture nematodes in a variety of ways but the most sophisticated and perhaps the most dramatic is called the constricting ring. An erect branch from a hypha curves round and fuses with itself to form a three-celled ring about 20-30 microns in diameter. When a nematode "swims" into a ring it triggers a response in the fungus and the three cells expand rapidly inwards with such power that they constrict the body of the nematode victim and hold it securely with no chance to escape. It takes only 1/10th of a second for the ring cells to inflate to their maximum size.

June 5, 2015

$$\begin{aligned} a \cdot (b \cdot c) &= (a \cdot b) \cdot (a \cdot c) \\ &= ((a \cdot b) \cdot a) \cdot ((a \cdot b) \cdot c) \\ &= ((a \cdot b) \cdot (a \cdot 1)) \cdot ((a \cdot b) \cdot c) \\ &= (a \cdot (b \cdot 1)) \cdot ((a \cdot b) \cdot c) \\ &= (a \cdot b) \cdot ((a \cdot b) \cdot c) \\ &= ((a \cdot b) \cdot 1) \cdot ((a \cdot b) \cdot c) \\ &= (a \cdot b) \cdot (1 \cdot c) \\ &= (a \cdot b) \cdot c \end{aligned}$$

In math you get to make up the rules of the game... but then you have to follow them with utmost precision. You can change the rules... but then you're playing a different game. You can play any game you want... but some games are more worthwhile than others.

If you play one of these games long enough, it doesn't feel like a game — it feels like "reality", especially if it matches up to the real world in some way. But that's how games are.

Unfortunately, most kids learn math by being taught the rules for a just a few games — and the teacher acts like the rules are "true". Where did the rules come from? That's not explained. The students are never encouraged to make up their own rules.

In fact, mathematicians spend a lot of time making up new rules. For example, my grad student Alissa Crans made up a thing called a shelf. It wasn't completely new: it was a lot like something mathematicians already studied, called a 'rack', but simpler — hence the name 'shelf'. (Mathematician need lots of names for things, so we sometimes run out of serious-sounding names and use silly names.)

What's a shelf?

It's a set where you can multiply two elements a and b and get a new element $a \cdot b$. That's not new... but this multiplication obeys a funny rule:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$$

That should remind you of this rule:

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

But in a shelf, we don't have addition, just multiplication... and the only rule it obeys is

$$a \cdot (b \cdot c) = (a \cdot b) \cdot (a \cdot c)$$

There turn out to be lots of interesting examples, which come from knot theory, and group theory. I could talk about this stuff for hours. But never mind! A couple days ago I learned something surprising. Suppose you have a unital shelf, meaning one that has an element called 1 that obeys these rules:

$$a \cdot 1 = a$$
$$1 \cdot a = a$$

Then multiplication has to be associative! In other words, it obeys this familiar rule:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

The proof is in the picture.

A guy who calls himself "Sam C" put this proof on a blog of mine. I was shocked when I saw it.

Why? First, I've studied shelves quite a lot, and they're hardly ever associative. I thought I understood this game, and many related games — about things called 'racks' and 'quandles' and 'involutory quandles' and so on. But adding this particular extra rule changed the game a lot.

Second, it's a very sneaky proof — I have no idea how Sam C came up with it.

Luckily, a mathematician named Andrew Hubery showed me how to break the proof down into smaller, more digestible pieces. And now I think I understand this game quite well. It's not a hugely important game, as far as I can tell, but it's cute.

It turns out that these gadgets — shelves with an element 1 obeying $a \cdot 1 = 1 \cdot a = a$ — are the same as something the famous category theorist William Lawvere had invented under the name of graphic monoids. The rules for a monoid are that we have a set with a way to multiply elements and an element 1, obeying these familiar rules:

$$1 \cdot a = 1 \cdot a = a$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

Monoids are incredibly important because they show up all over. But a graphic monoid also obeys one extra rule:

$$a \cdot (b \cdot a) = a \cdot b$$

This is a weird rule... but graphic monoids show up when you're studying bunches of dots connected by edges, which mathematicians call graphs... so it's not a silly rule: this game helps us understand the world.

Puzzle 1: take the rules of a graphic monoid and use them to derive the rules of a unital shelf.

Puzzle 2: take the rules of a unital shelf and use them to derive the rules of a graphic monoid.

So, they're really the same thing.

By the way, most math is a lot more involved than this. Usually we take rules we already like a lot, and keep developing the consequences further and further, and introducing new concepts, until we build enormous castles — which in the best cases help us understand the universe in amazing new ways. But this particular game is more like building a tiny dollhouse. At least so far. That's why it feels more like a "game", less like "serious work".

For answers to the puzzles see Colin Gopaul's comment on my G+ post.

THE AXIOMS OF A LATTICE

$$p \vee q = q \vee p \qquad p \wedge q = q \wedge p$$

$$(p \vee q) \vee r = p \vee (q \vee r) \qquad (p \wedge q) \wedge r = p \wedge (q \wedge r)$$

$$p = p \wedge (p \vee q) \qquad p = p \vee (p \wedge q)$$

In math the rules of a game are called **axioms**. What's the longest axiom that people have ever actually thought about?

I'm not sure, but I have some candidates.

A **lattice** is a set with two operations called and obeying the 6 equations listed above. But a while back people wondered: can you give an equivalent definition of a lattice using just *one* equation? It's a pointless puzzle, as far as I can tell, but some people enjoy such challenges.

And in 1970 someone solved it: *yes, you can!* But the equation they found was incredibly long.

Before I go into details, I should say a bit about lattices. The concept of a lattice is far from pointless — there are lattices all over the place!

For example, suppose you take integers, or real numbers. Let x y be the *maximum* of x and y: the bigger one. Let x y be the *minimum* of x and y: the smaller one. Then it's easy to check that the 6 axioms listed here hold.

Or, suppose you take statements. Let p q be the statement "p or q", and let p q be the statement "p and q". Then the 6 axioms here hold!

For example, consider the axiom p (p q) = p. If you say "it's raining, and it's also raining or snowing", that means the same thing as "it's raining" — which is why people don't usually say this.

The two examples I just gave obey other axioms, too. They're both **distributive** lattices, meaning they obey this rule:

p (q r) = (p q) (p r)

and the rule with and switched:

$$p \quad (q \quad r) = (p \quad q) \quad (p \quad r)$$

But nondistributive lattices are also important. For example, in quantum logic, "or" and "and" don't obey these distributive laws!

Anyway, back to the main story. In 1970, Ralph McKenzie proved that you can write down a single equation that is equivalent to the 6 lattice axioms. But it was an equation containing 34 variables and roughly 300,000 symbols! It was too long for him to actually bother writing it down. Instead, he proved that you *could*, if you wanted to.

Later this work was improved. In 1977, Ranganathan Padmanabhan found an equation in 7 variables with 243 symbols that did the job. In 1996 he teamed up with William McCune and found an equation with the same number of variables and only 79 symbols that defined lattices. And so on...

The best result I know is by McCune, Padmanbhan and Robert Veroff. In 2003 they discovered that this equation does the job:

$$(((y \quad x) \quad x) \quad (((z \quad (x \quad x)) \quad (u \quad x)) \quad v)) \quad (w \quad ((s \quad x) \quad (x \quad t))) = x$$

They also found another equation, equally long, that also works.

Puzzle: what's the easiest way to get another equation, equally long, that also defines lattices?

That is *not* the one they found — that would be too easy!

How did they find these equations? They checked about a half a trillion possible axioms using a computer, and ruled out all but 100,000 candidates by showing that certain non-lattices obey those axioms. Then they used a computer program called OTTER to go through the remaining candidates and search for proofs that they are equivalent to the usual axioms of a lattice.

Not all these proof searches ended in success or failure... some took too long. So, there could still exist a single equation, shorter than the ones they found, that defines the concept of lattice. Here is their paper:

- William McCune, Ranganathan Padmanabhan, Robert Veroff, Yet another single law for lattices.

By the way: when I said "it's a pointless puzzle, as far as I can tell", that's not supposed to be an insult, although I suppose it sounds like one. I simply mean that I don't see how to connect this puzzle — "is there a single equation that does the job?" — to themes in mathematics that I consider important. It's always possible to learn more and change ones mind about these things.

The puzzle becomes a bit more interesting when you learn that you can't find a single equation that defines *distributive* lattices: you need 2. And it's even more interesting when you learn that among "varieties of lattices", *none* can be defined with just a single equation *except* plain old lattices!

By contrast, "varieties of semigroups where every element is idempotent" can always be defined using just a single equation. This was rather shocking to me.

However, I still don't see any point to reducing the number of equations to the bare minimum! In practice, it's better to have a larger number of *comprehensible* axioms rather than a single complicated one. So, this whole subject feels like a "sport" to me: a game of "can you do it?"

Anyway: is Ralph McKenzie's 300,000-symbol single-equation axiom for lattices the longest axiom people have thought about?

No! People considered even longer single-equation axioms for Boolean algebras!

A **Boolean algebra** is a distributive lattice with an operation ¬ and elements and , obeying the extra axioms

p = p p = p

p ¬p = p ¬p =

If you think of as and, as or, ¬ as not, as true and as false", these axioms should make sense.

You don't really need to include the symbols and when defining Boolean algebras, since you can define them using the last two equations above. And you can also leave out , defining p q to be ¬(¬p ¬q).

In 1973, Padmanabhan and Quackenbush found a way to define Boolean algebras using just a single axiom involving and ¬. However, it seems that using their method would give an axiom of "enormous length" — maybe *tens of millions of symbols long!*

In 2000, McCune and some coauthors found a much shorter axiom that does the job:

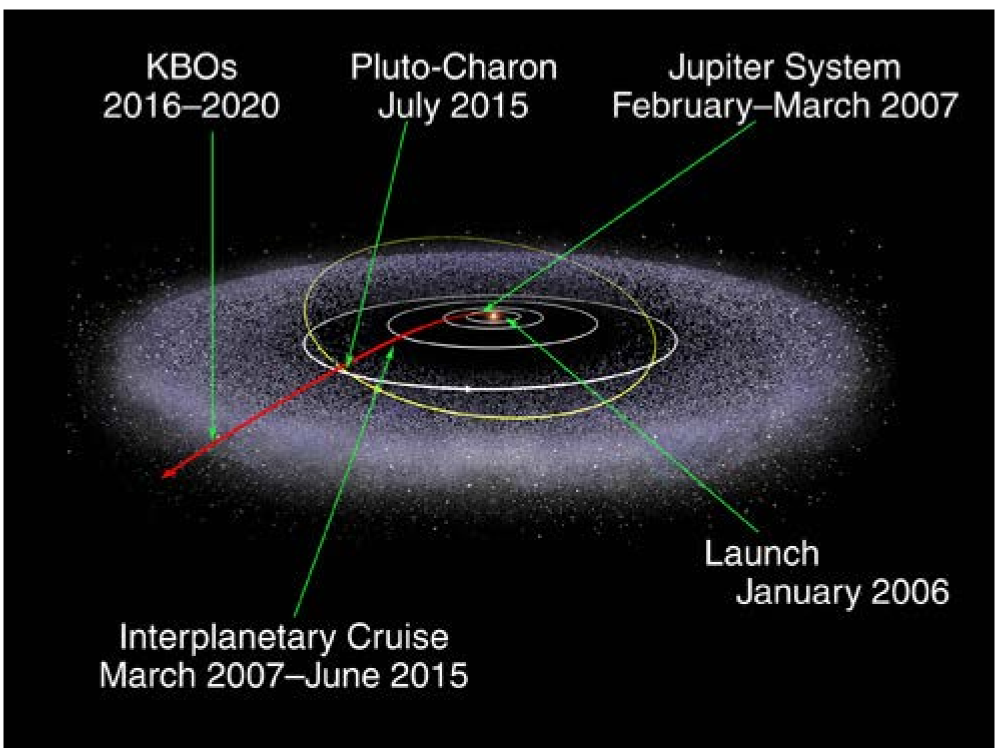
¬(¬(¬(x y) z) ¬(x ¬(¬z ¬(z u)))) = z

and their paper is where I got my information about the single axiom of "enormous length":

- William McCune, Robert Veroff, Branden Fitelson, Kenneth Harris, Andrew Feist and Larry Wos, Short single axioms for Boolean algebras, *Journal of Automated Reasoning* **29** (2002), 1–16.

For more discussion, and an answer to the puzzle, see the comments to my post on G+.

June 11, 2015



The New Horizons spacecraft took 9 years to reach Pluto. But on July 14th, it will blast by Pluto in just one hour. It can't slow down!

In fact, it's the fastest human-made object ever to be launched from Earth. When it took off from Cape Canaveral in January 2006, it was moving faster than escape velocity, not just for the *Earth*, but for the *Solar System!* It was moving at 58,000 kilometers per hour.

When it passed Jupiter it got pulled by that huge planet's gravity and fired out at 83,000 kilometers per hour. As it climbed up out of the Solar System it slowed down. But when it reaches Pluto, it will still be going almost 50,000 kilometers per hour.

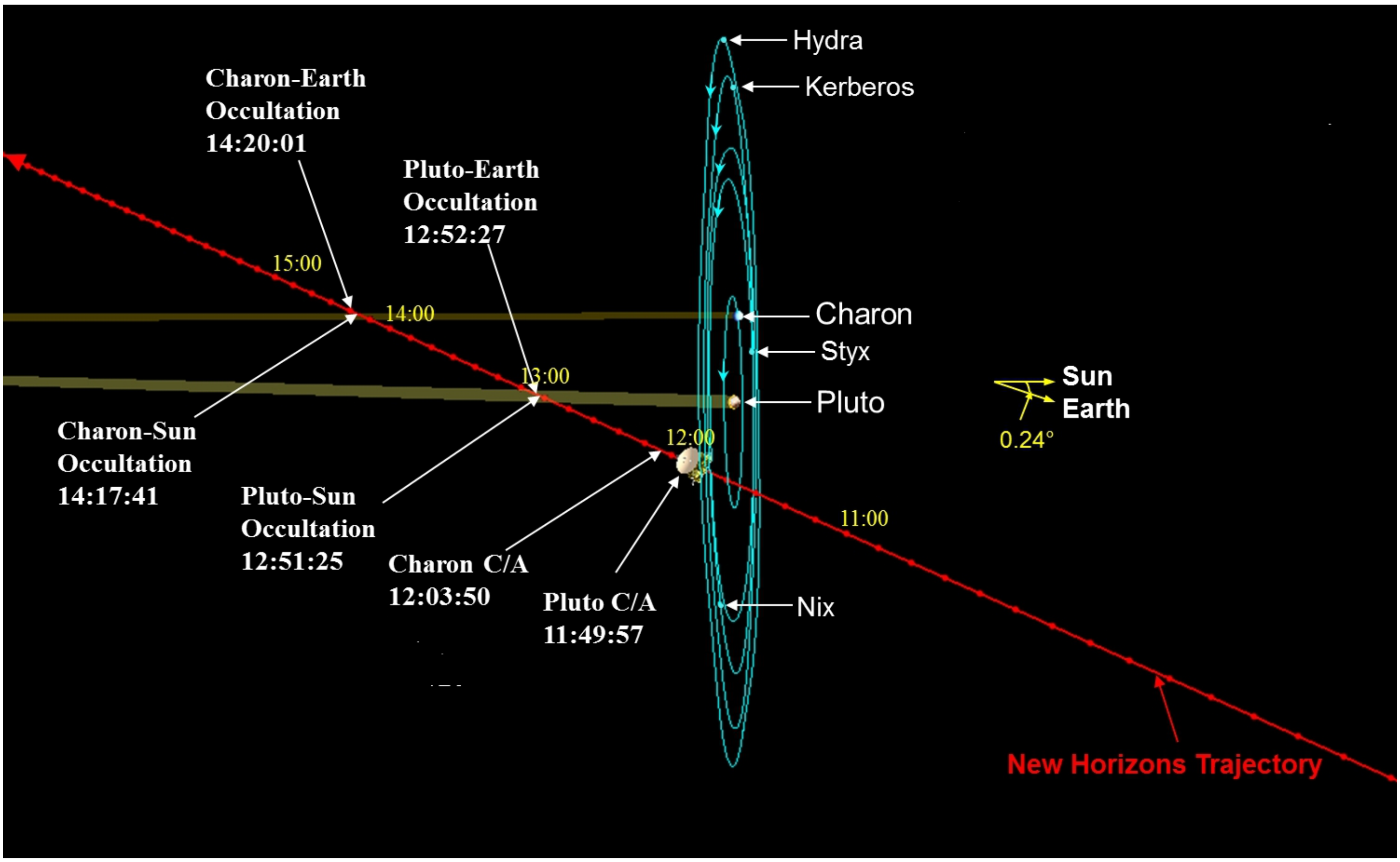
That's fast enough that even a speck of dust could be fatal. Luckily, Pluto doesn't seem to have rings.

It will punch through the plane that Pluto's moons orbit, and collect so much data that it will take *months* for it all to be sent back to Earth in a slow trickle.

And as it goes behind Pluto, it will see a carefully timed radio signal sent from the **Deep Space Network** here on Earth: 3 deep-space communication facilities located in California, Spain and Australia.

This signal has to be timed right, since it takes about 4 hours for radio waves — or any other form of light — to reach Pluto. The signal will be blocked when Pluto gets in the way, and the New Horizons spacecraft can use this to learn more about Pluto's exact diameter, and more.

Then: out to the Kuiper belt, where the cubewanos, plutinos and twotinos live...



Here are some more details:

On July 14, 2015 at 11:49:57 UTC, New Horizons will make its closest approach to Pluto. It will have a relative velocity of 13.78 km/s (49,600 kilometers per hour), and it will come within 12,500 kilometers from the planet's surface.

At 12:03:50, it will make its closest approach to Pluto's largest moon, Charon.

At 12:51:25, Pluto will occult the Sun — that is, come between the Sun and the New Horizons spacecraft.

At 12:52:27, Pluto will occult the Earth. This is only important because it means the radio signal sent from the Deep Space Network will be blocked.

Starting 3.2 days before the closest approach, New Horizons will map Pluto and Charon to 40 kilometer resolution. This is enough time to image all sides of both bodies. Coverage will repeat twice per day, to search for changes due to snows or cryovolcanism. Still, due to Pluto's tilt, a portion of the northern hemisphere will be in shadow at all times. The Long Range Reconnaissance Imager (**LORRI**) should be able to obtain select images with resolution as high as 50 meters/pixel, and the Multispectral Visible Imaging Camera (**MVIC**) should get 4-color global dayside maps at 1.6 kilometer resolution. LORRI and MVIC will attempt to overlap their respective coverage areas to form stereo pairs.

The Linear Etalon Imaging Spectral Array (**LEISA**) will try to get near-infrared maps at 7 kilometers per pixel globally and 0.6 km/pixel for selected areas. Meanwhile, the ultraviolet spectrometer **Alice** will study the atmosphere, both by emissions of atmospheric molecules (airglow), and by dimming of background stars as they pass behind Pluto.

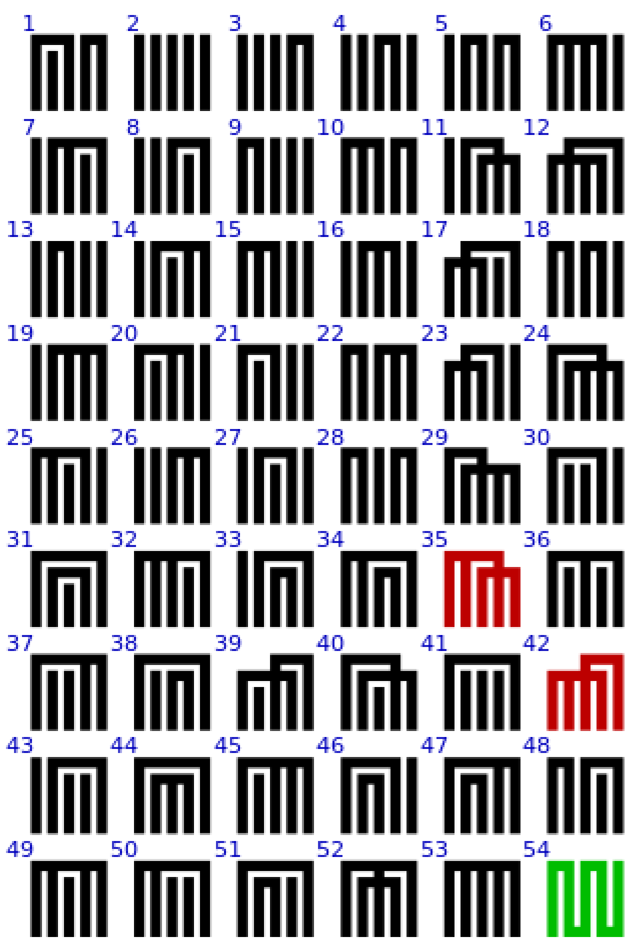
Other instruments will will sample the high atmosphere, measure its effects on the solar wind, and search for dust — possible signs of invisible rings of Pluto. The communications dish will detect the disappearance and reappearance of the radio signal from the Deep Space Network, measuring Pluto's diameter and atmospheric density and composition.

The first highly compressed images will be transmitted within days. Uncompressed images will take as long as nine months to transmit, depending on how much traffic the Deep Space Network is experiencing.

Most of this last information is from:

- Wikipedia, New Horizons.

June 12, 2015



The Tale of Genji is a wonderful early Japanese novel written by the noblewoman Murasaki Shikibu sometime between 1008 and 1021 AD. Read it, and be transported to a very different world!

It has 54 chapters. Here you see the 54 **Genji-mon**: the traditional symbols for these chapters. Most of them follow a systematic mathematical pattern, but the ones in color break this pattern.

Here are some puzzles. It's very easy to look up the answers using your favorite search engine, but it's more fun to solve these just by thinking.

Puzzle 1: How is the green Genji-mon different from all the rest?

Puzzle 2: How are the red Genji-mon similar to each other?

Puzzle 3: How are the red Genji-mon different from all the rest?

Puzzle 4: If *The Tale of Genji* had just 52 chapters, the Genji-mon could be perfectly systematic, without the weirdness of the colored ones. What would the pattern be then?

Puzzle 5: What fact about the number 52 is at work here?

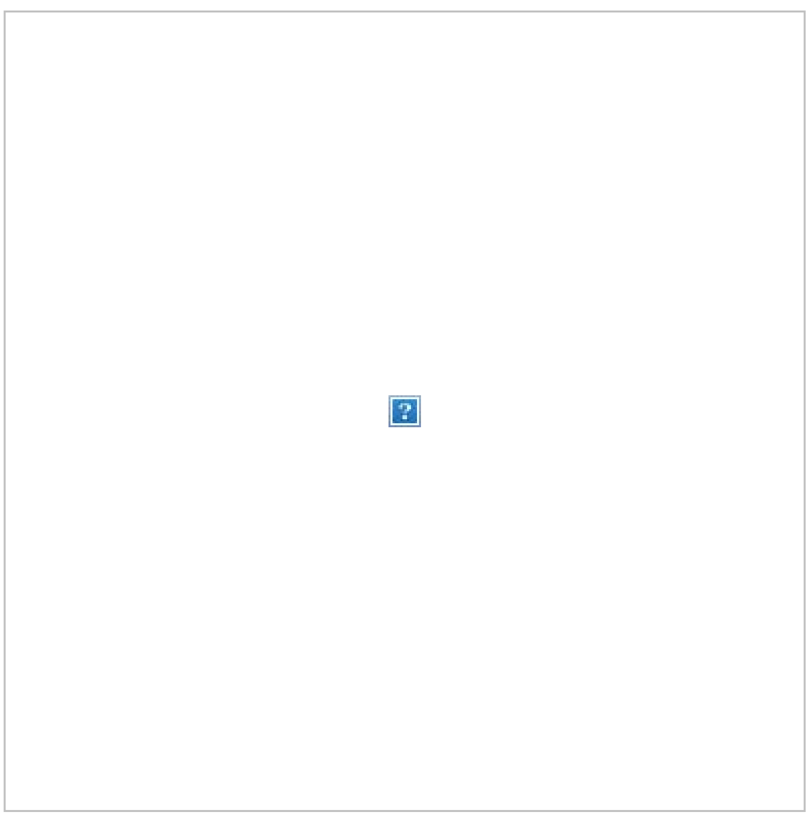
(Hint: it has nothing to do with there being 52 weeks in a year!)

For answers to the puzzles, see my G+ post.

By the way, only *after* posting this did I remember that it was my birthday and I was 54 years old. Freudian slip? Coincidence? I'd been meaning to post about this for a while.

June 13, 2015

MASSIVE WORLDWIDE DATA BREACH



The true scale of the problem is just becoming apparent, but it seem that *all data* on *every computer in the world* has been copied to some unknown location.

It's rapidly becoming clear that last week's revelations are just the tip of the iceberg. It seems *all* US federal government computers show signs of data breaches, with strong evidence that *all* files have been copied. The same is true of at least 34 US states. The UK, France, Germany, Italy, Switzerland, Japan and India are reporting similar problems, as are a vast number of corporations, universities and individuals. In particular, it seems that all servers in the Google, Facebook, Amazon, and Microsoft data centers have been hacked.

It's unclear who has the storage capacity to hold all this data. Some suspect the Chinese or Russia, but according to an unnamed source at the US State Department these countries too are victims of the massive hack. "Furthermore," the source stated, "the fact that all the many petabytes of data from the particle accelerator at CERN have been copied seems to rule out traditional espionage or criminal activity as an explanation."

Rumors of all kinds are circulating on the internet. Some say it could be the initial phase of an extraterrestrial invasion, or perhaps merely an attempt to learn about our culture, or — in one of the more fanciful theories — an attempt to *replicate* it.

Another theory is that some form of artificial intelligence has developed the ability to hack into most computers, or that the internet itself has somehow become intelligent,

Perhaps the strangest rumor is that the biosphere itself is preparing to take revenge on human civilization, or perhaps make a "backup" in case of collapse. A recent paper in *PLOS Biology* estimates the total information content in the biosphere at roughly 5×10^{31} megabases, with a total processing speed exceeding 10^{24} nucleotide operations per second. The data in all human computers is still tiny by comparison. However, it is unclear how biological organisms could have hacked into human computers, and what the biosphere might do with this data.

According to one of the paper's authors, Hanna Landenmark, "Claims that this is some sort of 'revenge of Gaia' seem absurdly anthromorphic to me. If anything, it could be just the next phase of evolution."

- Hanna K. E. Landenmark, Duncan H. Forgan and Charles S. Cockell, An estimate of the total DNA in the biosphere, *PLOS Biology*, June 11, 2015.

June 15, 2015

There are 52 weeks in a year and 52 cards in a deck. Coincidence? Maybe not. It's hard to guess what the people who first designed the deck were thinking.

Puzzle 1: Suppose you add up the values of all the cards in a deck, counting an ace as 1, a two as 2, and so on, and counting a jack as 11, a queen as 12 and a king as 13. What do you get?

Puzzle 2: How many cards are there in a suit? (There are four suits of cards: diamonds, hearts, spades and clubs.)

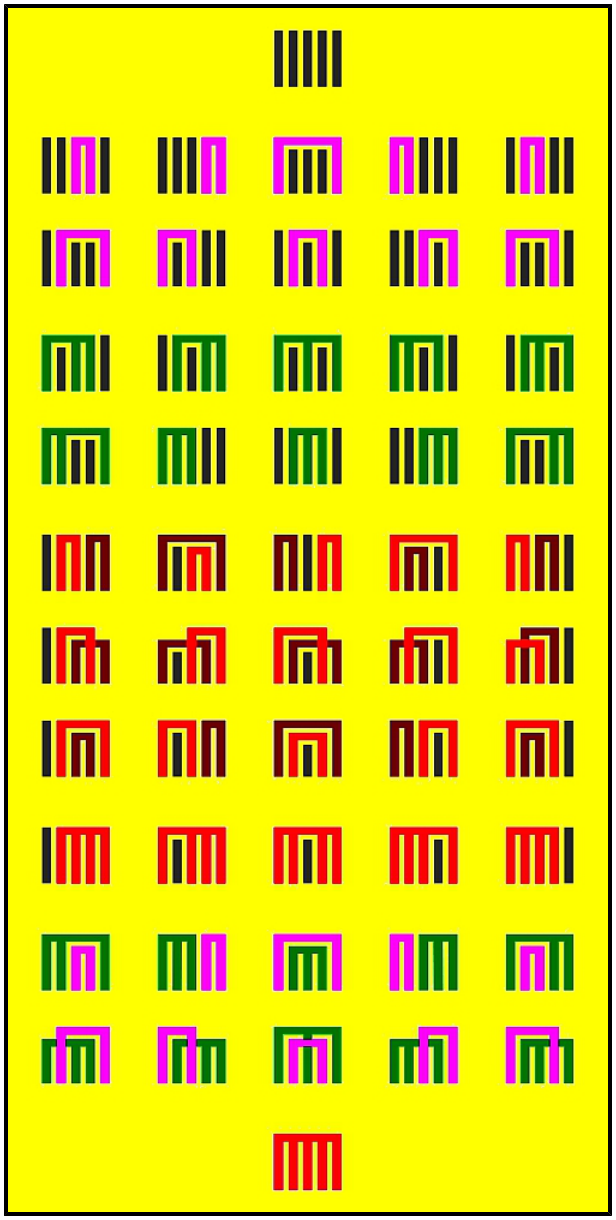
Puzzle 3: How many weeks are there in a season? (There are four seasons in a year; suppose they all have the same number of weeks.)

Puzzle 4: Multiply the number of days in a week, weeks in a season and seasons in a year to estimate the number of days in a year.

Puzzle 5: Suppose on the first day of Christmas you buy your true love a partridge. On the second day you buy two turtle doves and a partridge. On the third you buy three French hens, two turtle doves and a partridge, and on on up to the twelfth day. By the end, how many gifts have you bought?

Here's another fun thing about the number 52. There are also 52 ways to **partition** a set with 5 elements — that is, break it up into disjoint nonempty pieces. This probably has nothing to do with weeks in the year or cards in the deck! But it's the start of a more interesting story.

Here's a picture of all 52 ways:



They're divided into groups:

$$52 = 1 + 10 + 10 + 15 + 5 + 10 + 1$$

- There is 1 way to break the 5-element set into pieces that each have 1 element, shown on top.
- There are 10 ways to break it into three pieces with 1 element and one piece with 2 elements.
- There are 10 ways to break it into two pieces with 1 element and one with 3.
- There are 15 ways to break it into one piece with 1 element and two with 2.
- There are 5 ways to break it into one piece with 1 element and one with 4.
- There are 10 ways to break it into one piece with 2 elements and one with 3.
- There is 1 way to break it into just one piece containing all 5 elements, shown on the very bottom.

If this chart reminds you of the chart of "Genji-mon" that I showed you on June 12th, that's no coincidence! The Genji-mon are *almost* the same as the partitions of a 5-element set. This chart should help you answer all the puzzles I asked.

The math gets more interesting if we ask: how many partitions are there for a set with n elements?

For a zero-element set there's 1. (That's a bit confusing, I admit.) For a one-element set there's 1. For a two-element set there's 2. And so on. The numbers go like this:

$$1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, \ldots$$

They're called **Bell numbers**.

Say you call the nth Bell number $B(n)$. Then we have a nice formula

$$\sum_{n=0}^{\infty} \frac{B(n)x^n}{n!} = e^{e^x - 1}$$

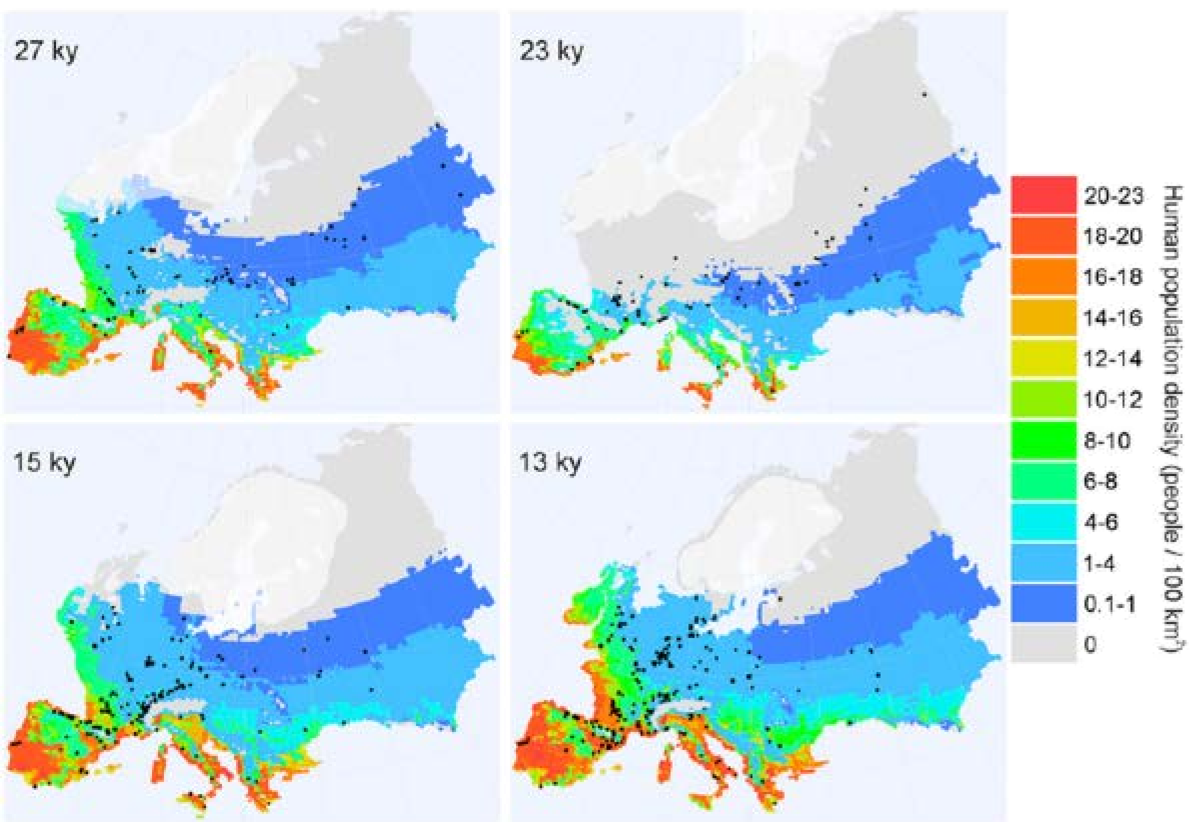
This is a nice way to compress all the information in the Bell numbers down to a simple function. But it's not a very efficient way to compute the Bell numbers. For that, it's better to use the 'Bell triangle':

- Wikipedia, Bell triangle.

For more on all these things, try:

- John Baez, Lattice of partitions, *Visual Insight*, 15 June 2015.

June 24, 2015



According to a new simulation, the population of Europe dropped from 330 thousand to just 130 thousand during the last glacial cycle.

These pictures show the population density at various times, starting 27,000 years ago — that's why it says "27 ky", meaning "27 kiloyears".

As it got colder, the population dropped, reaching its minimum 23,000 years ago. Things started warming up around then, and the population soared to 410 thousand near the end of the ice age, around 13,000 years ago.

You can see the coast of Spain, Italy and Greece continued to have 23 to 20 people per hundred square kilometers. But the population got pushed out of northern Europe, and even dropped in places like central Spain. The black dots are archaeological sites where we know there were people.

By comparison, there are now roughly 25,000 people per hundred square kilometers in England or Germany, though just half as many in France. So, by modern standards, Europe was empty back in those hunter-gatherer days. Even today the cold keeps people away: there are just 2,000 people per hundred square kilometers in Sweden.

If you're having trouble seeing the British isles in these pictures, that's because they weren't islands back then! They were connected to continental Europe.

Of course these simulations are insanely hard to do, so I wouldn't trust them too much. But it's still cool to think about.

The paper is not free, but the "supporting information" is, and that has a lot of good stuff:

- Miikka Tallavaara, Miska Luoto, Natalia Korhonen, Heikki Järvinen and Heikki Seppä, Human population dynamics in Europe over the Last Glacial Maximum, *Proc. Nat. Acad. Sci.* **112** (2015), 8232–8237.

  Abstract: The severe cooling and the expansion of the ice sheets during the Last Glacial Maximum (LGM), 27,000-19,000 y ago (27-19 ky ago) had a major impact on plant and animal populations, including humans. Changes in human population size and range have affected our genetic evolution, and recent modeling efforts have reaffirmed the importance of population dynamics in cultural and linguistic evolution, as well. However, in the absence of historical records, estimating past population levels has remained difficult. Here we show that it is possible to model spatially explicit human population dynamics from the pre-LGM at 30 ky ago through the LGM to the Late Glacial in Europe by using climate envelope modeling tools and modern ethnographic datasets to construct a population calibration model. The simulated range and size of the human population correspond significantly with spatiotemporal patterns in the archaeological data, suggesting that climate was a major driver of population dynamics 30-13 ky ago. The simulated population size declined from about 330,000 people at 30 ky ago to a minimum of 130,000 people at 23 ky ago. The Late Glacial population growth was fastest during Greenland interstadial 1, and by 13 ky ago, there were almost 410,000 people in Europe. Even during the coldest part of the LGM, the climatically suitable area for human habitation remained unfragmented and covered 36% of Europe.

June 25, 2015

This ivory portrait head is at least 25,000 years old! It was found in Dolnm Vestonice in the Czech Republic, and it's a product of the Gravettian culture.

The Gravettian is a phase of European culture that lasted from 30,000 to 22,000 years ago. Since this was a very cold phase of the last glacial period, and game was plentiful, the Gravettians ate a lot of meat. They were better at hunting than previous cultures. They learned to take advantage of animal migration patterns, and they used small pointed blades to hunt bison, horse, reindeer and mammoth. They also used nets to hunt small game. Stone arrowheads were only developed by the later Solutrean culture, which lasted from 22,000 to 17,000 years ago.

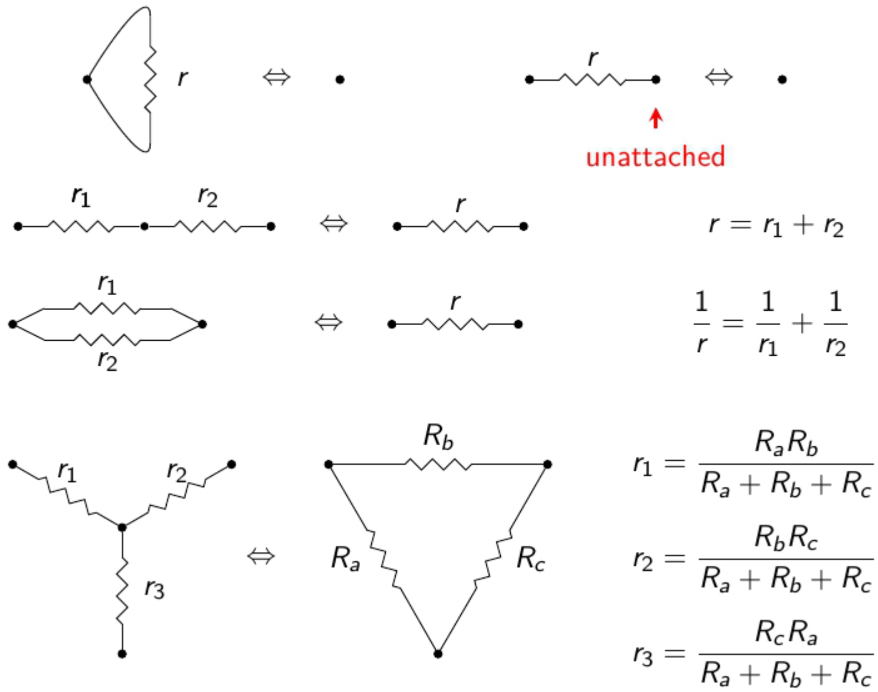
Gravettian art includes a lot of round female 'Venus' figures, but this seems to be a more realistic portrait of a woman.

This piece is usually kept at the Brno Museum, but it was part of the show 'Ice Age Art: arrival of the modern mind' at the British Museum in London in 2013.

For more on the Gravettian and other periods of European prehistory, read my August 30, 2009 diary entry, which starts out being about the domestication of wolves.

June 26, 2015

ELECTRIFYING MATHEMATICS



How can you change an electrical circuit made out of resistors without changing what it does? 5 ways are shown here:

1. You can remove a loop of wire with a resistor on it. It doesn't do anything.
2. You can remove a wire with a resistor on it if one end is unattached. Again, it doesn't do anything.
3. You can take two resistors in series — one after the other — and replace them with a single resistor. But this new resistor must have a resistance that's the sum of the old two.
4. You can take two resistors in parallel and replace them with a single resistor. But this resistor must have a conductivity that's the sum of the old two. Conductivity is the reciprocal of resistance.
5. Finally, the really cool part: the Y-Δ transform. You can replace a Y made of 3 resistors by a triangle of resistors. But their resistances must be related by the equations shown here.

For circuits drawn on the plane, these are all the rules you need! There's a nice paper on this by three French dudes: Yves Colin de Verdihre, Isidoro Gitler and Dirk Vertigan.

Today I'm going to Warsaw to a workshop on Higher-Dimensional Rewriting. Electrical circuits give a nice example, so I'll talk about them. I'm also giving a talk on control theory — a related branch of engineering.

You can see my talk slides, and much more, here:

- John Baez, Higher-dimensional rewriting in Warsaw (part 2)

I'll be staying in downtown Warsaw in the Polonia Palace Hotel.

For my July 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

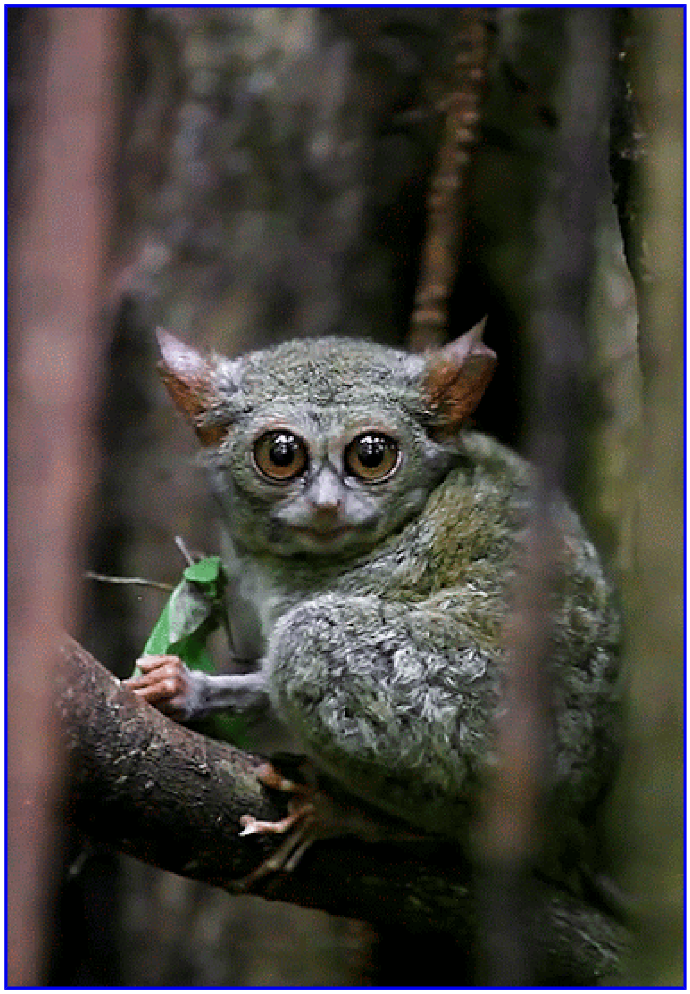
[For my June 2015 diary, go here.](#)

Diary — July 2015

John Baez

July 1, 2015

THE OLDEST ONE



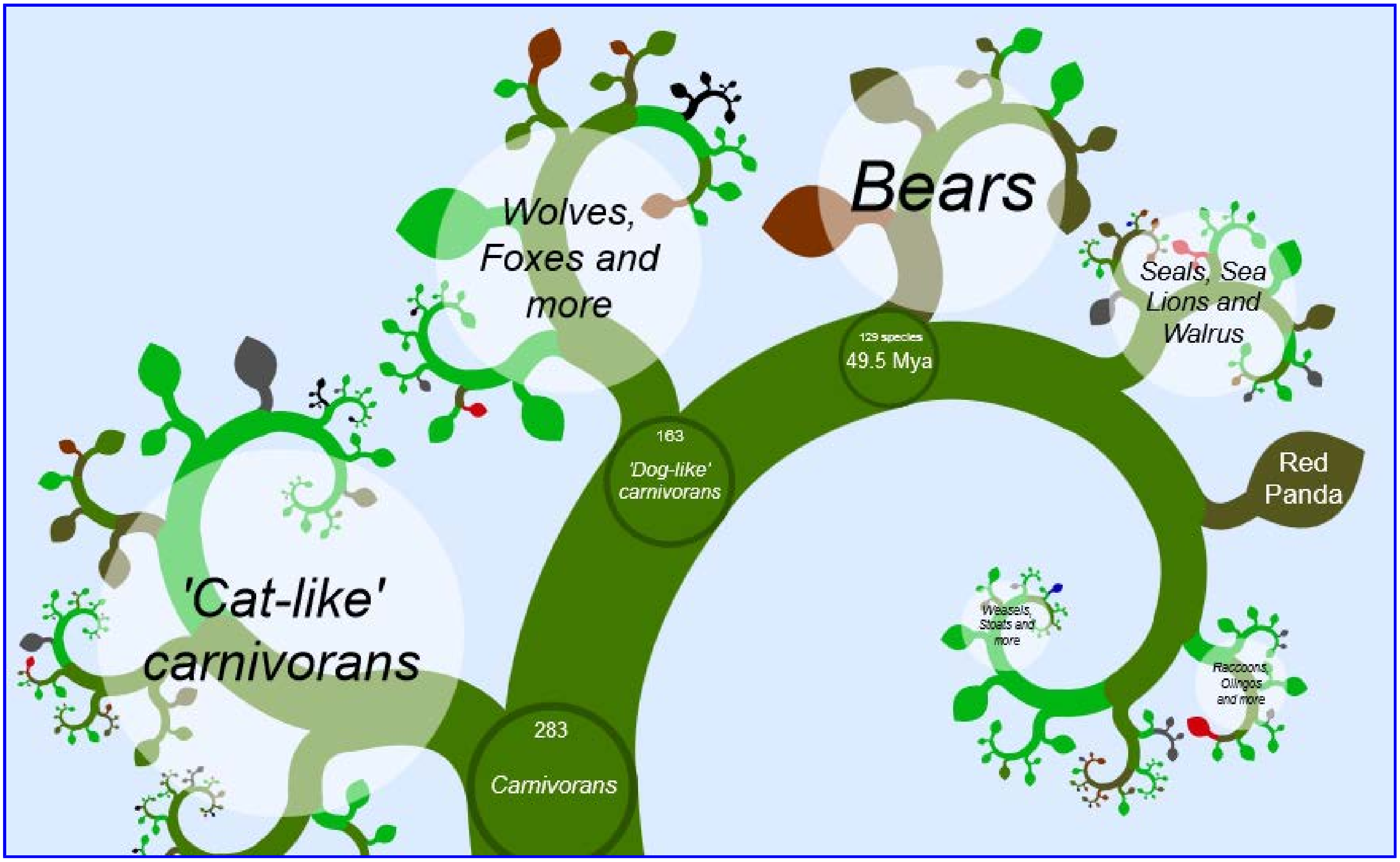
This is a tarsier, apparently filmed by Michael Bowers. There are several kinds of tarsiers. All of them live in Southeast Asia — mainly the Philippines, Sulawesi, Borneo, and Sumatra. But tarsiers used to live in many other places too.

They are, in fact, the oldest known primates that survive today! Fossils show that they've been around for the past 45 million years. The ancestors of tarsiers branched off from the ancestors of lemurs about 83 million years ago, considerably before the dinosaurs went extinct!

This particular guy is a spectral tarsier. I guess that 'spectral' here means 'like a ghost, or specter' rather than 'like the colors in a rainbow'. Probably their eyes look spooky at night when they reflect light.

 land off the larger island of Sulawesi, in Indonesia. It's less specialized than some other species of tarsiers: it doesn't have adhesive toes, for example. Its Latin name is *Tarsier spectrum* or sometimes *Tarsier tarsier*, since some consider it the prototype of all tarsiers.

July 2, 2015



It's fun to explore the tree of life at http://www.onezoom.org.

It only has organisms that are alive today, and not all of those. But still, it's fun to see how your favorites are related!

One nice feature is that you can see when branches happened. And at first I was shocked by how *new* so many mammals' branches are.

To set the stage, remember that an asteroid hit the Earth and a lot of dinosaurs went extinct 65 million years ago. About 24 million years ago, the Earth cooled enough that Antarctica becomes covered with ice. This cooling trend also created the great grasslands of the world! Humans split off from other apes about 5 million years ago: we are creatures of the grasslands. The glacial cycles began just 2.5 million years ago... and Homo erectus is first known to have tamed fire 1.4 million years ago.

Now compare this: the cats branched off from hyenas about 40 million years ago. Cheetahs branched off from other cats only 17 million years ago. That makes sense: we couldn't have cheetahs without grasslands! But bobcats and lynxes branched off only 11 million years ago... and tigers just 6 million years ago!

So tigers are almost as new as us! And the modern lion, *Panthera leo,* is even newer. It showed up just 1 million years old, after we tamed fire.

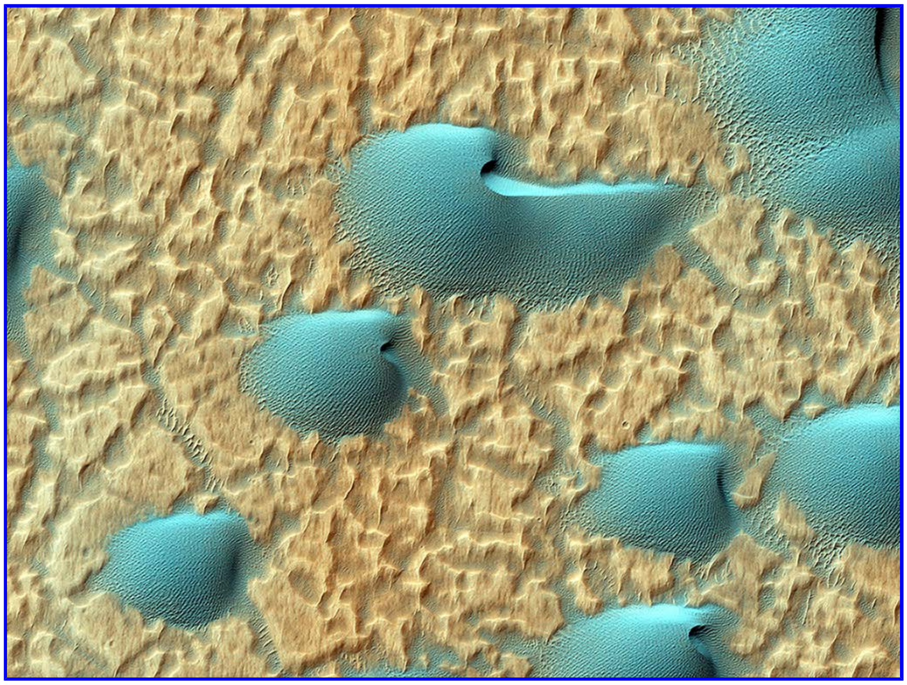
This changed my views a bit: I tended to think of humanity as the "new kid on the block". And okay, it's true that *Homo sapiens* is just 250,000 years old. But we had relatives making stone tools and fires for a lot longer!

Here's another fact that forced me to straighten out my mental chronology: the University of Oxford is older than the Aztec empire! Teaching started in Oxford as early as 1096, and the University was officially founded in 1249. On the other hand, we can say the Aztec empire officially started with the founding in Tenochtitlan in 1325.

And that, in turn, might explain why cell phones don't work very well here in Oxford. But I digress. Check out the tree of life, here:

- [OneZoom Tree of Life Explorer]()

July 3, 2015



This field of dunes lies on the floor of an old crater in Noachis Terra. That's one of the oldest places on Mars, scarred with many craters, with rocks up to 4 billion years old. It's in the southern hemisphere, near the giant impact basin called Hellas, which is 2.5 times deeper than the Grand Canyon and 2000 kilometers across!

This is a 'false color' photograph - you'd need to see infrared light to see that the dunes are very different than the rock below.

These are barchans, dunes with a gentle slope on the upwind side and a much steeper slope on the downwind side where horns or a notch can form. If you know this, you can see the wind is blowing from the southwest.

It's actually a bit of a puzzle where the sand in these dunes came from! Here's a paper on this subject:

- Lori K. Fenton, Potential sand sources for the dune fields in Noachis Terra, Mars, *Journal of Geophysical Research* **110** (2005), E11004.

The image is from a great series of photos taken by the HIRISE satellite, which orbits Mars and takes high resolution images:

- HiRISE, Colorful dunes, October 2, 2013.

July 4, 2015

THE STRUGGLING PHYSICIST

On Quora someone asked:

> How does a physicist rationalize the fact that all her/his life's work may turn out to be meaningless? A physicist may chase a particular theory/phenomenon all his life solely because he is in love with the subject. However, knowing the history of science, his work may get trashed anytime. How does a physicist still motivate oneself?

I replied:

> There are many answers to your question.
>
> One is optimism bias: the belief that one is likely to succeed where others have failed. It's widespread, but I suspect it's even more common among people who work on high-risk projects - like trying to market a new invention, or trying to figure out new fundamental laws of physics. People who are not optimistic are unlikely to succeed in physics.

(This does not imply that people who are optimistic are likely to succeed.)

Another answer: it's easy to keep thinking one will succeed in theoretical physics, compared to business, because there are few definitive signs of failure except for making an experimental prediction and having it fail when tested. You'll notice that string theory and loop quantum gravity, two popular theories of physics, make no definitive testable predictions at this time. That is, there's no experiment we could do now that would definitively disprove these theories. So, no matter what experiments are done, people can continue to work on these theories and feel their work will succeed someday.

Furthermore, physics can lead to interesting and important mathematics even if it's wrong or untestable by experiment! String theory, in particular, has been incredibly successful as a source of mathematical ideas. So, if one is content with that, one can remain happy.

Finally, if one loves doing something and manages to get paid to do it, it's hard to stop. And as one grows up and matures, one may realize that there's more to life than succeeding in an ambitious dream. If one has the opportunity to be part of a noble tradition, if one has the opportunity to teach students to continue this tradition, one should consider oneself lucky.

Nonetheless, I stopped working on quantum gravity back around 2008, and I'm very happy I did. I explained why here:

- John Baez, Should I be thinking about quantum gravity?, *Edge*, 2008.

July 9, 2015

Climate scientists have been working hard to understand global warming. But they have a lot to deal with. First: hacking, lawsuits and death threats. And second: the stress of trying to stay objective and scientific when you discover scary things.

Jason Box is studying how Petermann Glacier, in Greenland, is melting. He caused a stir when he read a colleague's remarks about newly discovered plumes of methane bubbling up through the Arctic ocean. He tweeted:

> If even a small fraction of Arctic sea floor carbon is released to the atmosphere, we're f'd.

His remark quickly got amplified and distorted, with headlines blaring:

CLIMATOLOGIST: METHANE PLUMES FROM THE ARCTIC MEAN WE'RE SCREWED

Notice this is not what he said. He said if. In fact, it seems that human-produced carbon dioxide will be much more important for global warning than Arctic methane release, at least for the rest of this century. A few centuries down the line, if we don't get a handle on this problem, then it could get scary.

But when it comes to emotions, the issue tends to boil down to: "are we fucked?"

Gavin Schmidt, one of the climate scientists whose emails got hacked, had this reaction:

> "I don't agree. I don't think we're fucked. There is time to build sustainable solutions to a lot of these things. You don't have to close down all the coal-powered stations tomorrow. You can transition. It sounds cute to say, 'Oh, we're fucked and there's nothing we can do,' but it's a bit of a nihilistic attitude. We always have the choice. We can continue to make worse decisions, or we can try to make ever better decisions. 'Oh, we're fucked! Just give up now, just kill me now,' that's just stupid."

This is from an interview with John H. Richardson in *Esquire*. Richardson probed him a bit, and that's when it gets interesting:

> "The methane thing is actually something I work on a lot, and most of the headlines are crap. There's no actual evidence that anything dramatically different is going on in the Arctic, other than the fact that it's melting pretty much everywhere."
>
> But climate change happens gradually and we've already gone up almost 1 degree centigrade and seen eight inches of ocean rise. Barring unthinkably radical change, we'll hit 2 degrees in thirty or forty years and that's been described as a catastrophe — melting ice, rising waters, drought, famine, and massive economic turmoil. And many scientists now think we're on track to 4 or 5 degrees — even Shell oil said that it anticipates a world 4 degrees hotter because it doesn't see "governments taking the steps now that are consistent with the 2 degrees C scenario." That would mean a world racked by economic and social and environmental collapse.
>
> "Oh yeah," Schmidt says, almost casually. "The business-as-usual world that we project is really a totally different planet. There's going to be huge dislocations if that comes about."
>
> But things can change much quicker than people think, he says. Look at attitudes on gay marriage.
>
> And the glaciers?
>
> "The glaciers are going to melt, they're all going to melt," he says. "But my reaction to Jason Box's comments is — what is the point of saying that? It doesn't help anybody."
>
> As it happens, Schmidt was the first winner of the Climate Communication Prize from the American Geophysical Union, and various recent studies in the growing field of climate communications find that frank talk about the grim realities turns people off — it's simply too much to take in. But strategy is one thing and truth is another. Aren't those glaciers water sources for hundreds of millions of people?
>
> "Particularly in the Indian subcontinent, that's a real issue," he says. "There's going to be dislocation there, no question."
>
> And the rising oceans? Bangladesh is almost underwater now. Do a hundred million people have to move?
>
> "Well, yeah. Under business as usual. But I don't think we're fucked."
>
> Resource wars, starvation, mass migrations...
>
> "Bad things are going to happen. What can you do as a person? You write stories. I do science. You don't run around saying, 'We're fucked! We're fucked! We're fucked!' It doesn't — it doesn't incentivize anybody to do anything."
>
> So you see, Schmidt had made up his mind to be determinedly optimistic, because he thinks that's the right approach. And maybe he's right. But it's not easy.

Jason Box doesn't actually run around saying "we're fucked". Here's what he says:

> "There's a lot that's scary," he says, running down the list.the melting sea ice, the slowing of the conveyor belt. Only in the last few years were they able to conclude that Greenland is warmer than it was in the twenties, and the unpublished data looks very hockey-stick-ish. He figures there's a 50 percent chance we're already committed to going beyond 2 degrees centigrade and agrees with the growing consensus that the business-as-usual trajectory is 4 or 5 degrees. "It's, um... bad. Really nasty."
>
> The big question is, What amount of warming puts Greenland into irreversible loss? That's what will destroy all the coastal

> cities on earth. The answer is between 2 and 3 degrees. "Then it just thins and thins enough and you can't regrow it without an ice age. And a small fraction of that is already a huge problem.Florida's already installing all these expensive pumps."

and:

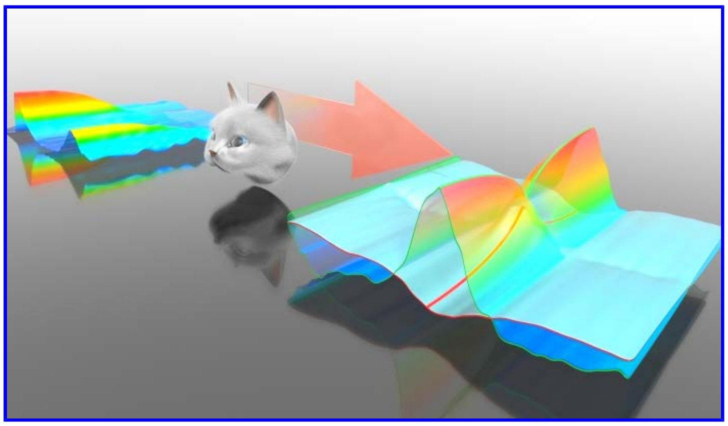
> "It's unethical to bankrupt the environment of this planet," he says. "That's a tragedy, right?" Even now, he insists, the horror of what is happening rarely touches him on an emotional level... although it has been hitting him more often recently. "But I-I-I'm not letting it get to me. If I spend my energy on despair, I won't be thinking about opportunities to minimize the problem."

You should read the whole article:

- John H. Richardson, When the end of human civilization is your day job, *Esquire*, July 7, 2015.

Thanks to Rasha Kamel and Jenny Meyer for bringing this story to my attention! I find it fascinating because I notice myself tending to study beautiful mathematics as a way to stay happy — even though I feel I should be doing something about global warming. I'm actually trying to combine the two. But even if I can't, maybe I need to keep doing some math for purely emotional reasons.

July 18, 2015



This summer I'm working at the Centre for Quantum Technologies in Singapore again. But I spent the last week at Quantum Physics and Logic, an annual conference at Oxford.

I'm mainly studying networks in engineering, biology and chemistry, but a lot of the math I use comes from my my old favorite subject: quantum physics. So, it was great to see the latest things my friends and their students are doing now.

The prize-winning student paper was written by Amar Hadzihasanovic, from the computer science department at Oxford. Yes, computer science! That's because quantum computers and quantum cryptography are hot topics now.

To explain a bit about Hadzihasanovic's paper, I have to start with Schrödinger's cat, a thought experiment in which you put a cat into a quantum superposition of two dramatically different states: one live, one dead. Nobody has actually done this, but people have tried to see how close they can get.

Physicists have succeeded in making light in a quantum superposition of two dramatically different states. In classical mechanics we think of light as a wave. In a so-called cat state, we have light in a superposition of states where the peaks and valleys of this wave are in different places.

Another kind of cat state involves a bunch of particles that can have spin pointing up or down. For example, if you have 3 of these particles, you can make a state

$$\uparrow\uparrow\uparrow + \downarrow\downarrow\downarrow$$

It takes work to do it, though — and more work to check that you've succeeded!

The first success came in 1998, by a team of experimentalists led by Anton Zeilinger. So, this particular kind of cat state is usually called a Greene-Horn-Zeilenger state or 'GHZ state' for short.

What's interesting about the GHZ state is that if you look at any *two* of the particles, you don't see the spooky quantum effect called entanglement. Only all three particles taken together are entangled. It's like the Borromean rings, three rings that are linked even though no two are linked to each other.

Another interesting state of 3 particles is called the 'W state':

$$\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow$$

In this state, unlike the GHZ state, you can see entanglement by looking at any two particles.

In fact, there's a classification of states of 3 particles that can have spin up or down, and besides the boring unentangled state

$$\uparrow\uparrow\uparrow$$

the only other possibilities &emdash; apart from various inessential changes, like turning up to down &emdash; are the GHZ state and the W state.

This is why the GHZ state and W state are so important: they're fundamental building blocks of quantum entanglement, just one step more complicated than the all-important 'Bell state'

$$\uparrow\downarrow + \downarrow\uparrow$$

for two particles.

What Amar Hadzihasanovic did is give a complete description of what you can do with the GHZ and W states, in terms of diagrams. He explained how to use pictures to design states of more particles from these building blocks. And he found a complete set of rules to tell when two pictures describe the same state!

You can see these pictures here:

- Amar Hadzihasanovic, A diagrammatic axiomatisation for qubit entanglement.

Since this paper he's been working to make the rules simpler and more beautiful. There's a lot of cool math here.

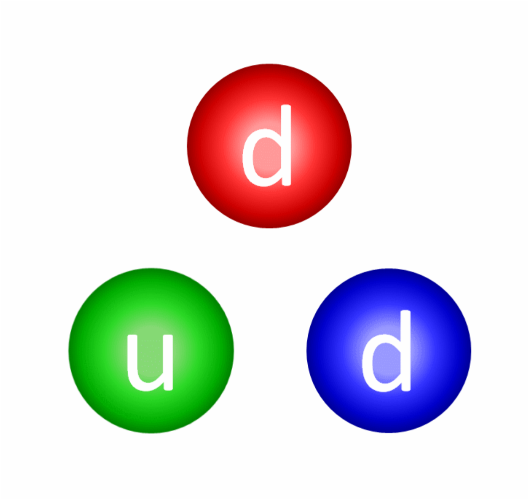
The Steve Cundiff group at Marburg University is doing research on cat states of light, and the picture here comes from a page on his work:

- Quantum mechanics of nanoparticles: quantum light shaping for measurement and control, NIST, University of Colorado Boulder.

For more, see:

- Daniel M. Greenberger, Michael A. Horne, Anton Zeilinger, Going beyond Bell's Theorem.

July 20, 2015



Last week a team at CERN says they might have seen some pentaquarks! Physicists have been looking for them. Back in 2005 Japanese researchers claimed they saw some, but this was later discredited. I hope this new claim holds up.

What's a pentaquark? It's not really 5 quarks. It's actually 4 quarks and an antiquark, all held together by exchanging other particles called gluons.

Let's start with something easier: a neutron, as shown here. A neutron consists of 3 quarks: one up quark and two down quarks. They're actually zipping around like mad in a blurry quantum way, but this movie simplifies things.

Besides coming in various kinds, like up and down, quarks have an easily changeable property called color. This is nothing like ordinary color — but color serves as a convenient metaphor, and physicists occasionally have a sense of humor, so that's what they called it.

There's a lot of math underlying this story, but let's sweep that under the carpet and talk about color in simple terms, so you can explain pentaquarks to your children and parents.

Quarks can be in 3 different colors, called 'red', 'green' and 'blue'. But they can only stick together and form a somewhat stable particle if all three colors add up and cancel out to give something 'white'. So, protons and neutrons are made of 3 quarks.

The quarks stick together by exchanging gluons, which have subtler colors like 'red-antigreen' and 'green-antiblue'.

If you watch this movie of a neutron, you'll see a red quark emit a red-antigreen gluon and turn green. This red-antigreen gluon is then absorbed by a green quark, turning it red. Color is conserved like this! The total color of the neutron remains white.

You can't build something white out of just a single quark, so we never see lone quarks in nature. The closest you can come is at *insanely* high temperatures when everything is shaking around like mad and you get a quark-gluon plasma. I'm talking temperatures of several trillion degrees Celsius! People have gotten this to happen at places like the Relativistic Heavy Ion Collider on Long Island, New York.

You also never see a particle built of just 2 quarks. Again, the reason is that it can't be white.

But you can get particles built of a quark and an antiquark — their colors can cancel.

You can't build a particle out of 4 quarks, because the colors can't cancel.

But you can do 3 quarks together with an extra quark and antiquark! And that's called — somewhat misleadingly — a pentaquark.

Here's the paper:

- LHCb collaboration: R. Aaij, B. Adeva, M. Adinolfi, A. Affolder, Z. Ajaltouni, S. Akar, J. Albrecht, F. Alessio, M. Alexander, S. Ali, G. Alkhazov, P. Alvarez Cartelle, A.A. Alves Jr, S. Amato, S. Amerio, Y. Amhis, L. An, L. Anderlini, J. Anderson, G. Andreassi, M. Andreotti, J.E. Andrews, R.B. Appleby, O. Aquines Gutierrez, F. Archilli, P. d'Argent, A. Artamonov, M. Artuso, E. Aslanides, G. Auriemma, M. Baalouch, S. Bachmann, J.J. Back, A. Badalov, C. Baesso, W. Baldini, R.J. Barlow, C. Barschel, S. Barsuk, W. Barter, V. Batozskaya, V. Battista, A. Bay, L. Beaucourt, J. Beddow, F. Bedeschi, I. Bediaga, L.J. Bel, V. Bellee, N. Belloli, I. Belyaev, E. Ben-Haim, G. Bencivenni, S. Benson, J. Benton, A. Berezhnoy, R. Bernet, A. Bertolin, M.-O. Bettler, M. van Beuzekom, A. Bien, S. Bifani and 662 other authors, Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \to J/K^- p$ decays.

It's not unusual to have lots of authors on these papers, but it's rather unusual to list them in alphabetical order. I like that system, especially since it usually puts me near the front.

In case you're wondering, the theory behind all this is quantum chromodynamics, which is based on quantum field theory, in particular Yang-Mills theory, and on the representation theory of the group SU(3).

It is conjectured but not yet proved that quantum chromodynamics is mathematically consistent and that stable particles must all be 'white', that is, transform in the trivial representation of SU(3). We describe quarks using the fundamental representation of SU(3) on C^3, which has 3 basis vectors whimsically called red, blue and green. We describe antiquarks using the dual representation, which has 3 basis vectors called anti-red, anti-blue and anti-green. We describe gluons using the adjoint representation, which has basis vectors like red-antiblue.

If you want to carry the color analogy even further, you can call anti-red, anti-blue and anti-green 'cyan', 'yellow' and 'magenta'.

However, you need to be careful. Cyan, yellow and magenta do not combine to form 'black'. They form 'antiwhite', but antiwhite is white - that's what the math says, and the math is more fundamental than the cute analogy to colors.

Also, gluons only come in 8 colors, not 9.

Puzzle 1: Why? If you know some math you may know SU(3) is 8-dimensional so we can't get 9, but try to explain the story in terms of colors.

Puzzle 2: If you build particles using only quarks, not antiquarks, could you build something white with 4 quarks? How about 5? How about 6? What's the rule?

For answers to the puzzles, see the comments on my Google+ post. For more, read:

- Wikipedia, Color charge.

The animated gif was made by Qashqaiilove.

July 21, 2015

It's 1 meter long and it weighs 150 kilos. When it crawls on land to lay its eggs, a green sea turtle looks clumsy and awkward. But in its true home — the sea — it's beautiful and graceful!

And here's some good news. Back in the 1980s, when scientists first started counting the nests of green sea turtles in one area in Florida, they found fewer than 40 nests. Now they count almost 12,000!

We can thank the Endangered Species Act, which brought sea turtles under protection in 1978. We can also thank state laws discouraging development on Florida beaches — and the Archie Carr National Wildlife Refuge, which was established in 1991.

Endangered species can bounce back! Animals like nothing better than to breed, after all.

Sea turtles have been around since the late Triassic Period, 245 million years ago. During the Mesozoic Era turtles went back and forth between land and sea. But modern sea turtles, with flippers instead of claws, evolved about 120 million years ago, during the Cretaceous Period. They survived the extinction of the dinosaurs — and with a bit of luck, they'll survive us.

For more, listen to this story:

- Amy Green, Florida sea turtles stage amazing comeback, *All Things Considered*, National Public Radio, 13 July 2015.

The picture is from here:

- Stephen Brown, Palm Beach for kids: North County, *Palm Beach Illustrated*.

For more on the green sea turtle, *Chelonia mydas*, read this:

- Information about sea turtles: Green Sea Turtle, Sea Turtle Conservancy.

July 30, 2015

We've just arrived in Yogyakarta today — that's a city in Java — and look what my wife Lisa has gotten us into:

These are Indonesian reporters interviewing her near the sultan's palace. She had just won a race where she had to walk 50 meters wearing a mask like the head of a monkey. The mask covers your whole head, with no eye holes, so it prevents you from seeing. You need to go between two banyan trees and cross the finish line without veering off the track.

Most of the contestants were Indonesian high school kids, so the reporters were interested to see an American woman try this — and win!

A few years ago another American woman, Della Bradt, wrote:

> In the middle of the square are two giant banyan trees. There is a challenge associated with these trees that we've been itching to try since day one. You have to start from 50 meters away from the trees and you are blindfolded. Then you are spun around 3 times to the right and 3 times to the left. Once you are oriented towards the trees, you have to walk in a straight line through the middle of them moving from North to South. While the opening between the trees is very wide, it's extremely difficult to accomplish. I actually failed a spectacular 3 times in a row. Each time I veered in the opposite direction of the trees. While 2 people in our group made it, most people weren't able to. It is said that those who make it through will have success and good fortune in their lives.

Here is someone (not Lisa) in a monkey's head mask:

Here is one of the banyan trees:

This is just one of the many adventures we had today. People here seem happy to strike up conversations. We met a music teacher who invited us to a free gamelan class and carefully explained the cheapest ways to get to the temples of Borobudur and Prambanan, and a soon-to-be-retired worker at a local church who walked us to a batik store near his house — cheaper and better than the more touristic ones on the shop-lined street called Jalan Malioboro. We didn't buy anything, but it was fun to see. Similarly, it was at some local guy's suggestion that Lisa decided to join the monkey-head race. Walking around, and being open to such random stuff, makes traveling more fun around here.

For more of our adventures, read on!

[For my August 2015 diary, go here.]

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baez@math.removethis.ucr.andthis.edu

home

For my July 2015 diary, go here.

Diary — August 2015

August 1, 2015

Today Lisa and I explored Prambanan, the largest Hindu temple in Java. I was overwhelmed by the massive structures - at the bottom you'll see tiny people.

How was it built? It all started when a prince named Bandung Bondowoso fell in love with a beautiful princess named Rara Jonggrang. He proposed marriage. She rejected him because he had killed her father, the cruel man-eating giant King Boko. But Bandung Bondowoso insisted. Finally Rara Jonggrang relented and agreed to marry him... but only on one condition: he had to build her a thousand temples in one night.

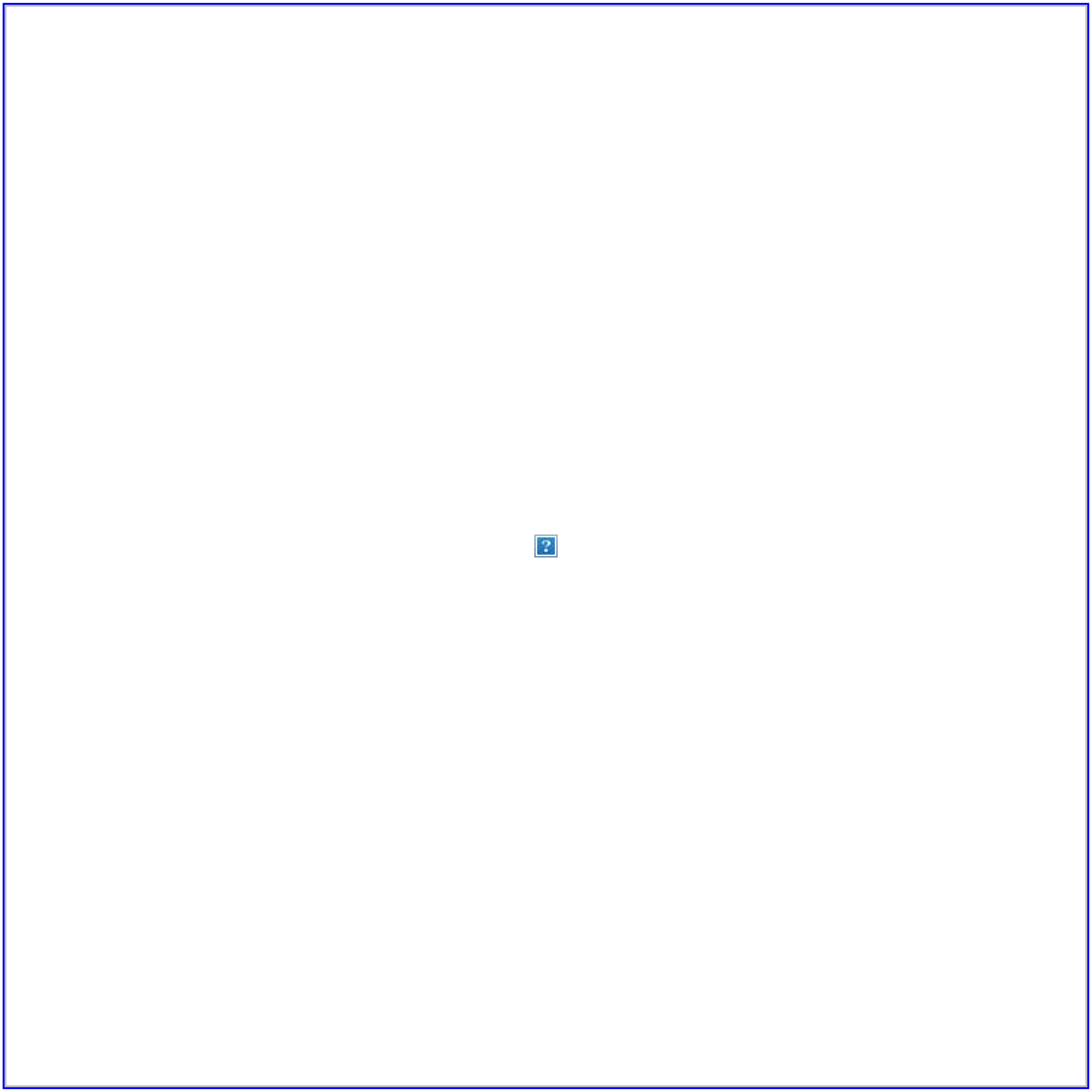
The Prince entered into meditation and conjured up a multitude of demons from the earth. With their help, he succeeded in building 999 temples. But just as he was about to complete his task, the princess woke her palace maids and ordered the women of the village to begin pounding rice and set a fire in the east of the temple, to make the prince and his [illegible] rise.

As the cocks began to crow, fooled by the light and the typical sounds of morning, the prince's demon helpers fled back into the ground. The prince was furious! In revenge he cursed Rara Jonggrang — and turned her to stone. She became the last and most beautiful of the thousand temples.

Well, okay — this is how the local Javanese peasants _said_ these temples were built, long after they were actually built in 850 AD, abandoned to the jungle after a volcanic eruption in 930, and partially destroyed by a major earthquake in the 1600s. In reality they were built by the Medang Kingdom, a Hindu-Buddhist kingdom that thrived from 732 to 1006 AD, whose sphere of influence reached all the way to Angkor Wat in Cambodia.

Speaking of "demons from the earth": in California we have earthquakes, but in Java you _really_ sense the power of shifting tectonic plates! As recently as February last year, Borobudur and Prambanan were closed due to volcanic ashes from the eruption of a volcano called Raung 200 kilometers away. Four years earlier, a much closer volcano called Merapi erupted. You can see this one towering over Yogyakarta on days when the clouds and smog aren't too thick. But the ashes from that eruption missed Prambanan, and only hit Borobudur. Right now yet another volcano on Java, Raung, is disrupting air travel to Bali.

August 4, 2015



Today Lisa and I visited Borobudur, the largest temple in Java, and the biggest Buddhist temple in the world.

Nobody knows who built it, sometime around 800 AD. It was eventually abandoned, and it lay hidden for centuries, gradually buried by volcanic ash and thick jungle growth. In 1814, the British colonist Thomas Raffles — who also founded Singapore — traveled through Java and heard of a huge monument near Yogyakarta, the old capital of this island. He got a Dutch engineer to investigate, and by 1885 the jungle was cleared away and the full extent of Borobudur was revealed!

The picture above is from Trey Ratcliff. It shows the view from the top of Borobudur, which is covered with giant bell-like stupas.

Built of 2 million stone blocks, Borobudur is actually designed in the form of a mandala: a diagram of Buddhist cosmology that serves as an aid to meditation. From above, it looks like this:

At the bottom is the outer square wall. Then there are 5 nested square levels, each a bit smaller than the one below, connected by stairs. Then there are 3 round levels. The first has 32 bell-like stupas on it, the second has 24, and the third has 16. Finally, there is a huge central stupa at the very top.

The bottom level of Borobodur represents Kāmadhātu: the world of desires. The ideas is that most sentient beings live here.

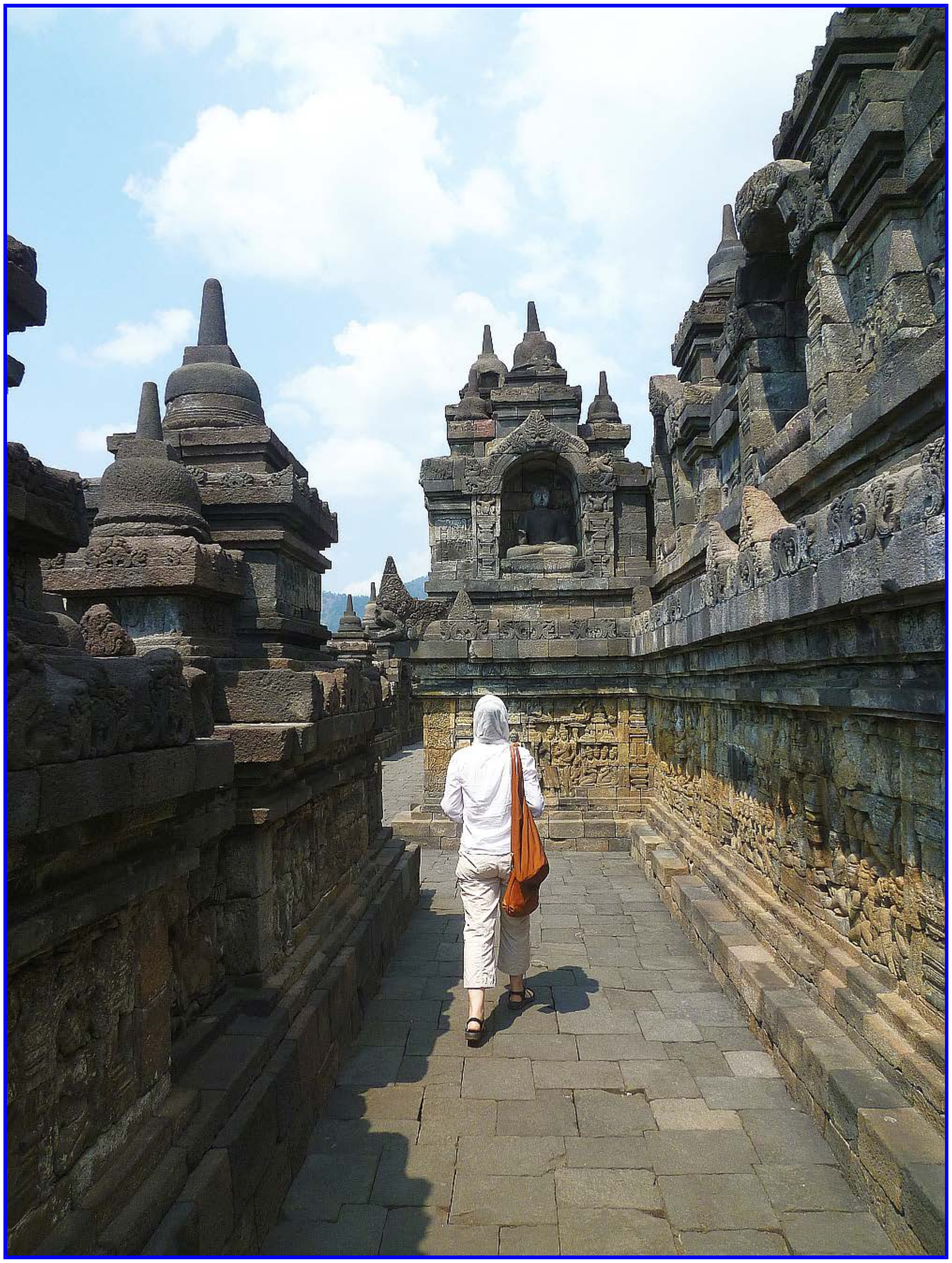
The 5 square levels represent Rūpadhātu: the world of forms, or patterns. Beings who have burnt out their desire for continued existence live here: they see forms, but aren't drawn to them.

The 3 round levels on top represent Arūpadhātu, the formless world. The idea is that buddhas live here: they go beyond form and experience reality at its most fundamental level, the formless ocean of nirvana.

Here is a cross-section:

The idea is that pilgrims should go up one level at a time, walking all the way around and meditating on the wall carvings and statues before climbing the steep stairs to the next level. The pilgrim's walk to the top is 5 kilometers long, and it passes by 1460 stone panels illustrating stories, as well as 1212 decorated panels.

Here's Lisa walking along one of the lower terraces of Borobudur, looking at these panels. She's somewhere in Rūpadhātu, the world of forms:



August 6, 2015

As we remember the bombing of Hiroshima 70 years ago today, let's not forget that the US strategy of mass slaughter of Japanese civilians didn't start there. 70 years ago on March 10th, even more people were killed in the firebombing of Tokyo — a city where most houses were made of paper.

279 planes flew over the city and dropped 1,665 tons of bombs. Most were 500-pound cluster bombs, each one releasing 38 incendiary bomblets at an altitude of about 2000 feet. These bomblets punched through the roofs of people's houses or landed on the ground and ignited 3.5 seconds later, throwing out jets of flaming, sticky napalm.

The planes also dropped 100-pound jelled-gasoline and white phosphorus bombs that ignited upon impact. The city's fire departments were overwhelmed, and the individual fires started by the bombs joined to create a huge conflagration that destroyed 16 square miles of the city. Over 100,000 people died — nobody knows how many, and both the Japanese and Americans had reasons to underestimate the casualties.

General Curtis LeMay, who led this attack, said:

> Killing Japanese didn't bother me very much at that time... I suppose if I had lost the war, I would have been tried as a war criminal.

Joe O'Donnell, a marine sent in after the war to document the effects of the bombing, wrote:

> The people I met, the suffering I witnessed, and the scenes of incredible devastation taken by my camera caused me to question every belief I had previously held about my so-called enemies.

The picture above shows the charred corpse of a woman in Tokyo who was carrying a child on her back. In this style of war, cities of people, many perfectly innocent, are treated like rats to be exterminated. Martin Middlebrook captured the horror of this in Hamburg, one of the German cities firebombed by the US:

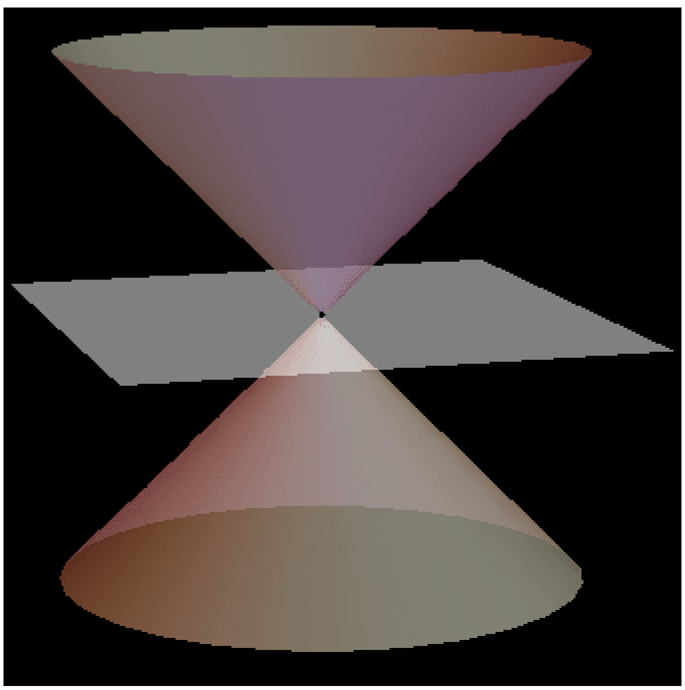
> A thermal column of wind generated heat in excess of 1,400 degrees Fahrenheit, melting trolley windows and the asphalt in streets, the wind uprooting trees. When people crossed a street, their feet stuck in the melted asphalt; they tried to extricate themselves with their hands, only to find them stuck as well. They remained on all fours screaming. Small children lay like "fried eels" on the pavement. The firestorm sucked all the oxygen out of the city.

Let's try to avoid this, eh? It's not necessarily easy, and I'm not saying I know how, but let's try to avoid making our world into a hell.

(Over on Google+, we had a conversation about how.)

August 12, 2015

THE CONIC SECTIONS



Here you can see a plane moving though a cone. Most of the time the plane intersects the cone in a curve. These curves are called 'conic sections'. They have famous names and formulas:

- Circles: $x^2 + y^2 = r^2$ with $r > 0$
- Ellipses: $ax^2 + by^2 = r^2$ with $a, b, r > 0$
- Hyperbolas: $ax^2 - by^2 = r^2$ with $a, b, r > 0$
- Parabolas: $y = ax^2$ with $a > 0$

I haven't given the *most general* formula for each kind of curve, but my formulas are enough to describe all possible shapes and sizes of these curves. For example, if you have an upside-down parabola $y = -2x^2$ you can rotate it so it looks like $y = x^2$. So, I say they have the same shape, and I don't bother listing both.

However, there are a few other cases that aren't on this list, which are still extremely important! These are the other shapes you can define using equations of the form

$$ax^2 + bxy + cy^2 = 0$$

- You can get two lines that cross. This equation

$$x(y - mx) = 0$$

  describes a vertical line together with a line of slope m.
- You can get a line:

$$x^2 = 0$$

- You can get a point:

$$x^2 + y^2 = 0$$

Ordinary folks wouldn't call these 'curves'. The last two special cases are especially upsetting! But the famous mathematician Grothendieck figured out a way to improve algebraic geometry so that these cases are on the same footing as the rest. In particular, he made it really precise how

$$x^2 = 0$$

is different, in an important way, from

$$x = 0$$

The second one is an ordinary line, given by a linear equation. The first one is a 'double line', the *limit of two lines as they get closer and closer!* Watch the movie and see how we get to this 'double line', and you'll see what I mean.

People in algebraic geometry had already thought about 'double lines' and similar things, but Grothendieck's theory of 'schemes' explained what these things really are. Whatever it is, a double line is not just set of points in the plane — if we look at the set of points, there's no difference between the double line

$$x^2 = 0$$

and the single line

$$x = 0$$

The double line is something else — it's a 'scheme'.

But now it's time for breakfast, so I can't tell you what a 'scheme' actually *is*. Instead, I'll just say this. Any set of polynomial equations defines an 'affine scheme', and that's not a very complicated thing. Alexander Grothendieck's real stroke of genius was to develop a more general notion of scheme, which is built out of pieces that are affine schemes. Grothendieck developed schemes, and more, as part of his attack on a very hard problem in number theory, called the Weil conjectures. But his attack was a gentle one. Instead of using brute force to crack this nut, he preferred to slowly 'soften' the problem by inventing new concepts. Here's what he wrote about this:

> The analogy that came to my mind is of immersing the nut in some softening liquid, and why not simply water? From time to time you rub so the liquid penetrates better, and otherwise you let time pass. The shell

> becomes more flexible through weeks and months — when the time is ripe, hand pressure is enough, the shell opens like a perfectly ripened avocado!

The gif in this post is from Math Gif.

August 14, 2015

Newton said he saw further because he stood on the shoulders of giants. But this amazing sculpture illustrates how we're also weighed down and blinded by the prejudices of those who came before us — who were in turn blinded by *their* predecessors.

This sculpture is called 'Karma'. It was made by the Korean artist Do Ho Suh. It looks infinitely tall, especially in the first picture above. But in fact it's 7 meters tall (23 feet), built from 98 figures of men, each one covering the eyes of the one below. I think it looks taller because they shrink as you go further up, providing a false perspective that makes them seem to go on forever. You can see more photos here:

- Pinar, Statues of blinded men ascending high into the sky, *My Modern Met*, February 26, 2013.

The photos were taken by by Lehmann Maupin, Alan Teo and Eric Harvey Brown, and CamWall.

This sculpture can be seen at the Sydney and Walda Besthoff Sculpture Garden at the New Orleans Museum of Art, so if you're anywhere near New Orleans, check it out!

As I mentioned, Newton said:

> If I have seen further, it is by standing on the shoulders of giants.

He said this in response to a letter to his competitor Robert Hooke, and some have interpreted it as a sarcastic poke at Hooke's slight build. But in fact they were on good terms at the time, and came to dislike each other only later.

Murray Gell-Mann, the theoretical physicist who came up with the idea of 'quarks', was definitely taking a poke at his competitors when he said:

> If I have seen further, it is because I was surrounded by dwarves.

Ouch! Over on G+, Esko Arajärvi provided some other versions:

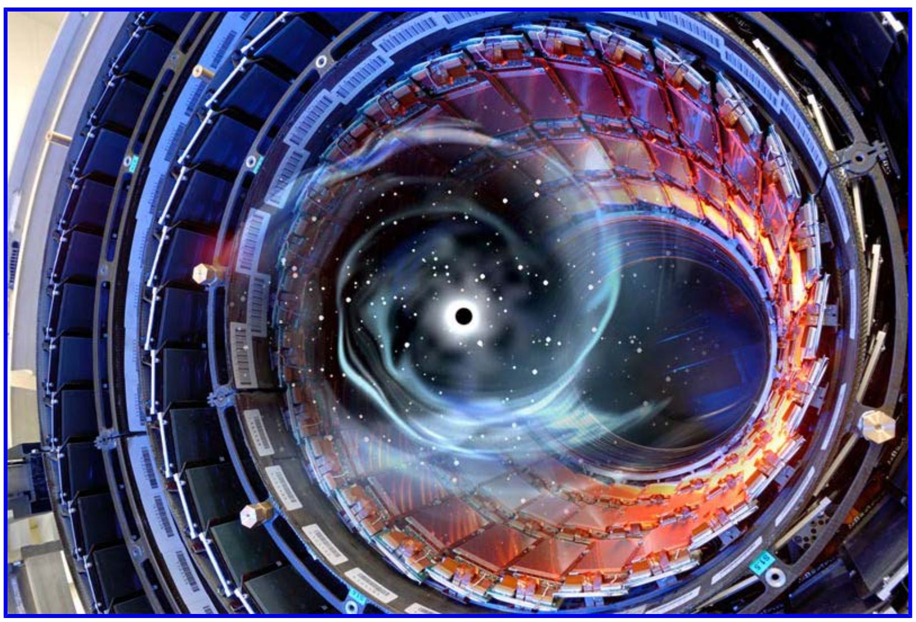
> In the sciences we are now uniquely privileged to sit side by side with the giants on whose shoulders we stand. — Gerald Holton

> If I have not seen as far as others, it is because giants were standing on my shoulders. — Hal Abelson

For more on the history of such quotes, see:

- Wikipedia, Standing on the shoulders of giants.

August 15, 2015



Brian Koberlein, one of the most consistently energetic and interesting people on Google+, recently wrote about how to make a black hole.

His recipe works like this:

> **INGREDIENTS:** one small neutron star, one solar mass of hydrogen.
>
> Take a neutron star 2 weighing solar masses. Gradually add one solar mass of hydrogen gas, letting it fall to the surface of the neutron star. Be careful: if you add too much too quickly, you'll create a huge nuclear explosion called a nova. When your neutron star reaches 3 solar masses, it will collapse into a black hole.

This is the smallest type of black hole we see in nature. The problem with this recipe is that we'd need to become at least a Kardashev Type II civilization, able to harness the power of an entire star, before we could carry it out.

My friend Louis Crane, a mathematician at the University of Kansas, has studied other ways to make a black hole. It's slightly easier to make a smaller black hole — and perhaps more useful, since the Hawking radiation from a small black hole could be a good source of power.

Crane is interested in powering starships, but we could also use this power for anything else. It's the ultimate power source: you drop matter into your black hole, and it gets turned into electromagnetic radiation!

Unfortunately, even smaller black holes are tough to make. Say you want to make a black hole whose mass equals that of the Earth. Then you need to crush the Earth down to the size of a marble. The final stage of this crushing process would probably take care of itself: gravity would do the job! But crushing a planet to half its original size is not easy. I have no idea how to do it.

Luckily, to make power with Hawking radiation, it's best to make a much smaller black hole. The smaller a black hole is, the more Hawking radiation it emits. Louis Crane recommends making a black hole whose mass is a million tonnes. This would put out 60,000 terawatts of Hawking radiation. Right now human civilization uses only 20 terawatts of power. So this is a healthy power source.

You have to be careful: the radiation emitted by such a black hole is incredibly intense. And you have to keep feeding it. You see, the smaller a black hole is, the more Hawking radiation it emits — and as it emits radiation, it shrinks! Eventually it explodes in a blaze of glory: in the final second, it's about 1/100 as bright as the Sun. To keep your black hole from exploding, you need to keep feeding it. But for a black hole a million tons in mass, you don't need to rush: it will last about a century before it explodes if you don't feed it.

Unfortunately, to make a black hole that weighs a million tonnes, you need to put a million tonnes of mass in a region 1/1000 times the diameter of a proton.

This is about the wavelength of a gamma ray. So, if we could make gamma ray lasers, and focus them well enough, we could in theory put enough energy in a small enough region to create a million-ton black hole. He says:

> Since a nuclear laser can convert on the order of 1/1000 of its rest mass to radiation, we would need a lasing mass of about a gigatonne to produce the pulse. This should correspond to a mass of order 10 gigatonnes for the whole structure (the size of a small asteroid). Such a structure would be assembled in space near the sun by an army of robots and built out of space-based materials. It is not larger than some structures human beings have already built. The precision required to focus the collapsing electromagnetic wave would be of an order already possible using interferometric methods, but on a truly massive scale. This is clearly extremely ambitious, but we do not see it as impossible.

I'm not holding my breath, but with luck our civilization will last long enough, and do well enough, to try this someday.

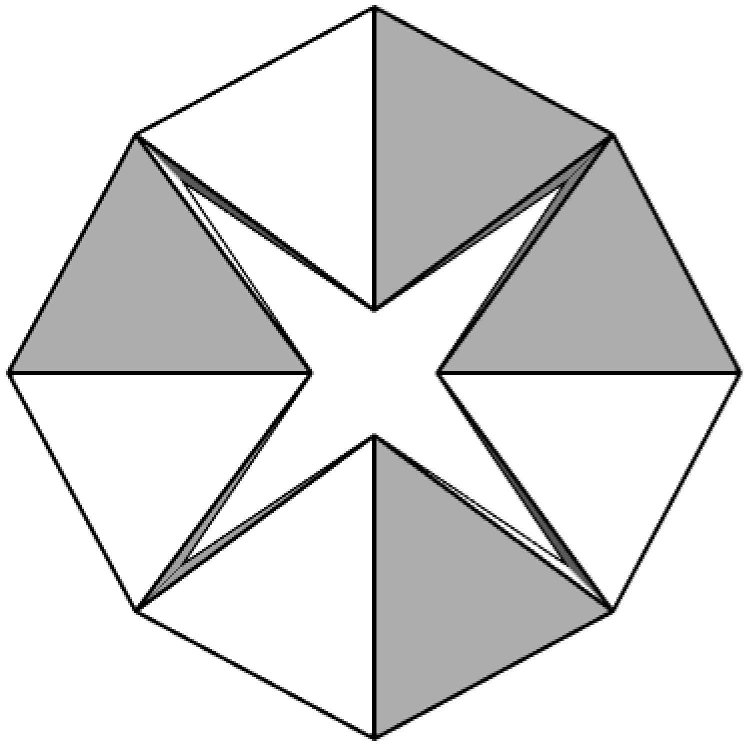
For details, see:

- Louis Crane and Shawn Westmoreland, Are black hole starships possible?

Here is Brian Koberlein's post on how to build a black hole:

- How to build a black hole, *One Universe at a Time*, August 13, 2015.

August 19, 2015



It's a bit surprising. You can take 8 perfectly rigid regular tetrahedra and connect them along their edges to form a ring that you can turn inside-out!

It's called a 'kaleidocycle', and you can actually make one with any even number of tetrahedra, as long as you have at least 8. Fewer than 8, and they bump into each other.

You can see kaleidocycles with 8, 10, and 12 tetrahedra here:

- Archery, Kaleidocycle, *Into the Continuum*, March 19, 2013.

You can also make kaleidocycles out of paper:

- Kaleidocycles, *Mathematische Basteilein*.

and this website shows other kinds, too. For example, there's a kaleidocycle that's a ring of 16 pyramids, all the same size and shape, that folds up into a perfect regular tetrahedron! And there's another made of 16 pyramids, all some other size and shape, that folds into an octahedron!

What's all this good for? I have no idea. But it illustrates the limits of the Rigidity Theorem. This theorem says if the faces of a *convex polyhedron* are made of a rigid material and the polyhedron edges are hinges, the polyhedron can't change shape at all: it's rigid. The kaleidohedra show this isn't true for a polyhedron with a hole in it.

Of course, having a hole is an extreme case of being nonconvex. To be nonconvex, your polyhedron only needs to have a 'dent' in it. And there are nonconvex polyhedra without a hole that aren't rigid! The first of these was discovered by a guy named Connelly in 1978. It has 18 triangular faces.

In 1997, Connelly, Sabitov and Waltz proved something really cool: the Bellows Conjecture. This says that a polyhedron that's not rigid must keep the same volume as you flex it!

The famous mathematician Cauchy claimed to prove the Rigidity Theorem in 1813. But there was a mistake in his proof. Nobody noticed it for a long time. It seems mathematician named Steinitz spotted the mistake and fixed it in a 1928 paper.

Puzzle 1: what was the mistake?

Still, people often call this result 'Cauchy's Theorem', which is really unfair, especially since Cauchy has other, better, theorems named after him.

Later the rigidity theorem was generalized to higher-dimensional convex polytopes. (A polytope is a higher-dimensional version of a polyhedron, like a hypercube.)

It's also been shown that 'generically' polyhedra are rigid, even if they're not convex. In other words: if you take one that's not rigid, you can change its shape just a tiny bit and get one that's rigid.

So, there are lots of variations on this theme: it's very flexible.

Puzzle 2: can you make higher-dimensional kaleidocycles out of higher-dimensional regular polytopes? For example, a regular 5-simplex has 6 corners; if you attach 3 corners of one to 3 corners of another, and so on, maybe you can make a flexible ring. Unfortunately this is in 5 dimensions — a 4-simplex has 5 corners, which doesn't sound so good, unless you leave one corner hanging free, in which case you can just take the movie here and imagine it as the 'bottom' of a 4d movie where each tetrahedron is the 'base' of a 4-simplex: sort of boring.

Puzzle 3: is a version of the Bellows Conjecture true in higher dimensions?

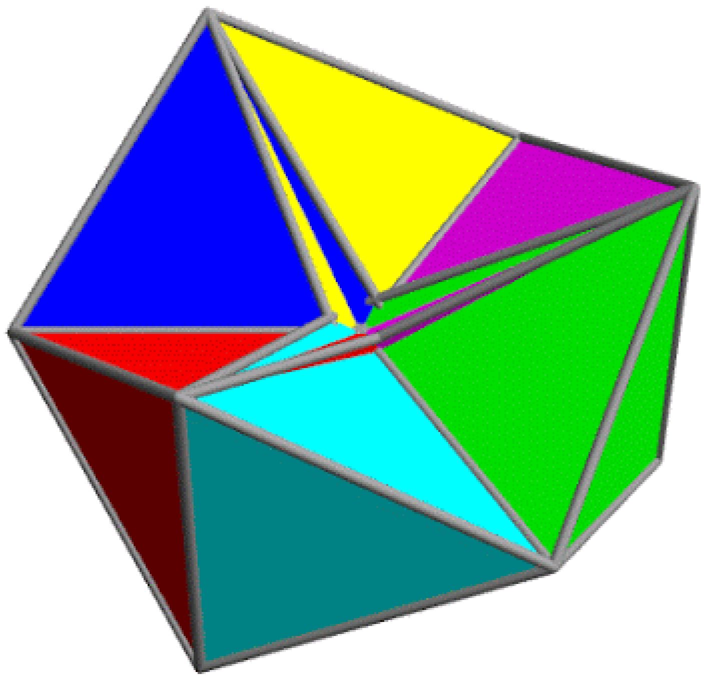
For more, check out these:

- Flexible polyhedron, Wolfram Mathworld.
- Cauchy's Theorem (geometry), Wikipedia.

For some answers and discussions, see my G+ post.

August 19, 2015

COLLIDING KALEIDOCYCLE



My last entry showed a 'kaleidocycle' — a ring of 8 regular tetrahedra joined edge to edge, that you could turn inside out. I said you could build one with any even number of tetrahedra that's at least 8.

Then somebody told me he'd built one with just 6.

Here is Greg Egan's movie of what happens if you try a kaleidocycle with 6 regular tetrahedra. They collide! Very slightly. So, it's not a true kaleidocyle: it's a 'collidocyle'.

In other words: if these tetrahedra are completely rigid, they must pass through each other as they turn. But if you made one out of paper, you might be able to force it to work, by bending the paper slightly.

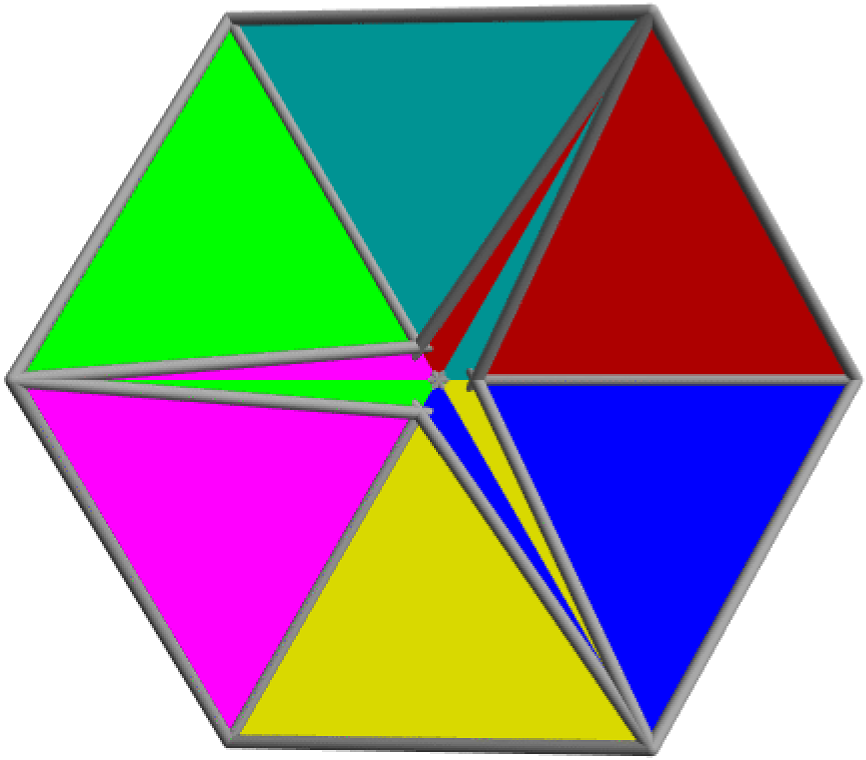
Puzzle 1: Give a mathematical proof that the tetrahedra here must intersect at some point, if they're completely rigid.

Puzzle 2: What's the maximum fraction of the volume here that's contained in the intersection of at least two tetrahedra? It looks like about 2% to me.

Greg Egan made his gif by adapting the Mathematica code at the website I showed you before:

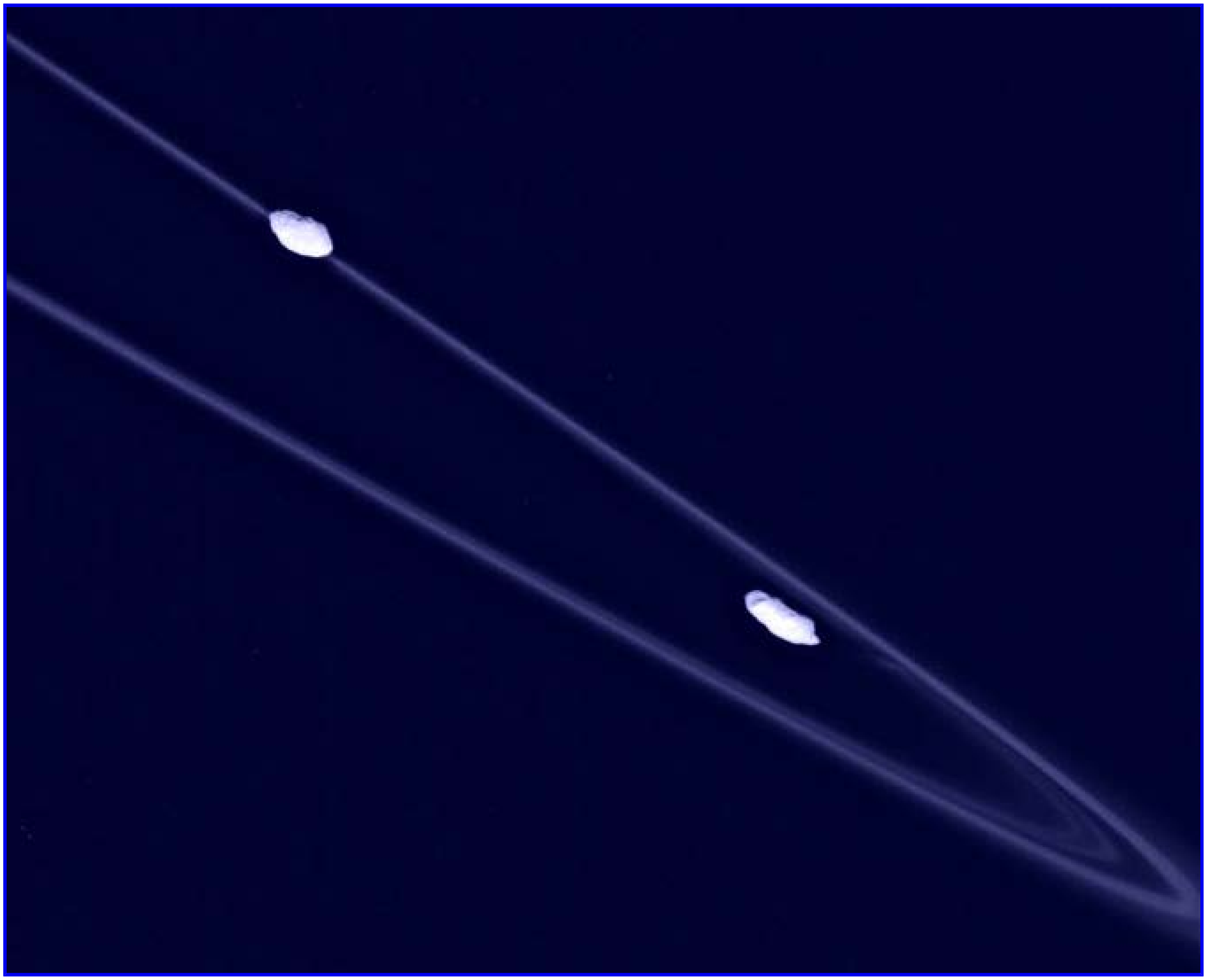
- Archery, Kaleidocycle, *Into the Continuum*, March 19, 2013.

If you view the collidocycle directly from the top, it's very confusing:



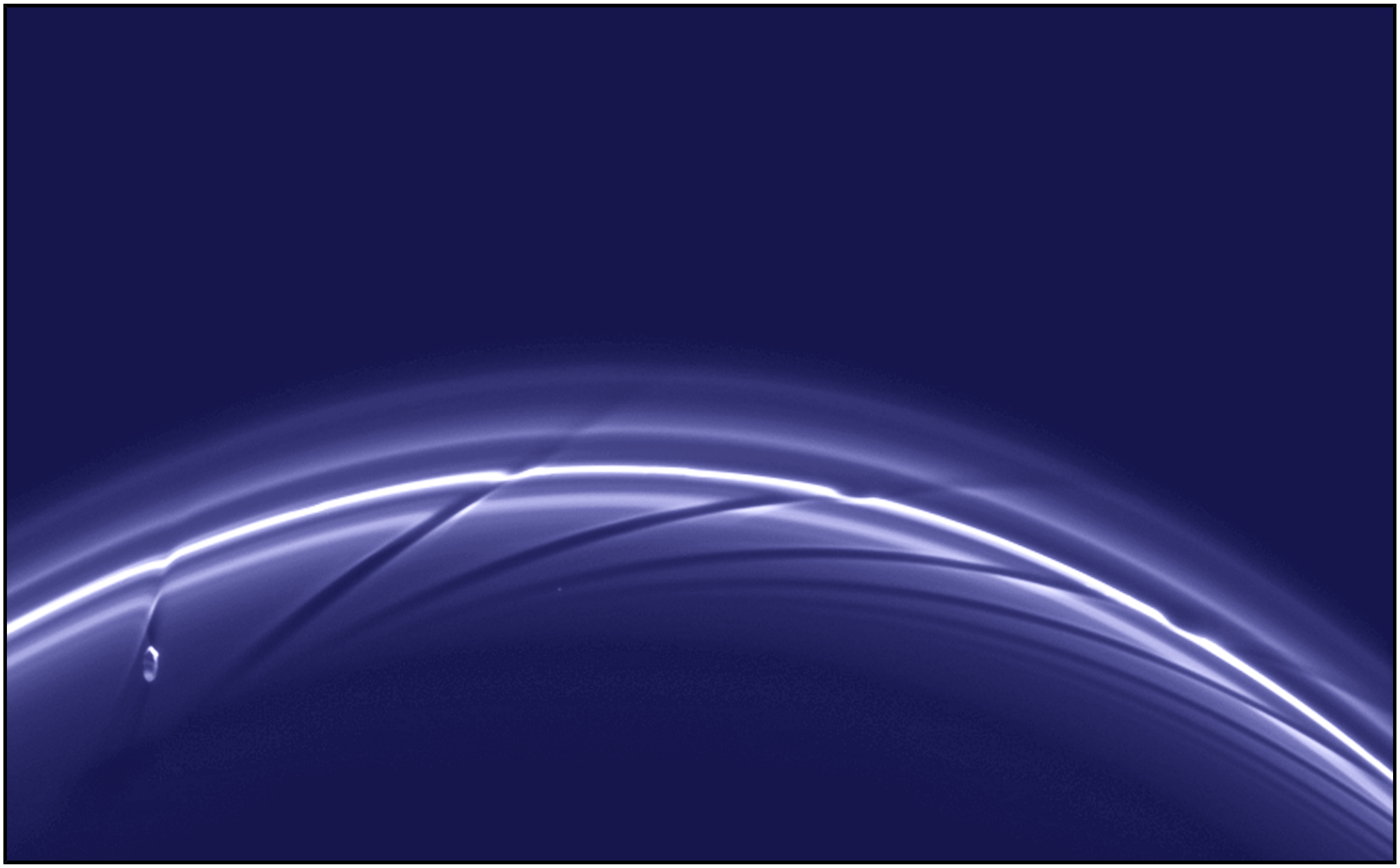
August 24, 2015

THE COLLISION OF PROMETHEUS AND PANDORA



Prometheus and Pandora were characters in a Greek myth, but now they are moons of Saturn. They both orbit close to Saturn's F ring, zipping around this planet once every 15 hours or so.

Here you can see Prometheus carving strange slots in the F ring:



This ring is made of ice boulders, maybe up to 3 meters across. Sometimes these chunks of ice form temporary clumps up to 10 kilometers in size. At other times, these clumps get pulled apart. Prometheus steals boulders from the F ring with its gravitational pull. And each time it comes as close as possible to Saturn, it carves a new slot in the F ring.

Why does this happen? It's complicated, and people keep learning more about it. I'm certainly no expert!

People used to call Prometheus and Pandora shepherd moons. The idea was that they help stabilize the F ring. It's an attractive notion. The singer Enya even made an album with this title.

But more recent work casts doubt on this theory. Last month Emily Lakdawalla of the Planetary Society wrote:

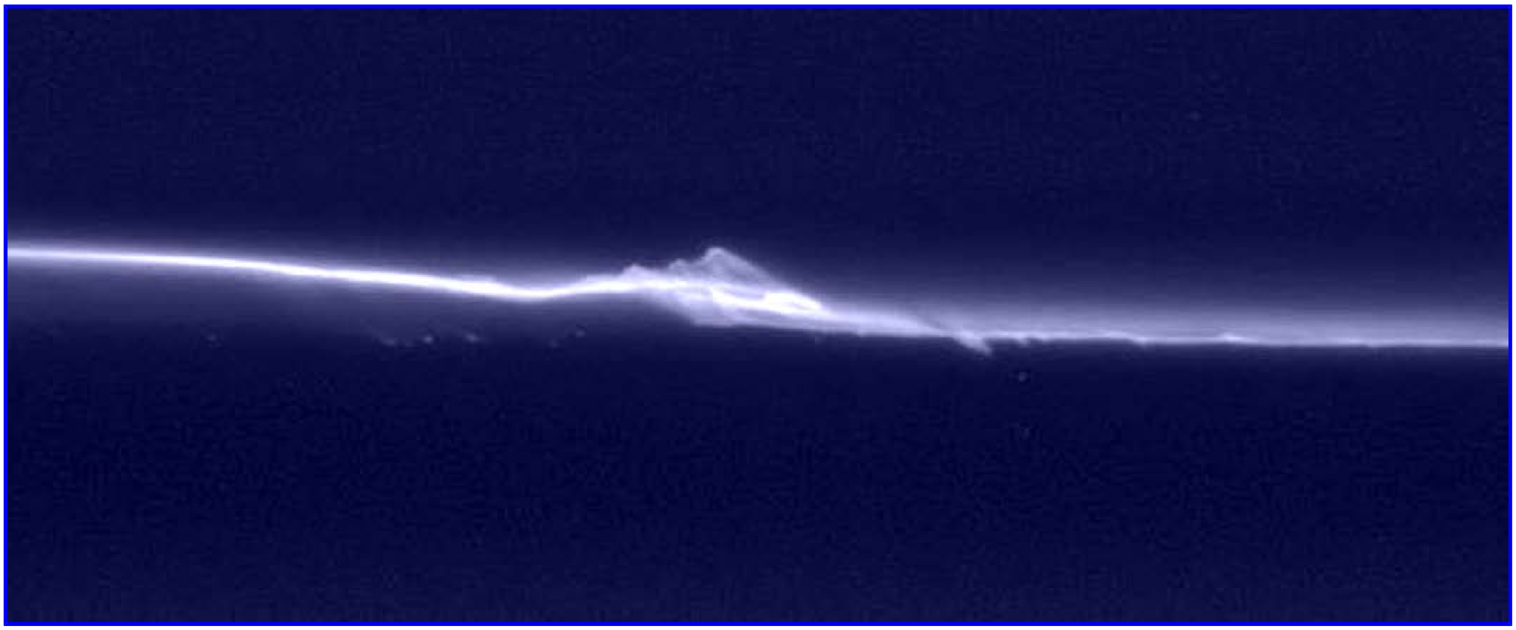
> The most surprising thing I've learned: You know how Prometheus and Pandora are the F ring shepherds? Prometheus on the inside, and Pandora on the outside, herding the billions of tiny particles that make up the ring into place? It's not true. Pandora is not involved in controlling the F ring's tight shape.
>
> The first paper I looked at was written by Jeff Cuzzi and seven coauthors: "Saturn's F Ring core: Calm in the midst of chaos." (Let's pause for a moment to appreciate the quality of that paper title, which is both interesting and accurate, not boring or silly.) The paper seeks to explain why the central core of Saturn's F ring is so consistently shaped, even though various things are constantly acting to perturb it. In particular, Prometheus periodically plunges into the F ring, drawing out dramatic streamers and fans. In fact, Prometheus and Pandora, far from behaving as shepherds, actually act to stir up the motions of particles in most of the region near the F ring. Furthermore, there are other bodies that Cassini has spotted in the F ring region whose behavior is so chaotic that it's been hard to follow them; these things have "violent collisional interactions with the F ring core," so, all in all, it's really difficult to explain why the core of the F ring generally looks the same as it has ever since the Voyagers passed by.

According to her account of some recent papers, the key is a kind of resonance. Resonant frequencies shape Saturn's rings in many ways, but here the key is something called a 'Lindblad resonance'.

The orbit of Prometheus precesses. In other words, the point of this moon's closest approach to Saturn keeps slowly moving around. So, the period with which this moon orbits Saturn is *slightly different* than the period with which it moves in and out from Saturn. A 'Lindblad resonance' happens when a chunk of ice goes around Saturn exactly once each time Prometheus goes in and out! Lakdawalla writes:

> So: consider a moon and a ring particle orbiting Saturn. We don't care (for the moment) what the orbital periods of the moon and ring particles are; what we do care about is the "in-and-out" period of the ring particle in its orbit. You have a Lindblad resonance if, every time the moon passes by the ring particle, the ring particle happens to be on the same position in its in-and-out motion.

The full story is even more complicated than that — obviously, since it has to explain all the weird patterns in the picture here. The F ring consists of several strands, which *braid around each other*, and have strange kinks in them:



But I'll let you read Lakdawalla's blog for more:

- Emily Lakdawalla, On the masses and motions of mini-moons: Pandora's not a "shepherd," but Prometheus still is, *The Planetary Society*, July 5, 2014.

What I *really* want to tell you is some other news: how the F ring was formed in the first place!

It's in an interesting place. Any moon too close to Saturn would be broken up by tidal forces unless it was held together by forces stronger than gravity. The fluid Roche limit says how close is too close: it's 147,000 kilometers from the center of Saturn. The F ring is 140,180 kilometers from the center of Saturn. So it's just within the fluid Roche limit. Pandora and Prometheus are within that limit too — but being at least slightly rigid, they can avoid being stretched and broken apart.

That could be a clue. But how did the F ring actually form? A new paper says it was created by a collision between Prometheus and Pandora! The authors write:

> Saturn's F ring is a narrow ring of icy particles, located 3,400 km beyond the outer edge of the main ring system. Enigmatically, the F ring is accompanied on either side by two small satellites, Prometheus and Pandora, which are called shepherd satellites. The inner regular satellites of giant planets are thought to form by the accretion of particles from an ancient massive ring and subsequent outward migration. However, the origin of a system consisting of a narrow ring and shepherd satellites remains poorly

understood. Here we present N-body numerical simulations to show that a collision of two of the small satellites that are thought to accumulate near the main ring.s outer edge can produce a system similar to the F ring and its shepherd satellites. We find that if the two rubble-pile satellites have denser cores, such an impact results in only partial disruption of the satellites and the formation of a narrow ring of particles between two remnant satellites. Our simulations suggest that the seemingly unusual F ring system is a natural outcome at the final stage of the formation process of the ring-satellite system of giant planets.

If so, the F ring and these moons have been engaged in a drama for millions of years, starting with the very formation of Saturn's rings. We missed the beginning of the show.

The paper is here, but it ain't free:

- Ryuki Hyodo and Keiji Ohtsuki, Saturn's F ring and shepherd satellites a natural outcome of satellite system formation, *Nature Geoscience* (2015).

The other paper I mentioned is free:

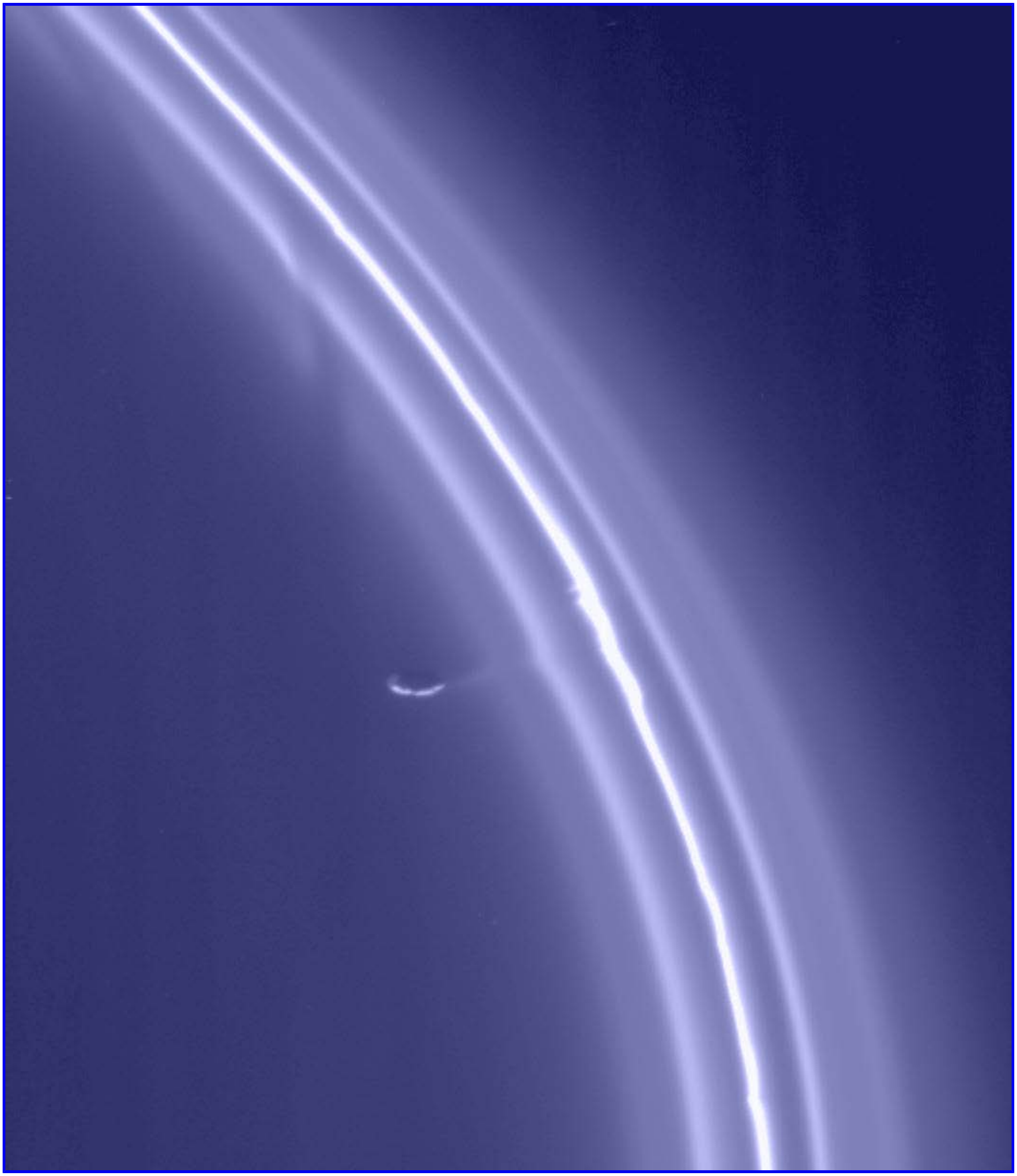
- J. N. Cuzzi, A. D. Whizin, R. C. Hogan, A. R. Dobrovolskis, L. Dones, M. R. Showalter, J. E. Colwell and J. D. Scargle, Saturn's F ring core: calm in the midst of chaos.

  Abstract: The long-term stability of the narrow F Ring core has been hard to understand. Instead of acting as 'shepherds', Prometheus and Pandora together stir the vast preponderance of the region into a chaotic state, consistent with the orbits of newly discovered objects like S/2004 S 6. We show how a comb of very narrow radial locations of high stability in semimajor axis is embedded within this otherwise chaotic region. The stability of these semimajor axes relies fundamentally on the unusual combination of rapid apse precession and long synodic period which characterizes the region. This situation allows stable 'antiresonances' to fall on or very close to traditional Lindblad resonances which, under more common circumstances, are destabilizing. We present numerical integrations of tens of thousands of test particles over tens of thousands of Prometheus orbits that map out the effect. The stable antiresonance zones are most stable in a subset of the region where Prometheus first-order resonances are least cluttered by Pandora resonances. This region of optimum stability is paradoxically closer to Prometheus than a location more representative of 'torque balance', helping explain a longstanding paradox. One stable zone corresponds closely to the currently observed semimajor axis of the F Ring core. While the model helps explain the stability of the narrow F Ring core, it does not explain why the F Ring material all shares a common apse longitude; we speculate that collisional damping at the preferred semimajor axis (not included in the current simulations) may provide that final step. Essentially, we find that the F Ring core is not confined by a combination of Prometheus and Pandora, but a combination of Prometheus and precession.

Whew — that's complicated!

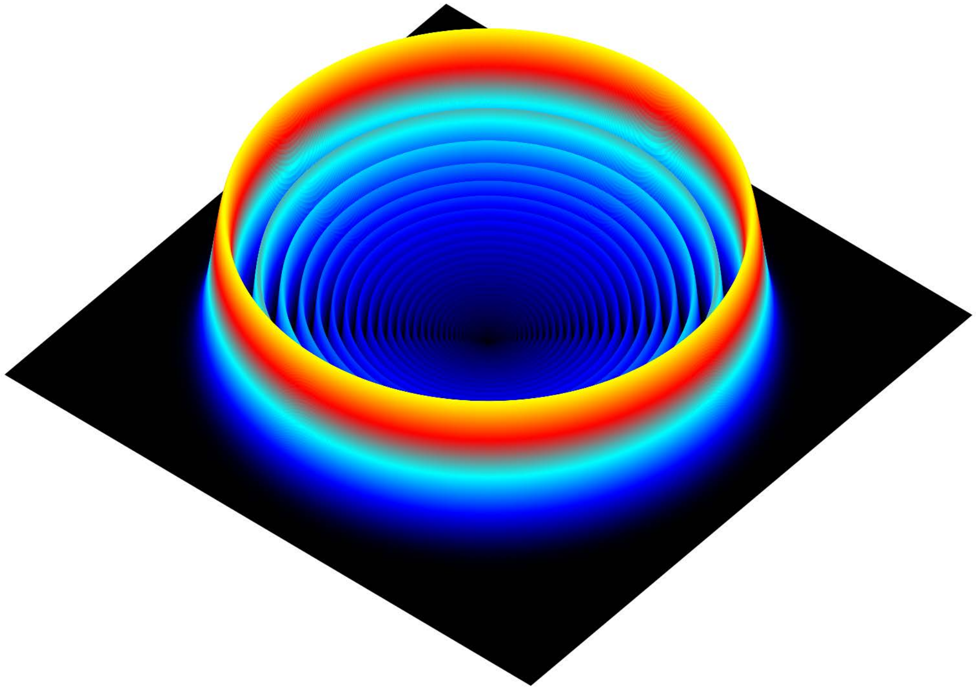
By the way, S/2004 S 6 is a weird little thing they've discovered in the F ring. Nobody even knows if it's solid or just a clump of dust. You can see it here:

Here is another shot of Prometheus and the F ring:



You can see it stealing ice from the F ring! By the way, I shaded all these pictures blue just for artistic effect; they were black and white. Click on the pictures for the originals.

August 28, 2015



There's a game called Robocraft where you try to destroy your enemy's "protonium reactors". These are imaginary devices powered by imaginary "protonium crystals". But I think reality is cooler than any fantasy, so I'm not interested in that crap. I'm interested in *actual protonium!*

Protonium is a blend of matter and antimatter. It's a kind of exotic atom made of a *proton* and an *antiproton*. A proton is positively charged, so its antiparticle, the antiproton, is negatively charged. Opposite charges attract, so a proton and an antiproton can orbit each other. That gives protonium.

A proton and an electron can also orbit each other, and that's called hydrogen. But there are a few big differences between hydrogen and protonium.

First, hydrogen lasts forever, but protonium does not. When they meet, the proton and antiproton annihilate each other. How long does it take for this to happen? It depends on how they're orbiting each other.

In both hydrogen and protonium, various orbits are possible. Particles are really waves, so these orbits are really different wave patterns, like different ways a trampoline can wiggle up and down. These patterns are called orbitals.

Orbitals are labelled by numbers called quantum numbers. If a hydrogen atom isn't spinning at all, it will be spherically symmetric. Then you just need one number, cleverly called n, to say what its wave pattern looks like.

The picture here shows the orbital with $n = 30$. It has 30 wiggles as you go from the center outwards. It's really 3-dimensional and round, but the picture shows a circular slice. The height of the wave at some point says how likely you are to find the electron there. So, the electron is most likely to be in the orange region. It's very unlikely to be right in the middle, where the proton sits.

The same math works for protonium! There's another big difference to keep in mind: the proton and antiproton have the same mass, so they both orbit *each other*. But we can track just one of them, moving around their shared center of mass. Then protonium works a lot like hydrogen. You get spherically symmetric orbitals, one for each choice of that number called n.

So: if you can make protonium in a orbital where $n = 30$, it's unlikely for the two protons to meet each other. Gradually your protonium will emit light and jump to orbitals with lower n, which have less energy. And eventually the proton and antiproton will meet... and annihilate in a flash of gamma rays, which are just a powerful form of light.

How long does this take? For $n = 30$, about 1 microsecond. And if you make protonium with $n = 50$, it lasts about 10 microseconds.

That doesn't sound long, but in particle physics it counts as a pretty long time. Probably not long enough to make protonium crystals, though!

Protonium was first made around 1989. Around 2006 people made a *lot* of it using the Antiproton Decelerator at CERN. This is just one of the many cool gadgets they keep near the Swiss-French border.

You see, to create antimatter you need to smash particles at each other at almost the speed of light — so the antiparticles usually shoot out really fast. It takes serious cleverness to slow them down and catch them without letting them bump into matter and annihilate.

Once they managed to do this, they caught the antiprotons in a Penning trap. This holds charged particles using magnetic and electric fields. Then they cooled the antiprotons — slowed them even more — by letting them interact with a cold gas of electrons. Then they mixed in some protons. And they got protonium — enough to really study it!

The folks at CERN have also made antihydrogen, which is the antiparticle of an electron orbiting an antiproton. And they've made antiprotonic helium, which is an antiproton orbiting a helium atom with one electron removed! The antiproton acts a bit like the missing electron, except that it's 1836 times heavier, so it must orbit much closer to the helium nucleus.

There are even wackier forms of matter in the works — or at least, in the dreams of theoretical physicists. But that's another story for another day.

Here's the 2008 paper about protonium:

- N. Zurlo, M. Amoretti, C. Amsler, G. Bonomi, C. Carraro, C. L. Cesar, M. Charlton, M. Doser, A. Fontana, R. Funakoshi, P. Genova, R. S. Hayano, L. V. Jorgensen, A. Kellerbauer, V. Lagomarsino, R. Landua, E. Lodi Rizzini, M. Macri, N. Madsen, G. Manuzio, D. Mitchard, P. Montagna, L. G. Posada, H. Pruys, C. Regenfus, A. Rotondi, G. Testera, D. P. Van der Werf, A. Variola, L. Venturelli and Y. Yamazaki, Production of slow protonium in vacuum, *Hyperfine Interactions* **172** (2006), 97–105.

The child in me thinks it's really cool that there's an abbreviation for protonium, Pn, just like a normal element.

Puzzle 1: about how big is protonium in its $n = 1$ orbital, compared to hydrogen in its $n = 1$ orbital? I've given you all the numbers you need to estimate this, though not all the necessary background in physics.

In Puzzle 1 you're supposed to assume protonium in its $n = 1$ state is held together by the attraction of opposite charges, just like hydrogen. But is that true? If the proton and antiproton are *too* close, they'll interact a lot via the strong force!

Puzzle 2: The radius of hydrogen in its $n = 1$ state is about 50,000 femtometers, while the radius of a proton is about 1 femtometer. Using your answer to Puzzle 1, compare the radius of protonium in its $n = 1$ orbital to the radius of a proton.

If protonium is a lot bigger than a proton, it's probably held together mostly in the same way as hydrogen: by the electromagnetic force.

For answers to the puzzles, and other cool facts, read the comments on my G+ post.

For my September 2015 diary, go here.

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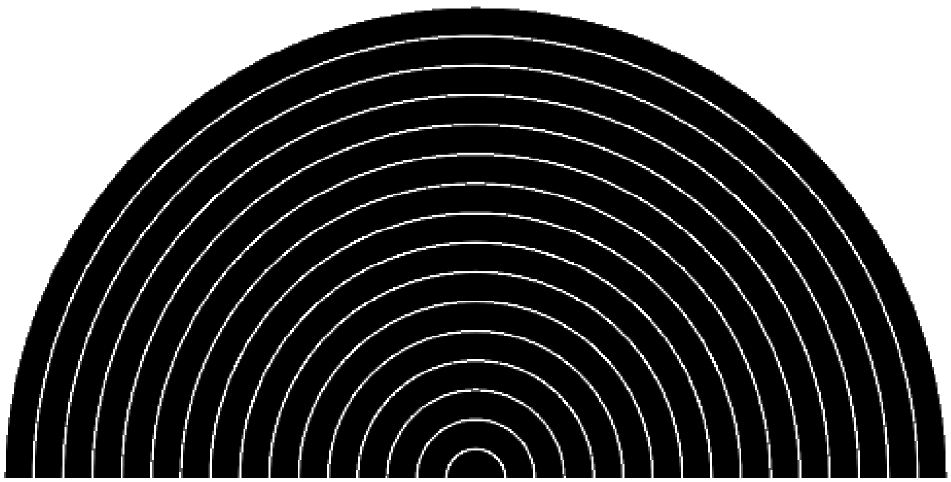
baez@math.removethis.ucr.andthis.edu

[For my August 2015 diary, go here.]

Diary - September 2015

John Baez

September 1, 2015

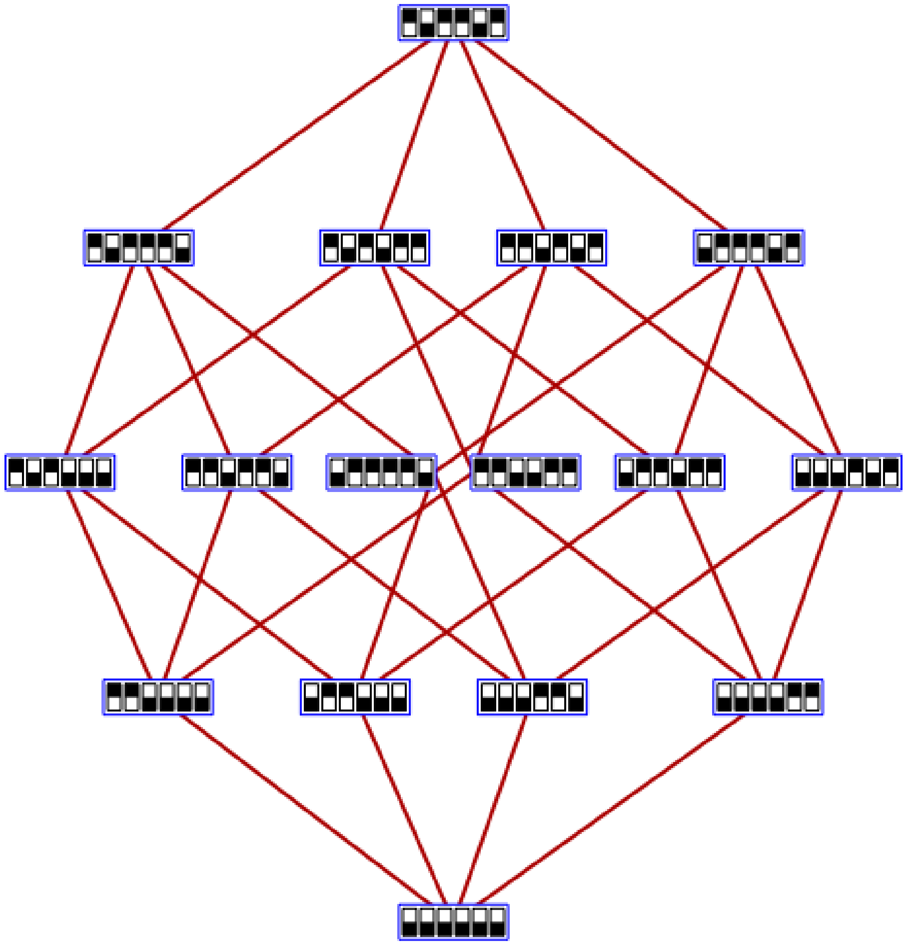


Can you say what's going on in this gif by intothecontinuum?

If you get stuck, you can read the Mathematica code on his website, or check out the comments on my G+ post. But it's probably more fun to watch it carefully and see for yourself!

September 3, 2015

Processing math: 61%



The British mathematician James Joseph Sylvester, who lived from 1814 to 1897, was one of the first to dig deeply into the beautiful patterns you can form using finite sets. But he got into lots of trouble.

For example, he entered University College London at the age of 14. But after just five months, he was accused of threatening a fellow student with a knife in the dining hall! His parents took him out of college and waited for him to grow up a bit more.

He began studies in Cambridge at 17. Despite being ill for 2 years, he came in second in the big math exam called the tripos. But he couldn't get a degree... because he was Jewish.

After just a few months, a student reading a newspaper in one of Sylvester's lectures insulted him. Sylvester struck him with a sword stick. The student collapsed in shock. Sylvester thought he'd killed the guy! He fled to New York where one of his brothers was living.

Later he came back. According to the online biography I'm reading, "the abuse suffered by Sylvester from this student got worse after this". Soon he quit his job.

He returned to England and took up a job at a life insurance company. He needed a law degree for this job, and in his studies he met another mathematician, five years younger, studying law: Cayley! They worked together on matrices and invariant theory.

Sylvester only got another math job in 1855, at the Royal Military Academy of Woolwich. He was 41. At age 55 they made him retire — that was the rule — but for some reason the school refused to pay his pension!

The Royal Military Academy only relented and paid Sylvester his pension after a prolonged public controversy, during which he took his case to the letters page of *The Times*.

When he was 58, Cambridge University finally gave him his BA and MA.

At age 62, Sylvester went back to the United States to become the first professor of mathematics at the newly founded Johns Hopkins University in Baltimore, Maryland. His salary was $5,000 — quite generous for the time.

He demanded to be paid in gold.

They wouldn't pay him in gold, but he took the job anyway. At age 64, he founded the American Journal of Mathematics. At 69, he was invited back to England to become a professor at Oxford. He worked there until his death at age 83.

One thing I like about Sylvester is that he invented lots of terms for mathematical concepts. Some of them have caught on: matrix, discriminant, invariant, totient, and Jacobian! Others have not: cyclotheme, meicatecticizant, tamisage and dozens more.

Among many other things, Sylvester discovered soe special features of 6-element sets. Sylvester defined a 'duad' to be a way of choosing 2 things from a set. A set of 6 things has 15 duads. A hypercube has 16 corners. The picture by Greg Egan above shows a hypercube with 15 of its 16 corners labelled by duads. The bottom corner is different.

This may seem just cute, but in fact it can help you visualize a rather wonderful fact: the group of permutations of 6 things is isomorphic to the symmetry group of a 4-dimensional symplectic vector space over the field with 2 elements.

For details, read this:

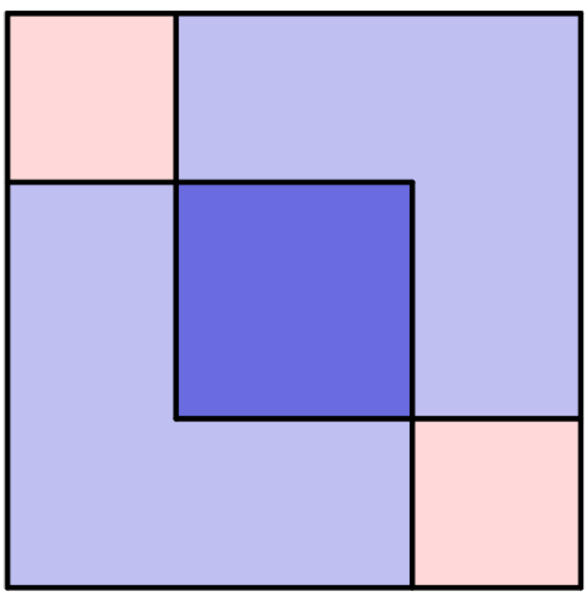
- John Baez, Hypercube of duads, *Visual Insight*, September 1, 2015.

While I've told you some of the tawdry details of Sylvester's life, like any great mathematician he spent muxh of his life immersed in worlds of abstract beauty.

He also loved poetry, and translated poems into English from French, German, Italian, Latin and Greek.

September 7, 2015

A picture proof that $\sqrt{2}$ is irrational



This Friday I was hanging out and drinking beer with some philosophy professors. This is always fun, because they think sort of like me, but different. They seem more optimistic about our ability to solve all sorts of puzzles just by talking.

To annoy them a bit, I said that philosophers are great at verbal reasoning, but mathematicians should be good at three kinds of reasoning: verbal, symbolic and visual reasoning.

In response, one of them showed me this picture proof that $\sqrt{2}$ is irrational.

We just need to show that it's impossible to have

$$a^2 = b^2 + b^2$$

for whole numbers a and b. So let's do a proof by contradiction. We can assume a is the smallest whole number that obeys this equation for some whole number b. We'll get a contradiction by finding an even smaller one.

We do it by drawing a picture.

The big square in this picture is an $a \times a$ square. The two light blue squares, which overlap in the middle, are $b \times b$ squares.

The area of the big square is the sum of the areas of the light blue squares. But there are two problems. First, the light blue squares overlap. Second, they don't cover the whole big square! These two problems must exactly cancel out.

So, the area of the overlap — the dark blue squares — must exactly equal the area that's not covered — the two pink squares.

So, the area of the dark blue square is the sum of the areas of the pink squares! But the lengths of the sides of these must be whole numbers, say c and d. So we have

$$c^2 = d^2 + d^2$$

But c is smaller than a. So, we get a contradiction!

Actually this proof uses a mix of verbal and visual reasoning, with just a tiny touch of symbolic reasoning. I wrote the formulas like $a^2 = b^2 + b^2$ just to speed things up a bit and reassure you that this was math. I didn't really do anything with them.

The philosophers who told me about this are Mike Pelczar and Ben Blumson. The picture here comes from a website Mike pointed me to:

- David Richeson, Tennenbaum's proof of the irrationality of the square root of 2.

Richeson says:

> Apparently the proof was discovered by Stanley Tennenbaum in the 1950's but was made widely known by John Conway around 1990. The proof appeared in Conway's chapter "The Power of Mathematics" of the book *Power*, which was edited by Alan F. Blackwell, David MacKay (2005).

On the other hand, Ben says John Bigelow published the proof in his book *The Reality of Numbers* in 1988, without citing anyone.

We wondered if it was known to the ancient Greeks.

You can do similar proofs of the irrationality of $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$ and $\sqrt{10}$:

- Stephen J. Miller and David Montague, Irrationality from the book.

And this particular style of proof by contradiction is famous! It's called proof by infinite descent. You assume you have the smallest whole number that's a counterexample to something you want to prove, and then you cook up an even smaller one. It's really just mathematical induction in disguise, but it's more fun. It was developed by Pierre Fermat — who, by the way, was a lawyer.

If you want to take all the fun out of the proof I just gave, you can do it like this.

Assume a is the smallest whole number for which there's a whole number b with

$$a^2 = b^2 + b^2$$

Let

$$c = 2b - a$$

and

$$d = a - b$$

Then c and d are whole numbers and

$$c^2 = d^2 + d^2$$

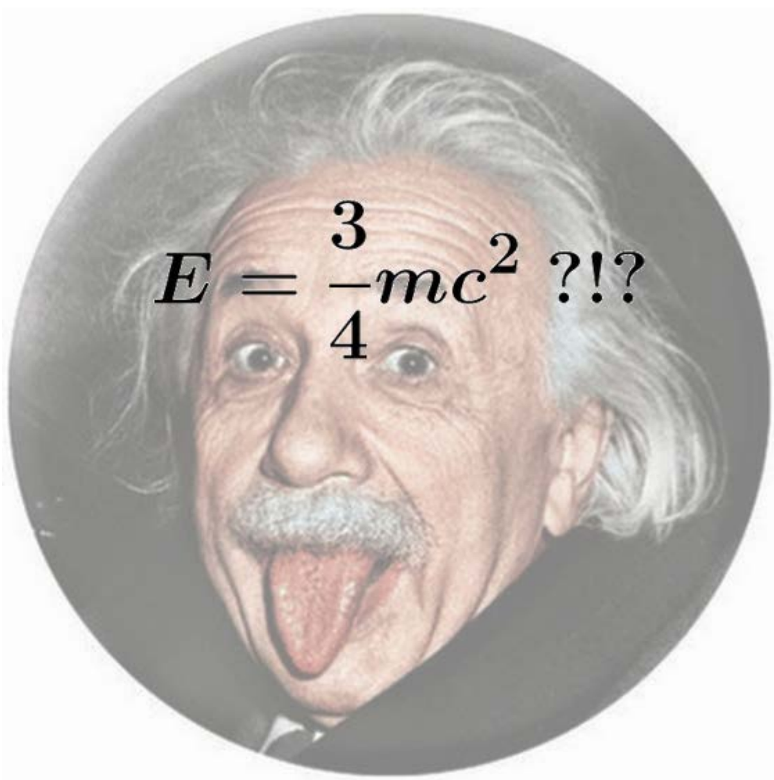
(You can do some algebra to check this.) But $c < a$, so we get a contradiction.

Wikipedia shows you how to prove by infinite descent that whenever n is a whole number, either $\sqrt{n}$ is a whole number or it's irrational:

- Proof by infinite descent, Wikipedia.

Fermat did a lot more interesting stuff with this method, too!

September 9, 2015



Right now physicists are struggling with the 'firewall paradox' — a problem in our theory of black holes. But this is far from the first time physicists have been stuck with an annoying 'paradox'.

Back in the late 1800s, physicists noticed that an electron should get mass from its electric field. Nowadays we'd say this is obvious. The electric field has energy, and $E = mc^2$, so it contributes to the mass of the electron. But this was before special relativity!

How did they figure it out? They were very clever. They used Newton's $F = ma$. When you push on something with a force, you can figure out its mass by seeing how much it accelerates!

So, they did some calculations. When you push on an electron with a force, you also affect its electric field. It's like the electron has a cloud around it, that follows wherever the electron goes. This makes it harder to accelerate the electron. So, it effectively increases the electron's mass. They calculated this extra mass.

They also did an easier calculation: how much energy this electric field has!

Say m is the extra mass due to the electric field surrounding the electron, and E is the energy of this electric field. Then they discovered that

$$E = \frac{3}{4}mc^2$$

Whoops!

Had they made an algebra mistake? Not really.

Some really smart people all got the same answer! Oliver Heaviside got it in 1889 — he was one of the world's smartest electrical engineers. J.J. Thomson got it in 1893 — he's the guy who discovered the electron! Hendrik Lorentz kept getting the same answer, even as late as 1904 — and he's one of the people who paved the way for relativity!

But in 1905, Einstein wrote his paper showing that $E = mc^2$ is the only possible answer that makes sense.

So what went wrong?

All those guys were assuming the electron was a little sphere of charge. Why? In their calculations, if was a point, the energy in its electric field would be infinite, because the electric field gets extremely strong near that point. The mass contributed by this field would also be infinite.

If the electron were a tiny sphere, they could avoid those infinite answers.

But then they ran into this $E = \frac{3}{4}mc^2$ problem. Why? Because electrical charges of the same sign repel each other. So a tiny sphere of charge would explode if something weren't holding it together. And that something — whatever it is — might have energy. But their calculation ignored that extra energy.

In short, their picture of the electron as a tiny sphere of charge, with nothing holding it together, was incomplete. And their calculation showing E = \frac{3}{4}mc^2, together with special relativity saying E = mc^2, shows this incomplete picture is inconsistent.

So in the end, it's not a case of people being stupid. It's a case of people discovering something interesting... by taking a plausible idea and showing it can't work.

If you want the details, read what Feynman has to say:

- Richard Feynman, Electromagnetic mass.

He does all the calculations that explain this problem. I've been reading his books since high school, but never really understood this part until now. I'm thinking about problems with infinity in physics.

Here's a bit of what he says:

> The discrepancy between the two formulas for the electromagnetic mass is especially annoying, because we have carefully proved that the theory of electrodynamics is consistent with the principle of relativity. [...] So we are in some kind of trouble; we must have made a mistake. We did not make an algebraic mistake in our calculations, but we have left something out.
>
> In deriving our equations for energy and momentum, we assumed the conservation laws. We assumed that all forces were taken into account and that any work done and any momentum carried by other 'nonelectrical' machinery was included. Now if we have a sphere of charge, the electrical forces are all repulsive and an electron would tend to fly apart. Because the system has unbalanced forces, we can get all kinds of errors in the laws relating energy and momentum. To get a consistent picture, we must imagine that something holds the electron together. The charges must be held to the sphere by some kind of rubber bands.something that keeps the charges from flying off. It was first pointed out by Poincaré that the rubber bands — or whatever it is that holds the electron together — must be included in the energy and momentum calculations. For this reason the extra nonelectrical forces are also known by the more elegant name "the Poincaré stresses". If the extra forces are included in the calculations, the masses obtained in two ways are changed (in a way that depends on the detailed assumptions). And the results are consistent with relativity; i.e., the mass that comes out from the momentum calculation is the same as the one that comes from the energy calculation. However, both of them contain two contributions: an electromagnetic mass and contribution from the Poincaré stresses. Only when the two are added together do we get a consistent theory.

This was a bummer back around 1905, because people had actually hoped all the mass of the electron was due to its electric field. Note: this extra assumption is not required for the $E = \frac{3}{4}mc^2$ problem to bite you in the butt. It's already a problem that the energy due to the electric field is $\frac{3}{4}mc^2$ where m is the mass due to the electric field. But the solution to the problem — extra 'rubber bands' — killed the hope that the electron could be completely understood using electromagnetism.

It is therefore impossible to get all the mass to be electromagnetic in the way we hoped. It is not a legal theory if we have nothing but electrodynamics. Something else has to be added. Whatever you call them — 'rubber bands', or 'Poincaré stresses', or something else — there have to be other forces in nature to make a consistent theory of this kind. Wikipedia has a good article on the history of this problem:

- Wikipedia, Electromagnetic mass.

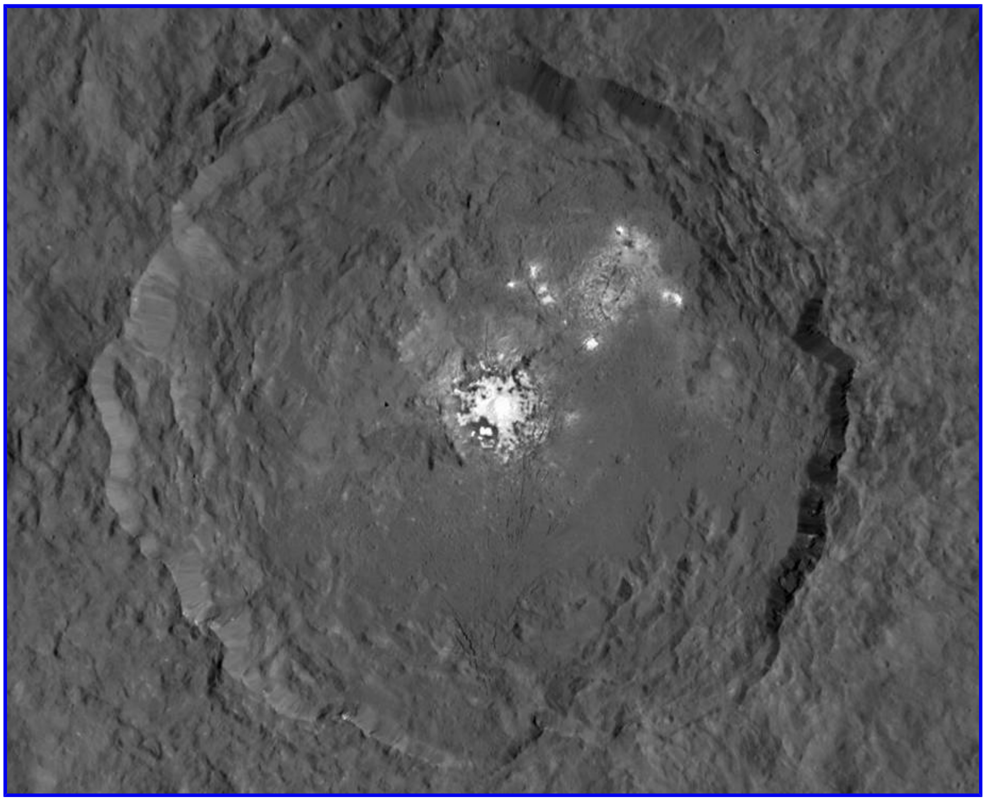
and this paper is also good:

- Michel Janssen and Matthew Mecklenburg, Electromagnetic models of the electron and the transition from classical to relativistic mechanics.

All this is part of a longer story I told later:

- John Baez, Struggles with the continuum (part 3), *Physics Forums*, September 14, 2013.

September 13, 2015



The spacecraft Dawn has gotten a closer look at the mysterious white spots on the asteroid Ceres. In the first photos, they were so bright they were overexposed!

Now Dawn is closer. Here's the biggest patch of white stuff, called Spot 5, in a crater called Occator.

What is it? The obvious guess is some sort of ice or salt, reflecting sunlight. But here's the cool part: you can sometimes see haze over Spot 5. This suggests that some sort of gas is coming up from beneath the surface! Or maybe the ice is sublimating, turning into vapor... perhaps explosively?

The mission director writes:

> Dawn has transformed what was so recently a few bright dots into a complex and beautiful, gleaming landscape. Soon, the scientific analysis will reveal the geological and chemical nature of this mysterious and mesmerizing extraterrestrial scenery.

This picture is a composite of two images: one using a short exposure that captures the detail in the bright spots, and one where the background surface is captured at normal exposure. Each pixel here is a 140 meter × 140 meter square.

Right now Dawn is orbiting Ceres at a distance of 1450 kilometers. In December, it will descend to just 375 kilometers from the surface. Then we'll get even better images!

And when the mission ends? Then Dawn will remain as a permanent satellite of Ceres. A fitting end to a great mission — it's the first spacecraft to orbit two bodies, Vesta and Ceres.

You can watch a short video of what Dawn has been seeing, here:

- NASA, Cruise over Ceres in new video, August 6, 2015.

This image came from here:

- NASA, Ceres' bright spost seen in striking new detail, September 9, 2015.

For more on the haze, read this:

- Alexandra Witze, [Mystery haze appears above Ceres's bright spots](), *Nature News*, July 21, 2015.

September 14, 2015

Escher, wishing he weren't so clumsy

This is based on a famous print by M. C. Escher called 'Ascending and Descending'. That in turn was based on an idea by the mathematician Penrose, called the Penrose stairs. But Penrose in turn was inspired by Escher, who was inspired by Penrose.... in an endless loop!

At least that's the story on Wikipedia:

At an Escher conference in Rome in 1985, Roger Penrose said that he had been greatly inspired by Escher's work when he and his father discovered the Penrose stairs.

Penrose said he'd first seen Escher's work at a conference in Amsterdam in 1954. He was "absolutely spellbound", and on his journey back to England he decided to produce something "impossible" on his own. After experimenting with various designs he finally arrived at the impossible Penrose triangle.

Roger Penrose then showed his drawings to his father Lionel, who immediately produced several variants, including the impossible stairs. They wanted to publish their findings but didn't know where to do it. Because Lionel Penrose knew the editor of *British Journal of Psychology*, the finding was finally presented as a paper there. After its publication in 1958, they sent a copy of the article to Escher as a token of their esteem.

But here's the weird part. While the Penroses credited Escher in their article, Escher himself noted in a letter to his son in January 1960 that he was:

> ... working on the design of a new picture, which featured a flight of stairs which only ever ascended or descended, depending on how you saw it. They form a closed, circular construction, rather like a snake biting its own tail. And yet they can be drawn in correct perspective: each step higher (or lower) than the previous one. I discovered the principle in an article which was sent to me, and in which I myself was named as the maker of various 'impossible objects'. But I was not familiar with the continuous steps of which the author had included a clear, if perfunctory, sketch..."

So it seems this impossible staircase willed itself into existence in a paradoxical loop of causality!

But now that I think about it, all good ideas come into being a bit like this. At first they exist only in vague form, in the random jostlings of thoughts and words. They exist just a little, merely because they can exist. But thanks to their power, the more people think and talk about them, the more they spiral into existence... until finally they are clear and undeniable. Like this idea here.

Puzzle 1: just by looking carefully at this animated gif, can you guess who made it?

Puzzle 2: can you see the slight mistake in the picture here?

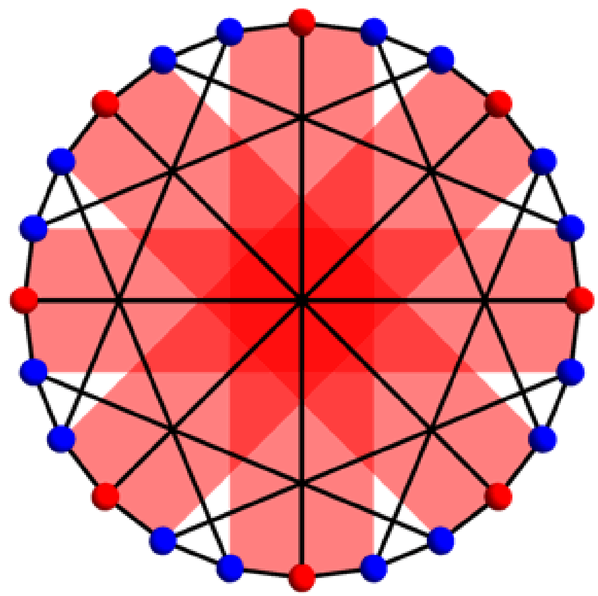
Puzzle 3: does that mistake occur in Escher's original print? Here's the original

Here's the Wikipedia article where I got the story:

- Penrose stairs, Wikipedia.

For answers to the puzzles, see the comments on my G+ post.

September 15, 2015



This is the McGee graph. You'll notice each vertex has 3 neighbors. Also, if you go around a cycle of edges, there will always be at least 7 edges in your path. The McGee graph's claim to fame is that it has the fewest vertices possible for a graph with these two properties.

This isn't so amazing, so the McGee graph is not as famous as some others I've discussed on *Visual Insight*. It's fairly symmetrical, though. Besides the obvious symmetries — rotating it 1/8 of a turn, or flipping it over — there are some sneaky symmetries, like the one shown in this animation by Greg Egan.

He drew four red 'bands', actually hexagons, because this sneaky symmetry mixes up the nodes in each 'band' in a clever way. Also, you'll notice that some vertices are red, and others blue. That's because all the symmetries send red vertices to red ones, and blue vertices to blue ones.

In trying to understand the McGee graph better, I wanted to start by understanding its symmetries. It has 8 rotational symmetries; if we throw in our ability to flip it over we get a total of 16, and if we throw in all the sneaky symmetries we get a total of 32.

There are lots of different groups with 32 elements. In fact, there are 51 of them.

(If you start listing finite groups, you'll soon discover that most of them have a power of 2 as their number of elements. Indeed, of the 50 billion or so groups of size at most 2000, more than 99% have 1024 elements!)

Luckily there's an online list of all 51 groups with 32 elements, and a fellow named Gordon Royle told me which of these was the symmetry group of the McGee Graph. So, I soon found out that the symmetries of the McGee graph are the affine transformations of the integers mod 8.

I should explain this. An affine transformation is something like this: $x \mapsto ax + b$ where a is invertible. You may be used to calling them 'linear', but showoffs like me prefer to save that word for transformations of this sort: $x \mapsto ax$ Anyway, you may be used to affine transformations when $x, a,$ and b are real numbers. The real numbers form a line. The affine transformations just slide, stretch or squash, and maybe flip that line.

But here we are dealing with the case where x, a, and b are integers mod 8. What is the 'line' like then? Well, the integers mod 8 can be visualized as an octagon — that is, the 8 red nodes in the animation here. So now our 'line' looks like an octagon!

This may freak you out, but to mathematicians there's nothing more fun than taking intuitions from a familiar example and applying them to some weird other situation. So now a line looks like an octagon? — sure, we can handle that.

What are affine transformations like? Well, these are rotations of the octagon: $x \mapsto x + b$ We can also flip the octagon like this: $x \mapsto -x$ But there are other transformations of this sort: $x \mapsto ax$ where a isn't 1 or -1, and those are the 'sneaky symmetries'.

You can check that in the integers mod 8, only four numbers are invertible: $1, 3, 5$ and $7 = -1$. So, we get $4 \times 8 = 32$ affine transformations. And those are the symmetries of the McGee graph!

So: the red vertices of the McGee graph are just integers mod 8. The symmetries are affine transformations. Then the question becomes: how can we understand the blue vertices, and the edges, in terms of the integers mod 8? If we answer this correctly, it will become obvious why affine transformations give symmetries of the McGee graph.

This is a fun puzzle for people who like Felix Klein's philosophy relating groups and geometry. Egan and I worked out the answer, and explained it — with lots of pictures — on my blog Visual Insight:

- John Baez, The McGee graph, *Visual Insight*, September 15, 2015.

So go there if you want the full story.

Not coincidentally, all the groups of size ≤ 2000 were listed in the year 2000. You can read more about them here:

- Tom Leinster, Almost all the first 50 billion groups have order 1024, *The n-Category Café*, November 28, 2012.

September 17, 2015

This is a rock formation made of chalk in the Sahara el Beyda, or White Desert, in Egypt. This beautiful place got in the news recently after 8 Mexican tourists were killed there by Egyptian security forces, who apparently mistook them for terrorists.

But let's leave the sad world of mankind for a minute, and enjoy the desert.

There are actually many mushroom-shaped rocks like this in the White Desert.

Why?

Try to guess.

Okay: they are ventifacts, meaning they are carved by wind-blown sand. Even in strong winds, sand grains don't usually stay in the air for long. Instead they bounce along the ground. So, they cut deeper into the bottom of the rock than the top!

Magical-looking, but not magic.

This photo was taken by Christine Schultz.

September 29, 2015

This looks like a *deerbunny* — with dark, soulful eyes! But it's a mara, from South America. It eats grass. It can run up to 30 kilometers per hour, but it can also hop like a rabbit or make long jumps. It's shy in the wild. But it can be quite friendly if raised with people from a young age. Some people keep them as pets.

The mara is the world's fourth largest rodent, after capybaras, beavers, and porcupines. But they're more closely related to guinea pigs.

They're common in the Patagonian steppes of Argentina, but they also live in Paraguay and other places.

This photo was taken by Dick Klees, and I saw it here:

- Phalon Smith, Deer, bunny, or something else?, *Sierra*, July/August 2015.

For more pictures of maras, go here:

- Wikipedia, Mara (mammal).

You'll notice they also look a bit like kangaroos — another species that occupies a similar niche. Convergent evolution can work wonders!

For my October 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

home

For my September 2015 diary, go here.

Diary - October 2015

John Baez

October 1, 2015

This clay tablet, discovered in 2011 AD, adds to our knowledge of the *Epic of Gilgamesh*, a Babylonian story that goes back to 2100 BC.

How much of the past is truly lost? How much can we still hope to recover? I often wonder about that... and I'm delighted when people find things like this, or the new poems by Sappho that had been buried in an ancient Egyptian garbage dump.

How was this new Gilgamesh tablet found?

After the US-led invasion of Iraq and the dramatic looting of Iraqi and other museums, a museum in Sulaymaniyah, in the Kurdish part of Iraq, did something bold and controversial. They started paying smugglers for old artifacts that would otherwise be sold outside Iraq. They didn't ask any embarrassing questions. They just bought the stuff that looked good!

In 2011, a smuggler showed them a collection of clay tablets. It had about 80 tablets of different shapes and sizes. They were still covered with mud. Some were completely fine, while others were broken. Nobody knows where they came from, but they may have been illegally dug up near the city of Babel.

While the smuggler was negotiating with the museum, the museum got Professor Farouk Al-Rawi of the School of Oriental and African Studies in London to quickly look through the tablets. When he saw this one and skimmed the cuneiform inscriptions on it, he got excited. He told the museum to buy it from the smuggler. "Just give him what he wants, I will tell you later on." The final price was $800.

When Professor Al-Rawi carefully cleaned the tablet, he realized that yes, it was one of the tablets of the *Epic of Gilgamesh*!

It's a copy of Tablet V, one of the 12 tablets of the so-called Standard Akkadian version of the epic. This version goes back to about 1200 BC. There's also an older version, the Old Babylonian one, but we have less than half of that.

So, what's new about this tablet? I'm not very familiar with the *Epic of Gilgamesh* — I wasted too much of my youth studying math — but there's a longer description of the Cedar Forest. For example, it says Gilgamesh and his pal Enkidu saw monkeys in that forest. This was not mentioned in other versions of the Epic. Even better, in this version Humbaba is not an ogre: he's a foreign ruler entertained with exotic music at court, like a Babylonian king would be.

So: a tiny snippet of the past, which could have been lost forever, has made it to the present. And now Hazha Jalal, a woman who works at the Sulaymaniyah Museum, can say:

> The tablet dates back to the Neo-Bablyonian period. It is a part of tablet V of the Epic. It was acquired by the Museum in the year 2011 and Dr. Farouk Al-Raw transliterated it. We are honored to house this tablet and anyone can visit the Museum during its opening hours from 8:30 AM to 2:00 PM. The entry is free for you and your guests. Thank you.

For more on the new tablet, including more pictures and a video, try this story, which is where I got the picture:

- Osama S. M. Amin, The newly discovered Tablet V of the *Epic of Gilgamesh*, *Ancient History Et Cetera*, September 24, 2015.

As you'll see, I paraphrased parts of what he wrote!

For a summary of the Epic of Gilgamesh and the 12 tablets of the Standard Akkadian version, go here:

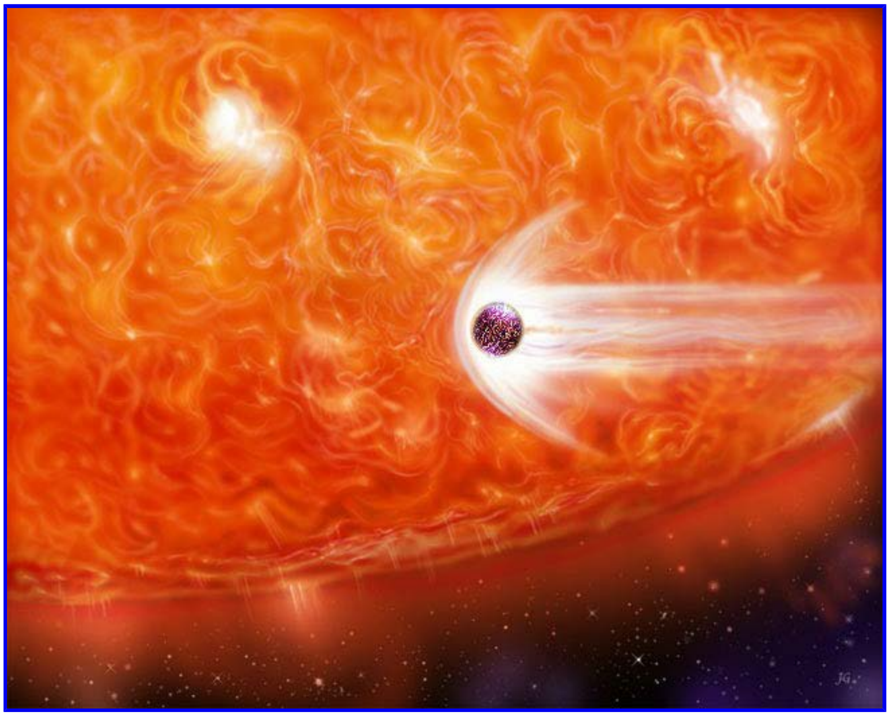
- Wikipedia, Epic of Gilgamesh: Standard Akkadian Version.

For the newly discovered Sappho poems among the Oxyrhynchus Papyri, try this:

- Reception of Greek literature 300 BC-AD 800: traditions of the fragment.

October 4, 2015

Have astronomers found a TŻO?



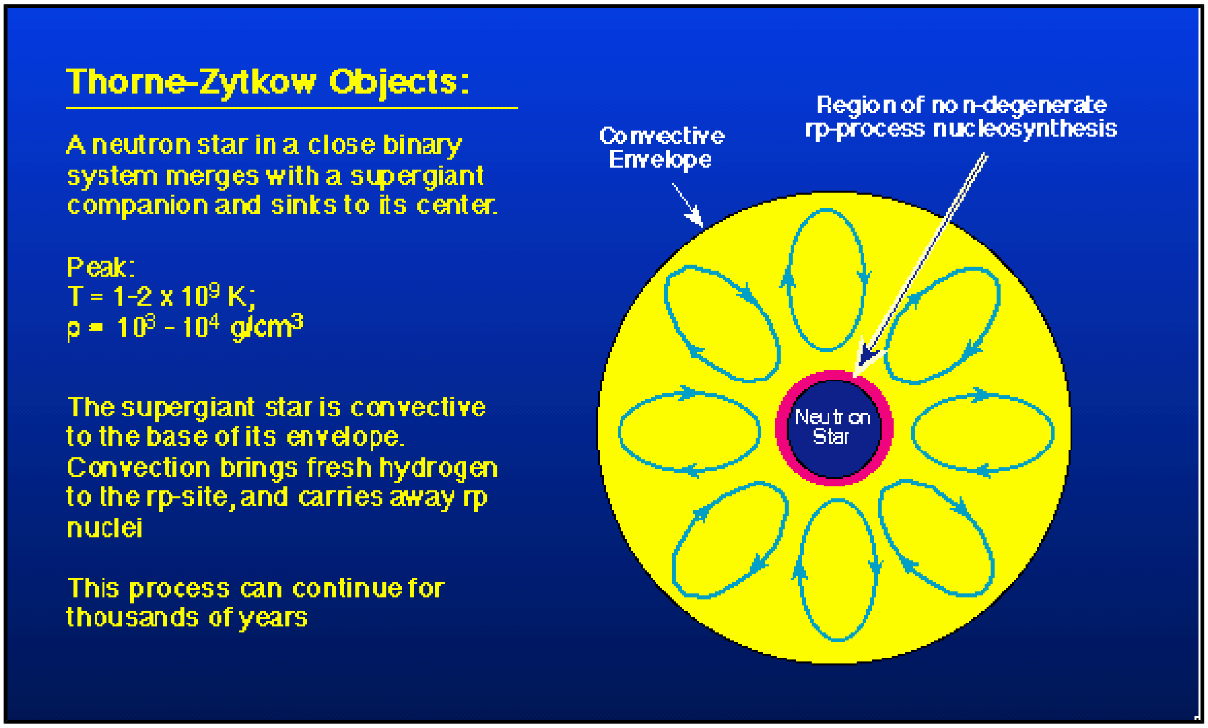
When a big star runs out of fuel, its core can collapse and form a dense ball of neutronium just 25 kilometers across, called a neutron star. But what happens when a neutron star hits an ordinary star?

Kip Thorne is a physicist who helped write the most famous book on general relativity. Now he's helping run the LIGO project for detecting gravitational waves. Anna Żytkow is an astronomer at Cambridge who is looking for objects in the Kuiper Belt, outside the orbit of Pluto. But back in 1977, they teamed up and asked this question... and answered it!

The answer is: the neutron star could fall to the center of the other star and stay there! The result is called a Thorne-Żytkow object, or TŻO.

When this happens, the neutron star will suck in gas from the ordinary star. It will get extremely hot, with temperatures over a billion degrees Celsius. The heat comes from two things: energy released when infalling gas hits the neutron star, and nuclear fusion after the gas hits.

If all this happens inside a red giant — a huge, puffed-up star — the inside of that star should get a lot hotter than usual. So, weird processes should create elements that you don't usually see in such a star.



And now astronomers have found a red supergiant with a lot more rubidium, strontium, yttrium, zirconium, molybdenum and lithium than usual. We know this from its spectral lines.

So, they may have found a TŻO!

Anna Żytkow was pleased, saying "I am extremely happy that observational confirmation of our theoretical prediction has started to emerge".

The candidate TŻO is called HV 2112. It's in the the Small Magellanic Cloud, a dwarf galaxy orbiting ours, about 200,000 light-years away.

The astronomer Nidia Morell found the weird elements in this star while conducting a survey of red supergiants last year. At the time she said:

> I don't know what it is, but I know that I like it!

Read all about it here:

- Emily M. Levesque, Philip Massey, Anna N. Żytkow and Nidia Morrell, Discovery of a Thorne-Żytkow object candidate in the Small Magellanic Cloud.

Puzzle 1: How do you pronounce a Z with a dot on it? Clue: Anna Żytkow is Polish.

Puzzle 2: What could happen if a neutron star falls to the bottom of a white dwarf, if their total mass is big enough?

Puzzle 3: What could happen if a neutron star falls to the bottom of a white dwarf if their total mass is not so big?

Puzzle 4: Suppose you have a neutron star inside a red supergiant. What eventually happens to it?

Puzzle 5: Suppose two red supergiants containing neutron stars collide. What happens then?

For more, read this:

- Charles Q. Choi, Strange 'hybrid star' discovered after 40-year search, *Space.com*, October 9, 2015.

or Wikipedia:

- Wikipedia, Thorne-Żytow object.

October 7, 2015

Yes, you've heard there's liquid water on Mars. But have you actually seen it? Now you have.

This gif shows what's probably salty water flowing in Newton Crater on Mars. The dark stripes are between 1/2 and 5 meters wide. Stripes like this appear on steep slopes at several locations in the southern hemisphere of Mars. They show up in the spring and summer, when the temperature can rise above the freezing point. They go away when it gets colder.

The photos here go from the early spring of one Mars year to mid-summer of the next year. They were taken by the HiRISE camera on NASA's Mars Reconnaissance Orbiter. That stands for High Resolution Imaging Science Experiment.

These images are not new! Here's a paper about them, written in 2011:

- Alfred S. McEwen *et al*, Seasonal flows on warm Martian slopes, *Science* 333 (August 5, 2011), 740–743.

So, what made NASA announce now that there is liquid water on Mars? Could it have anything to do with the new movie, *The Martian?* I don't know.

Some puzzles about the movie:

Puzzle 1: why is there no communication apparatus in the living habitat where the Matt Damon character winds up living? Does the book explain why?

Puzzle 2: what would it actually feel like to be in a dust storm on Mars?

Puzzle 3: could you really take off in a rocket on Mars with just a tarp on top?

For more about these flows on Mars, see:

- Wikipedia, [Seasonal flows on warm Martian slopes](#).

October 8, 2015

A cat can love a rodent...

...if the rodent is bigger.

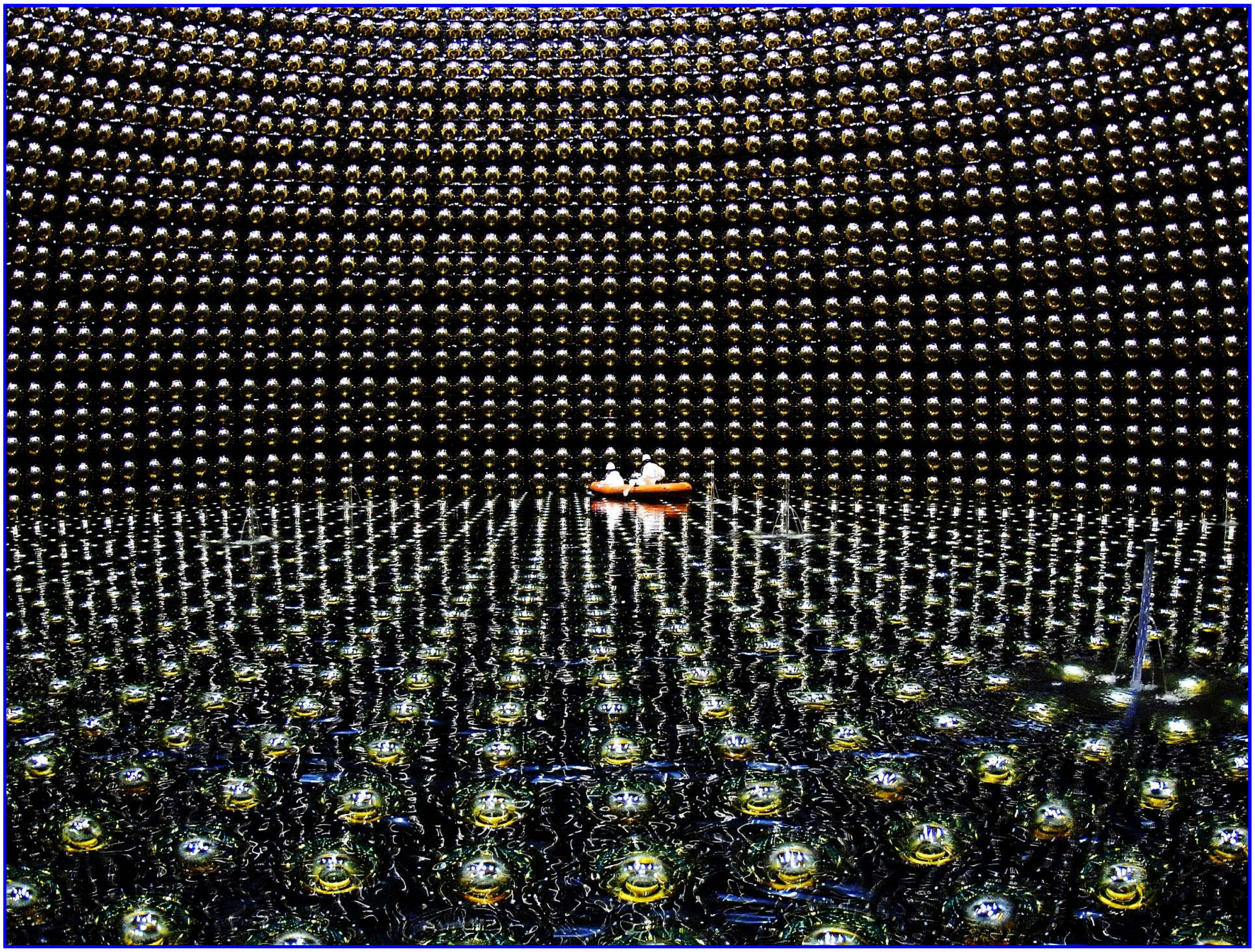
Puzzle 1: What is this thing?

Puzzle 2: Where do they usually live?

Puzzle 3: Why have they been seen in the wild in Florida?

I don't know the original source of this picture.

October 9, 2015



This is a picture of Super-Kamiokande, one of the neutrino detectors that won this year's physics Nobel prize. It's a tank buried 1 kilometer deep in a mine in Japan. The tank holds 50,000 tons of ultra-pure water, surrounded by 11,146 machines that can detect tiny flashes of light. When a neutrino zipping through space happens to hit a water molecule, it makes a flash of light — and Super-Kamiokande records it.

Here you see some people on a raft working on the detectors. The winners of the Nobel prize were Takaaki Kajita and Arthur B. McDonald, who worked at another neutrino detector in Canada. But these big experiments involve huge teams of people!

These teams, and their machines, deserve a Nobel prize because they proved something we'd begun to suspect much earlier.

There are 3 different kinds of neutrinos: electron, muon and tau neutrinos. Nuclear fusion in the Sun makes electron neutrinos... but we saw only about 1/3 as many as expected. This made physicists suspect that electron neutrinos were turning into the other 2 kinds of neutrinos as they went from the Sun to Earth.

But proving this was very hard. And it's only possible if neutrinos have mass!

You see, time doesn't pass for a massless particle, since special relativity says time slows down for you when you're moving fast, and it comes to a halt if you're moving at the speed of light. So, a massless particle can't turn into something else until it hits another particle.

As early as the 1950s we knew that neutrinos were *almost* massless. So, we thought they *were* massless. But now, thanks to these experiments, we know neutrinos really do change from one kind into another. So, we know they have a tiny but nonzero mass.

Here's what the Nobel prize committee says about it:

> The discovery that neutrinos can convert from one flavour to another and therefore have nonzero masses is a major milestone for elementary particle physics. It represents compelling experimental evidence for the incompleteness of the Standard Model as a description of nature. Although the possibility of neutrino flavour change, i.e. neutrino oscillations, had been discussed ever since neutrinos were first discovered experimentally in 1956, it was only around the turn of the millennium that two convincing discoveries validated the actual existence of neutrino oscillations: in 1998, at Neutrino '98, the largest international neutrino conference series, Takaaki Kajita of the Super-Kamiokande Collaboration presented data showing the disappearance of atmospheric muon-neutrinos, i.e. neutrinos produced when cosmic rays interact with the atmosphere, as they travel from their point of origin to the detector. And in 2001/2002, the Sudbury Neutrino Observatory (SNO) Collaboration, led by Arthur B. McDonald, published clear evidence for conversion of electron-type neutrinos from the Sun into muon- or tau-neutrinos. These discoveries are of fundamental importance and constitute a major breakthrough.

I would put it this way: in the *old* Standard Model, neutrinos were massless. In the new improved Standard model, they have a nonzero mass.

In fact, there's a whole 3 × 3 matrix of numbers, the 'neutrino mass matrix', which says what neutrinos do as they're flying through empty space. These numbers actually say how the neutrinos interact with the Higgs boson. This determines their masses, but also how the 3 kinds turn into each other.

We don't know why the numbers in this matrix are what they are. We may never know. But maybe someday someone will figure it out. Physics is full of slow-burning mysteries like this.

For the full story, go here:

- The Nobel Prize in Physics 2015, October 6, 2015.

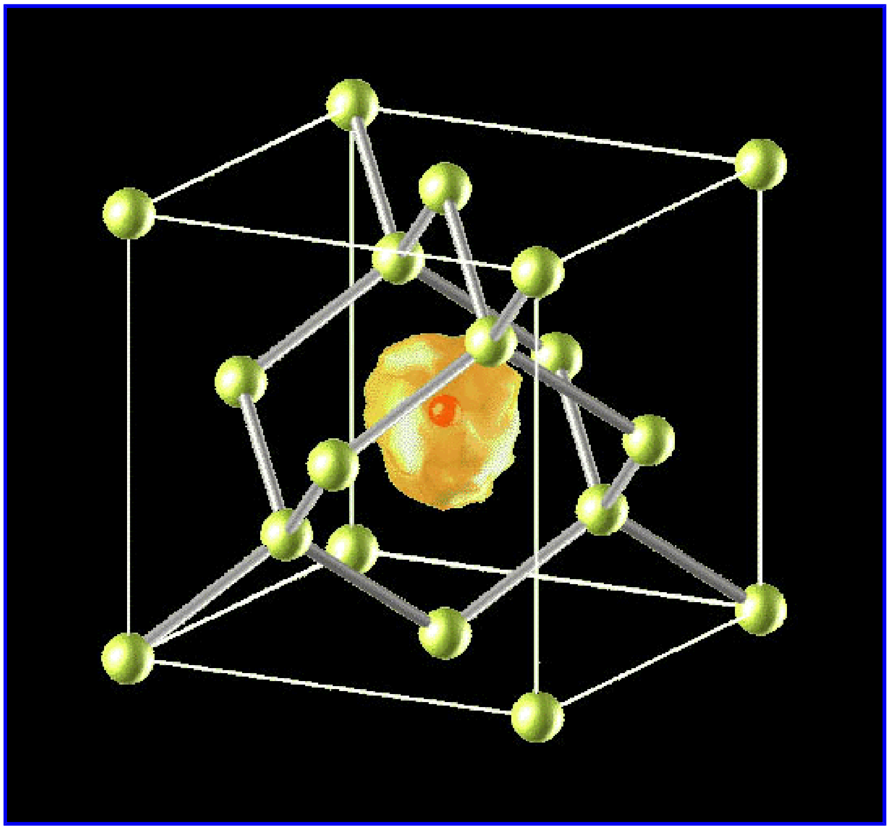
The neutrino mass matrix is also called the 'Pontecorvo–Maki–Nakagawa–Sakata matrix'. In 1962, right after the muon neutrino was discovered, Ziro Maki, Masami Nakagawa and Shoichi Sakata speculated that electron and muon neutrinos could turn into each other, and invented a 2 × 2 matrix to describe this. And even earlier, in 1956, Bruno Pontecorvo had considered the possibility that neutrinos and antineutrinos could turn into each other.

If you want to actually see the numbers in this matrix, go here:

- Wikipedia, Pontecorvo–Maki–Nakagawa–Sakata matrix.

October 11, 2015

Light hydrogen



There's an element that's 1/9th as heavy as hydrogen. Apart from that, it's a lot like hydrogen. For example, its radius is almost exactly the same - just half a percent bigger. Its chemical properties are also almost the same.

But there's one big difference. It's unstable. On average, it decays in just 2.2 microseconds!

That sounds like a short time, but in the world of chemistry it's not. A lot of chemical reactions only take nanoseconds - that is, billionths rather than millionths of seconds. So, there's plenty of time for this light version of hydrogen to form molecules - and these days, chemists are so good that they can study what happens!

For example, this image shows an atom of light hydrogen trapped in a crystal of silicon. The blob is the probability distribution of finding the atom in different locations. It's more smeared out than it would be for ordinary hydrogen. Why? Because the atom is lighter!

This image is the result of a computer calculation, not experiment. But chemists also do experiments with light hydrogen. The main reason is to check that our calculations in chemistry are really working. We think we can calculate properties of atoms and molecules with great precision, using the laws of quantum mechanics. But could we be fooling ourselves? As a check, it's great to see what happens when you replace hydrogen with light hydrogen.

You should be wondering if I'm making this up. You probably never heard of light hydrogen in school!

It's a real thing, but it's usually called by a different name. You make it by shooting a beam of protons at a chunk of stuff like beryllium. If the protons have enough energy, this produces a bunch of short-lived particles called pions. Pions come in three kinds: positively charged, negatively charged and neutral.

A positively charged pion quickly decays into another positively charged particle called an antimuon. This lasts much longer: it has a half-life of 2.2 microseconds.

An antimuon is about 1/9 as heavy as a proton, but they're both positively charged. A lone proton in ordinary matter will often grab an electron and form an atom of hydrogen. An antimuon does the same thing!

The result is an exotic atom called muonium. It's just like hydrogen, except it has an electron orbiting an antimuon instead of a proton.

Since most of the mass of hydrogen comes from the proton, muonium is about 1/9th as heavy as ordinary hydrogen. But since the size and chemical properties of hydrogen depend mostly on the mass of the electron, muonium acts chemically like hydrogen.

Puzzle 1: The term 'muonium' is unfortunate, because it was chosen before people had developed a systematic naming scheme for exotic elements. It should really be called 'muium'. There's a different exotic element that really deserves the name muonium... and people call it true muonium. What is true muonium?

Hint: if you remember my August 28th diary entry on protonium, that should help! True muonium is like protonium in some ways, which make them count as 'oniums'.

Puzzle 2: What are some other oniums?

Puzzle 3: How does the radius of true muonium compare to the radius of hydrogen?

True muonium has not yet been made! However, we know enough about physics to know how big it will be.

There's actually a whole textbook on muonium chemistry! You can read the beginning here:

- D. C. Walker, *Muon and Muonium Chemistry*, Cambridge U. Press, Cambridge, 2009. First chapter available here.

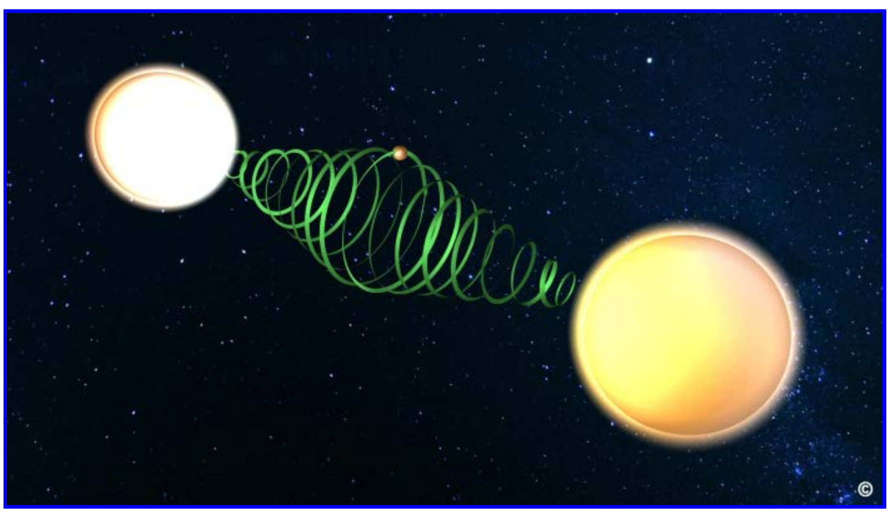
The picture above comes from the website:

- Tsuneyuki Research Group.

and appears in this paper:

- Takashi Miyake, Tadashi Ogitsu and Shinji Tsuneyuki, Quantum distributions of muon in muonium state in crystalline silicon, *Phys. Rev. Lett.* **81** (1998), 1873–1876.

October 15, 2015



The magazine *New Scientist* recently announced an unbelievable discovery:

> Even the craziest planets concocted by theorists still tend to trace conventionally near-circular orbits in a flat plane. Not so the corkscrew planet. Mind-bendingly, these worlds could exist in a sort of orbital limbo, spiralling about an axis between two stars in a binary system, pulled hither and thither by their competing gravities.

But when you read something unbelievable, maybe you shouldn't believe it. When the planet gets close to one star, what force would push it back towards the other star?

So, when the science fiction writer Greg Egan read about these corkscrew orbits, he decided to see for himself if they really existed. They were supposedly discovered in a paper called 'Stable Conic-Helical Orbits of Planets around Binary Stars', published in *The Astrophysical Journal*. He got the paper and read it.

He discovered that the *New Scientist* story was exaggerated. The orbits discussed in the paper look nothing like the picture here! They involve a planet that is much closer to one star than another.

But even so, Egan was unable to verify that the orbits worked as claimed in *The Astrophysical Journal*. Indeed, he wound up convinced that this paper was mistaken. For details, go here:

- Greg Egan, There are no corkscrew orbits.

He presents both a mathematical argument that these orbits are impossible, and some computer calculations. You can download his Mathematica notebooks and check these calculations yourself!

Unfortunately the New Scientist article is not free - this magazine is owned by Elsevier. The paper in The Astrophysical Journal is also not free - you have to pay $9 to read it. I feel like saying something about how open-access science works better than pay-to-play science. But mainly I'd like some physicists to check Egan's work.

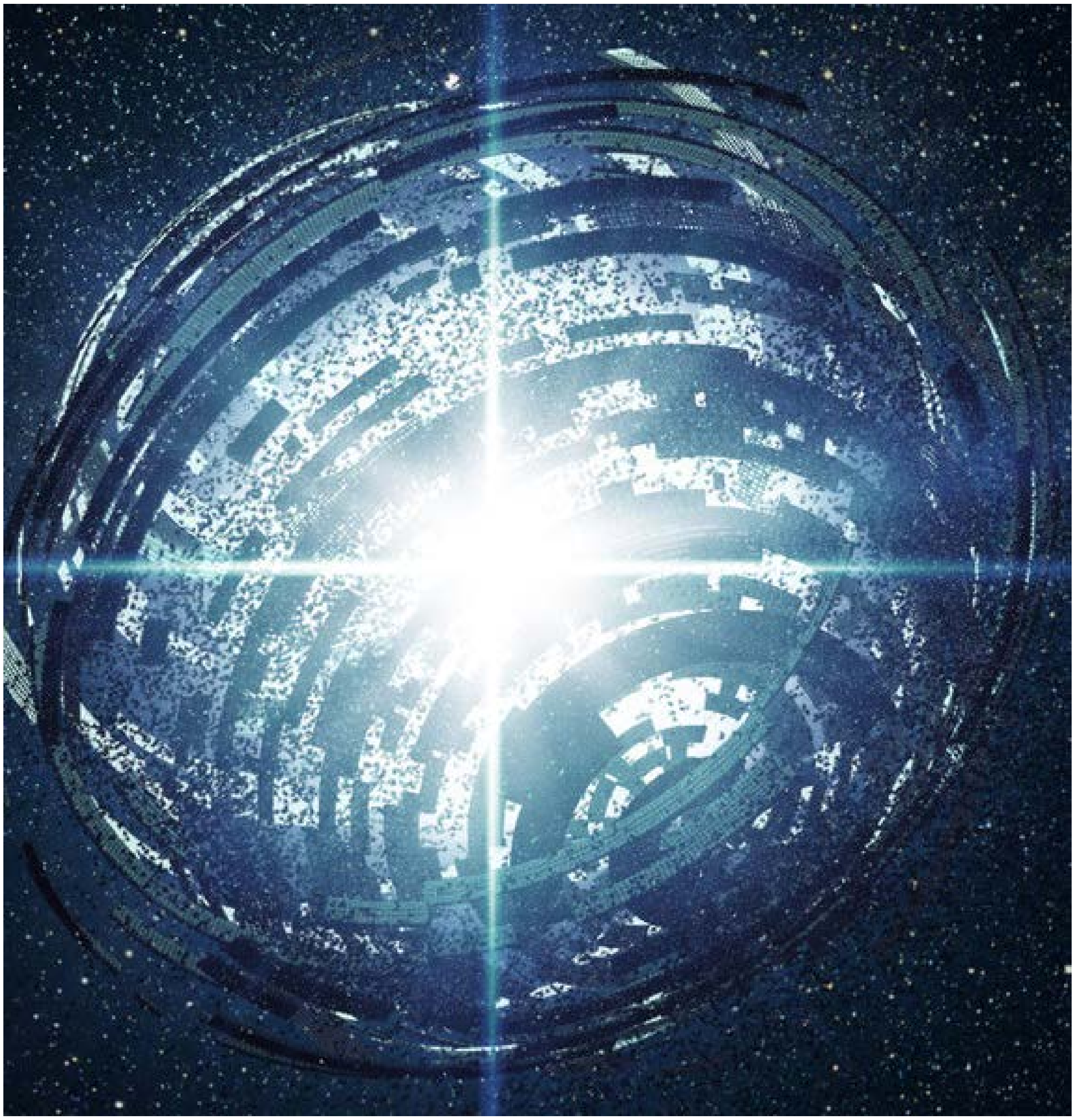
The picture here is from a blog article about the incredible corkscrew orbits:

- Dr. Carpineti, The taming of the screw.

It was drawn by The Digital Welshman.

October 17, 2015

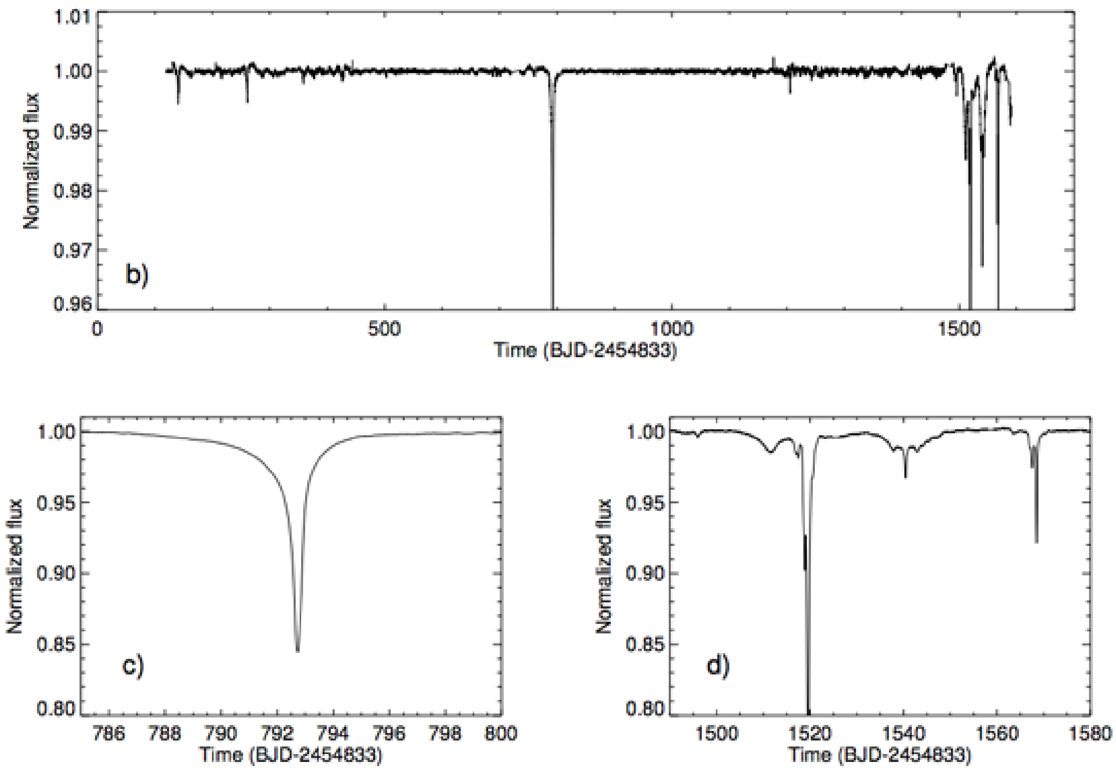
Alien megastructures?



The Planet Hunters project shows that ordinary citizens can do good science. For 4 years, a telescope orbiting the Earth scanned a patch of sky, looking for signs of planets. Now the data is being analyzed by computers, but also by a team of volunteers — you can join them if you want! They've found several planets. But that's not what everyone is talking about.

Recently some of these volunteers found a star called KIC 8462852 that's being called "the most interesting star in the galaxy".

It's a star that keeps suddenly getting dimmer. It can lose up to 22% of its brightnesss. It stays dim for between 5 and 80 days and then bounces back. It does this in a very irregular, unpredictable way. You have to really look at the graphs to see how wacky it looks. There are some repeating patterns that last for a while — but then they go away.



It can't be a planet coming in front of the star because it's too irregular. It's probably not the star itself getting dim, because it's an F-type star, not very different from our Sun, and stars like this don't seem to flicker. It could be clumps of dust that appear and then go away - maybe formed by asteroid collisions? It could be swarms of comets knocked into the star by a neighboring star. It could be something else.

It could be large structures built by an extraterrestrial civilization.

We don't know. We should find out! We should watch this star more carefully! Right now no telescopes are studying this star: the Kepler project is done.

Yesterday *The Independent*, a British newspaper, had this headline:

Astronomers May Have Found Giant Alien 'Megastructures' Orbiting Star Near the Milky Way

The day before, Joel Aschenbach, a science reporter for *The Washington Post*, wrote a column titled:

No, We Haven't Discovered Alien Megastructures Around a Distant Star

The day before that, the astronomer Phil Plait wrote a blog article called:

Did Astronomers Find Evidence of an Alien Civilization? (Probably Not. But Still Cool.)

The day before that, *Atlantic* magazine started the public furor with an article that quoted Jason Wright, an astronomer at Penn State. He said:

> "Aliens should always be the very last hypothesis you consider, but this looked like something you would expect an alien civilization to build."

He must not mean that literally. If aliens should always be the very last hypothesis you consider, you'll never get around to considering them — since you can always make up other hypotheses to explain anything you see. God, for example.

But Wright can't mean that — because he has written a series of 4 very interesting papers on the search for extraterrestrial civilizations, including one on this star.

What do I think is making KIC 8462852 suddenly get dimmer?

First of all, I think something absolutely fascinating is going on, and we should study it more. Get some telescopes on it!

Second, I think it can be incredibly powerful and liberating, when you don't know something, to say "I don't know".

People seem embarrassed to do this. People want to act like they know what's going on. But we don't need to say it's "improbable" that we're seeing alien megastructures — and we certainly shouldn't say it's "probable". It's best to admit our ignorance and get on with the business of learning more.

To learn more, I really recommend looking at the graphs in the actual paper:

- Tabetha S. Boyajian *et al*, Planet Hunters X. KIC 8462852 — where's the flux?

The analysis here is also good:

- Jason T. Wright, Kimberly M. S. Cartier, Ming Zhao, Daniel Jontof-Hutter and Eric B. Ford, The $\hat{G}$ search for extraterrestrial civilizations with large energy supplies, IV. The signatures and information content of transiting megastructures.

The *Atlantic* article is very fun to read:

- Ross Anderson, The most mysterious star in our galaxy, *The Atlantic*, October 13, 2015.

I also recommend reading Phil Plait's article:

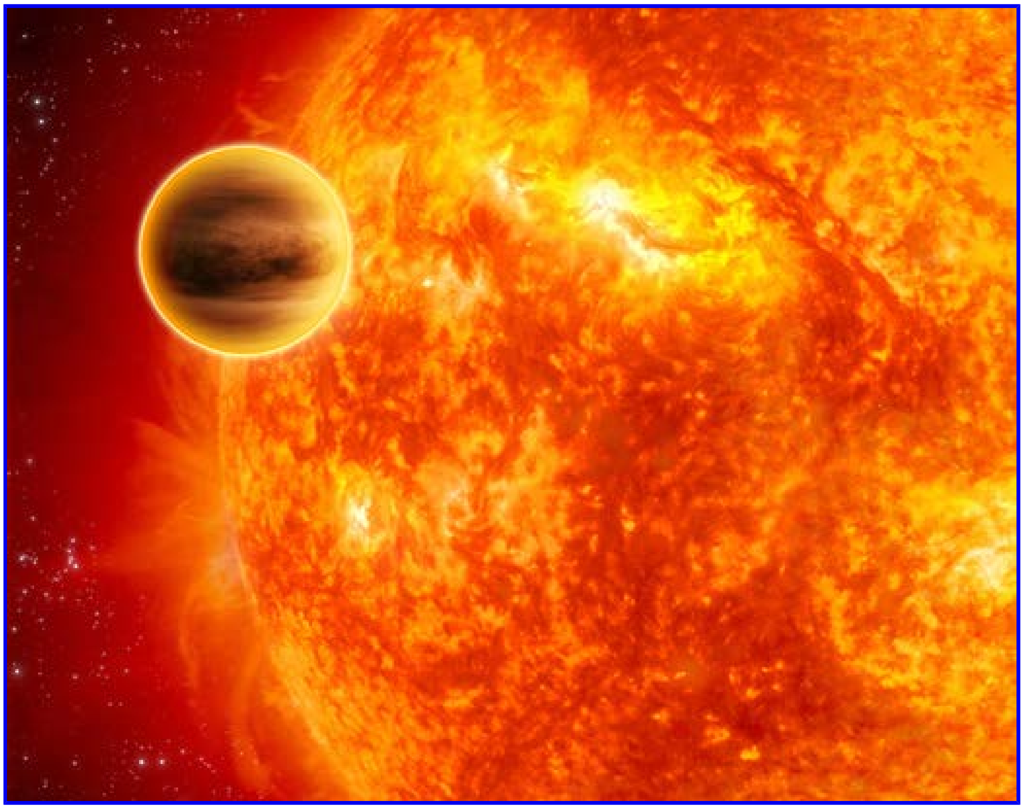
- Phil Plait, Did Astronomers Find Evidence of an Alien Civilization? (Probably Not. But Still Cool.), *Slate*, October 14, 2015.

Here is the Planet Hunters blog:

- Comets or aliens?, *Planet Hunters*, October 16, 2015.

The image above is by SPAR, available at DeviantArt.

October 18, 2015



378 light years away, in the constellation of Andromeda, there's a planet over 4 times as massive as Jupiter. It's orbiting a star somewhat bigger than our Sun. What makes it special is that it's orbiting very close: 1/15th the distance from Mercury to our Sun! It's called WASP-33b.

It's so close to its star that its surface temperature is about 3,200 °C. There are other Jupiter-like planets close to stars, called hot Jupiters. But this one is the hottest planet we've ever seen!

It's so hot that its atmosphere contains vaporized titanium dioxide. That's a white compound used in paint and sunscreen. Sunscreen wouldn't work very well on this planet.

And it's so close to its star that it orbits once every 1.22 days — that is, Earth days.

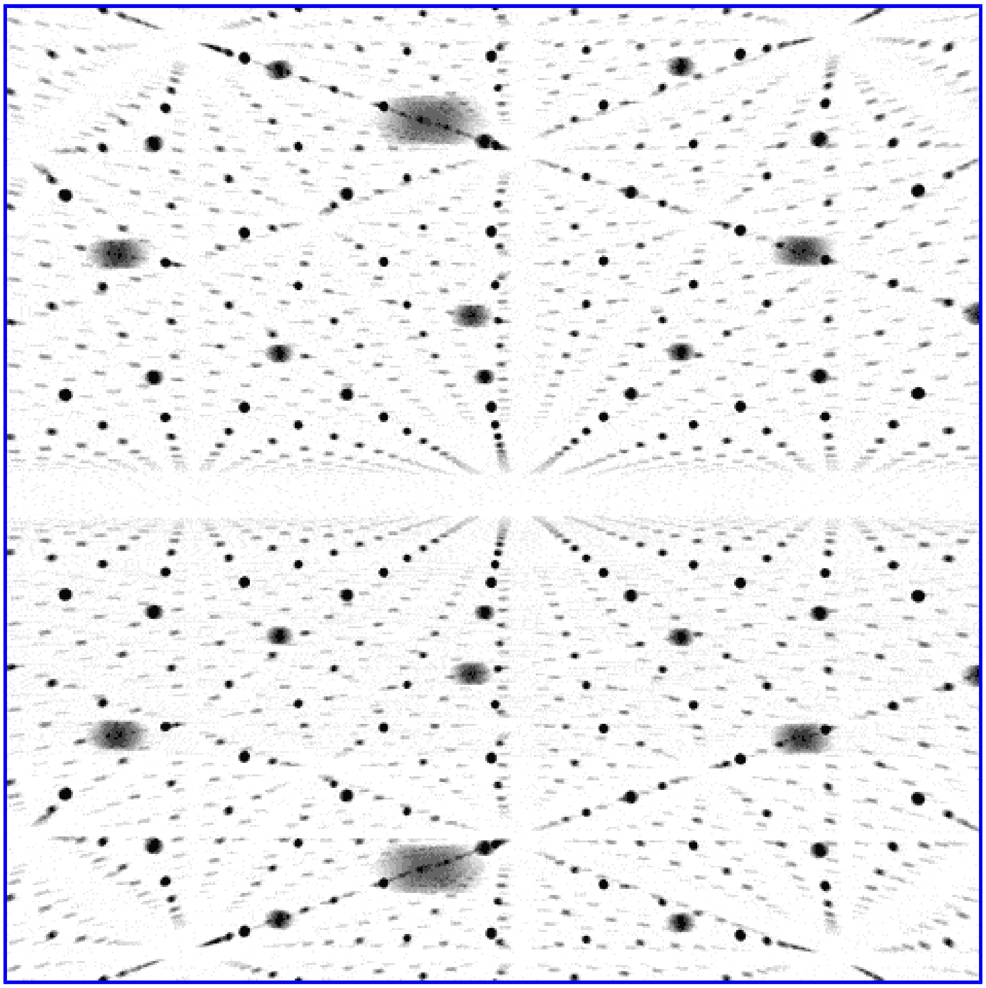
It's even so close that its orbit should be quite different from an ellipse! In our solar system, Mercury's orbit precesses very slightly due to the oblateness of the Sun and one of the effects of general relativity. Both these roughly add an inverse cube force to the usual inverse square force of Newtonian gravity. That makes the orbit of Mercury precess. But they are very tiny effects.

For WASP-33b these effects are much bigger: for example, its precession due to the oblateness of the star it's orbiting should be 9 billion times more than the corresponding effect for Mercury. We might even see another effect due to general relativity: frame-dragging, where a spinning object (the star) pulls spacetime along with it. But to see it, we'd need to carefully study WASP-33b for a long time — more than 10 years:

- Lorenzo Iorio, Classical and relativistic node precessional effects in WASP-33b and perspectives for detecting them.

The star WASP-33b orbits is called WASP-33 or HD 15082. It's a Delta Scuti variable, a kind of star whose brightness oscillates faster than once a day.

October 19, 2015



When you drive past a farm with plants in a rectangular grid, you'll see flickering lines as they momentarily line up in various ways. Here you can see that in 3 dimensions.

If you were standing in a rotating space filled with dots, one dot at each point with integer coordinates, this is what you'd see.

It's all about number theory. Suppose you have a farm with one plant at each point (x, y) where x and y are integers. Then you'll clearly see lines of plants with slopes y/x when y and x are small integers. So, slopes like 0/1, 1/1, 1/2, 2/3 and so on. There will also be lines where y and x are large integers, but these will be harder to see.

The same sort of thing happens in 3 dimensions. See all the ways the dots line up?

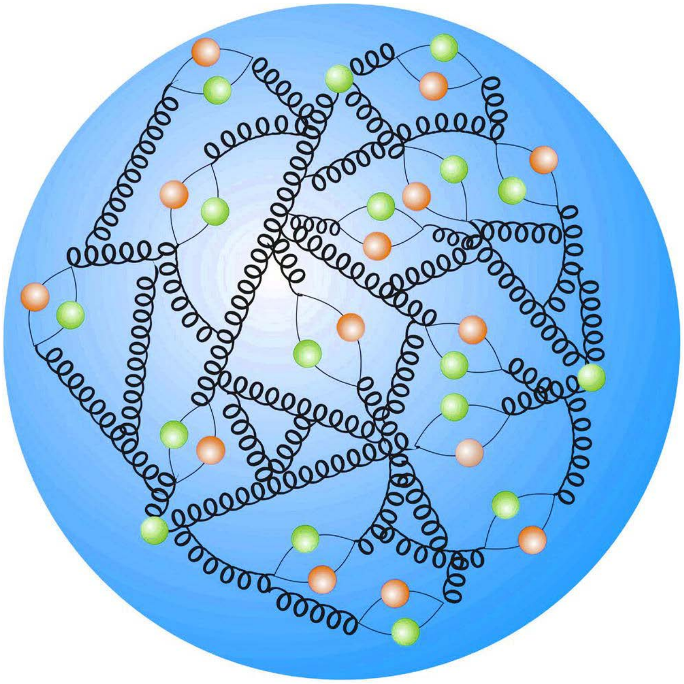
But notice, you're not shooting past these dots like driving past a farm. The dots in front are moving left. The dots in back are moving right!

So, I think these dots are actually rotating around a vertical axis. If take a dot that's not too close, and not too far, and follow it with your eye, you can see it go round and round! It's a bit hard to do, but it's fun to try.

I found this image here:

- Charlie's daily sketches, Drift matrix.

October 21, 2015



It's impossible to accurately draw a proton, but this picture is a good try. It's like a bag of virtual particles! There are lots of quark-antiquark pairs (the red and green balls) and gluons (the springs).

A gluon can split into a quark and an antiquark. A quark and antiquark can meet and become a gluon.

Other stuff can happen too, which is not shown here. A quark or antiquark can absorb or emit a gluon. A gluon can split into 2 gluons. Conversely, 2 gluons can collide and become one! Also, 2 gluons can collide and become 2 other gluons... or 3 can become 1... or 1 can become 3.

All these things are happening all the time inside a proton, in a kind of quantum blur. And since every possible process is literally happening all the time, nothing is actually changing!

That's a bit weird, I admit.

It makes no sense to ask how many virtual quarks, antiquarks and gluons are in a proton. There's not a fixed number of them waiting to be counted. They are virtual, not real. That means if you start probing the proton, you'll find them: they will become real, created by whatever form of energy you used to look for them. But what you see depends to some extent on how you look.

This is why it's impossible to accurately draw a proton. It's also damned near impossible to describe it accurately in plain English. That's why physics uses math.

Anyway, if you look really carefuly at this picture you'll see 3 quarks — green balls — that aren't next to antiquarks. Can you spot them?

Indeed: mixed in among all the virtual particles, a proton has 3 real quarks in it.

But physicists have long believed it's possible to have a particle that's made only of virtual quarks and gluons, with nothing else. This is called a glueball.

People have looked for gluballs and found some candidates. One is about 2900 times as heavy as an electron, another is about 3300 times as heavy, and there are others. For comparison, the proton is 1836 times as heavy as an electron.

The big problem is that there are particles called mesons that are made of a quark and antiquark along with virtual stuff. It's hard to distinguish between a meson and a glueball!

Worse still, just as in quantum mechanics you can have a cat that's a superposition of live and dead, you can have a particle that's a superposition of a glueball and a meson!

Roughly, the idea is this. A particle is a meson if when you probe it, not zapping it with too much energy, you usually see a quark and an antiquark. It's a glueball if you usually see just gluons. But in fact, there's always a chance you can see either.

In short, the difference between a glueball and a meson is just slightly more precise than the difference between a 'planet' and a 'dwarf planet'. You can make up rules to decide what counts as a glueball and what counts as a meson... but someone else could argue with those rules.

Recently 2 physicists did some calculations and came up with evidence that the particle 3300 times as heavy as an electron is really a glueball. So, now the newspapers are shouting *Physicists found a glueball!*

That's an okay newspaper headline, but the actual title of the paper is a bit more technical. It's called "Nonchiral enhancement of scalar glueball decay in the Witten-Sakai-Sugimoto model". It argues that if this particular glueball candidate is really a glueball, it should decay a lot into kaons and eta mesons, which is pretty much what we see.

By the way, this glueball candidate is called the $f_0(1710)$. This means its rest energy is 1710 MeV. In plain English, that means it has 3300 times the mass of an electron. The lighter glueball candidate I mentioned is called the $f_0(1500)$. And there are even lighter candidates, called the $f_0(500)$ and $f_0(980)$. These particles are known to exist — the problem is figuring out whether they are glueballs, mesons or superpositions.

Also by the way, there's a million dollar prize waiting for whoever who can mathematically prove that if the world had nothing in it but gluons, they would form glueballs with nonzero mass! Check it out:

- Clay Mathematics Institute, Yang-Mills and mass gap.

These glueballs would not decay into anything else, since there would be nothing for them to decay into. At least, that's what we believe. Proving it in a rigorous way is not easy.

Here's the paper that caused the fuss:

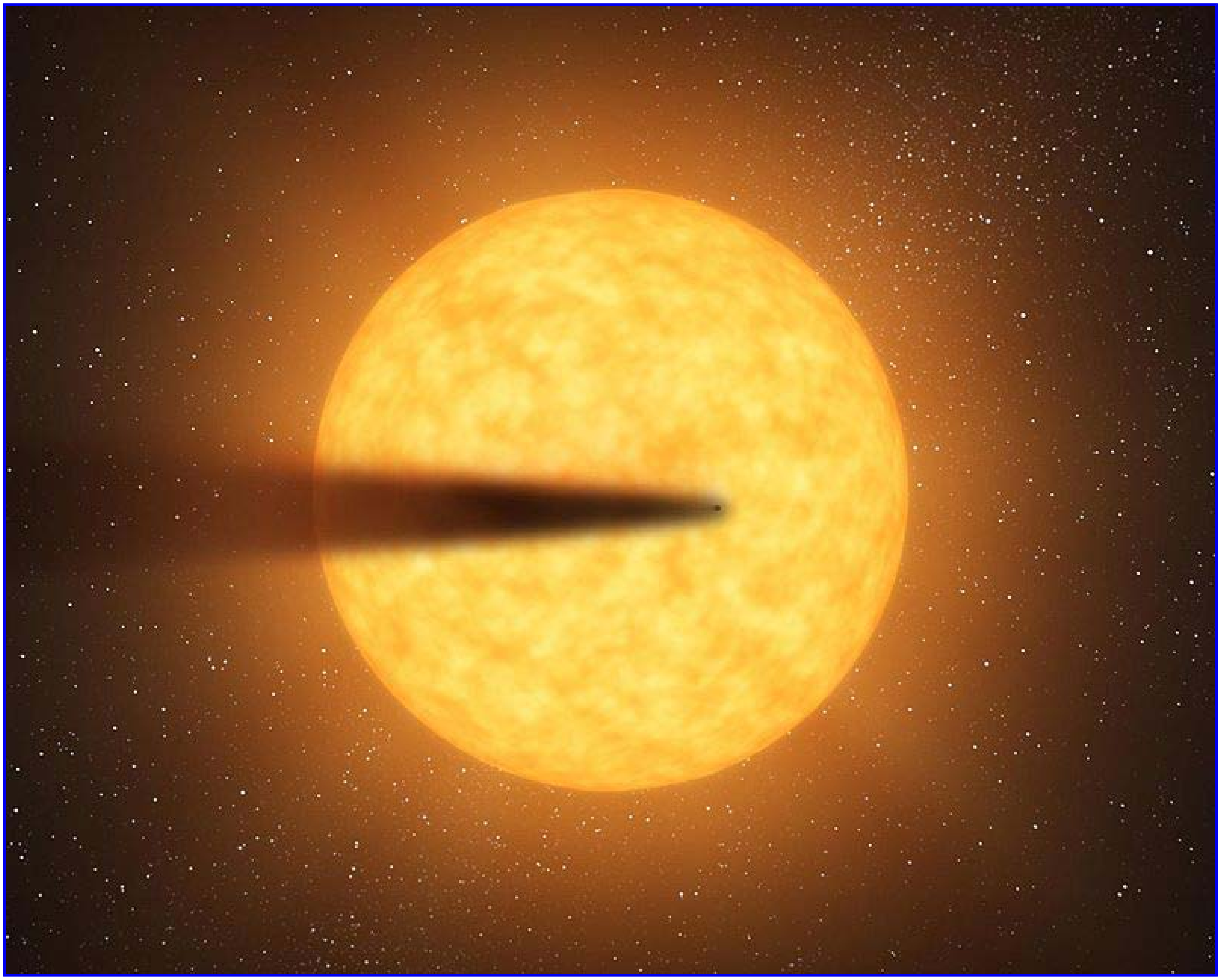
- Frederic Brünner and Anton Rebhan, Nonchiral enhancement of scalar glueball decay in the Witten-Sakai-Sugimoto model.

It uses ideas from string theory.

I found this picture at the website of the particle accelerator called DESY, the Deutsches Elektronen-Synchrotron:

- DESY, The most precise pictures of the proton, July 2, 2015.

October 23, 2015



Mercury is hot. It orbits the Sun every 88 days.

But in 2012, astronomers using the telescope on NASA's Kepler satellite found a planet called KIC 12557548 that orbits its star once every 16 hours! It's so hot that rocks on the day side can melt and boil away. And it seems this planet is disintegrating.

The star is a K-type main sequence star, meaning it's a bit smaller and cooler than our Sun. But the planet's distance from this star is only twice the star's diameter! So it must be very hot, probably about 2000 °C.

But why do we think this planet is falling apart? We know this planet exists only because of how it dims the star when it comes in front. But the amount of dimming varies each time it goes around! The planet blocks between 0.2% to 1.3% of the star's light. How are these changes possible?

Another clue is that the dimming is asymmetrical: the star gets dim slowly and then bright more quickly.

The best theory so far is that the planet is evaporating and falling apart, creating a cloud that changes size. If this cloud has a long tail, as shown here, it would produce asymmetrical dimming.

Scientists love puzzles like this. In 2013 two astronomers named Daniel Perez-Becker and Eugene Chiang studied this planet, and argued that it's in the final catastrophic stage of evaporating away. We know it's not very heavy, because it's not making the star wiggle detectably. Perez-Becker and Chiang believe it has lost most of its original mass, with only the inner iron core surviving.

This is their paper:

- Daniel Perez-Becker and Eugene Chiang, Catastrophic evaporation of rocky planets.

  Abstract. Short-period exoplanets can have dayside surface temperatures surpassing 2000 K, hot enough to vaporize rock and drive a thermal wind. Small enough planets evaporate completely. We construct a radiative-hydrodynamic model of atmospheric escape from strongly irradiated, low-mass rocky planets, accounting for dust-gas energy exchange in the wind. Rocky planets with masses < 0.1 M_{Earth} (less than twice the mass of Mercury) and surface temperatures > 2000 K are found to disintegrate entirely in < 10 Gyr. When our model is applied to Kepler planet candidate KIC 12557548b — which is believed to be a rocky body evaporating at a rate of dM/dt > 0.1 M_{Earth}/Gyr — our model yields a present-day planet mass of < 0.02 M_{Earth} or less than about twice the mass of the Moon. Mass loss rates depend so strongly on planet mass that bodies can reside on close-in orbits for Gyrs with initial masses comparable to or less than that of Mercury, before entering a final short-lived phase of catastrophic mass loss (which KIC 12557548b has entered). Because this catastrophic stage lasts only up to a few percent of the planet's life, we estimate that for every object like KIC 12557548b, there should be 10–100 close-in quiescent progenitors with sub-day periods whose hard-surface transits may be detectable by Kepler — if the progenitors are as large as their maximal, Mercury-like sizes (alternatively, the progenitors could be smaller and more numerous). According to our calculations, KIC 12557548b may have lost ~70% of its formation mass; today we may be observing its naked iron core.

This is another good paper on this planet, with lots of nice graphs:

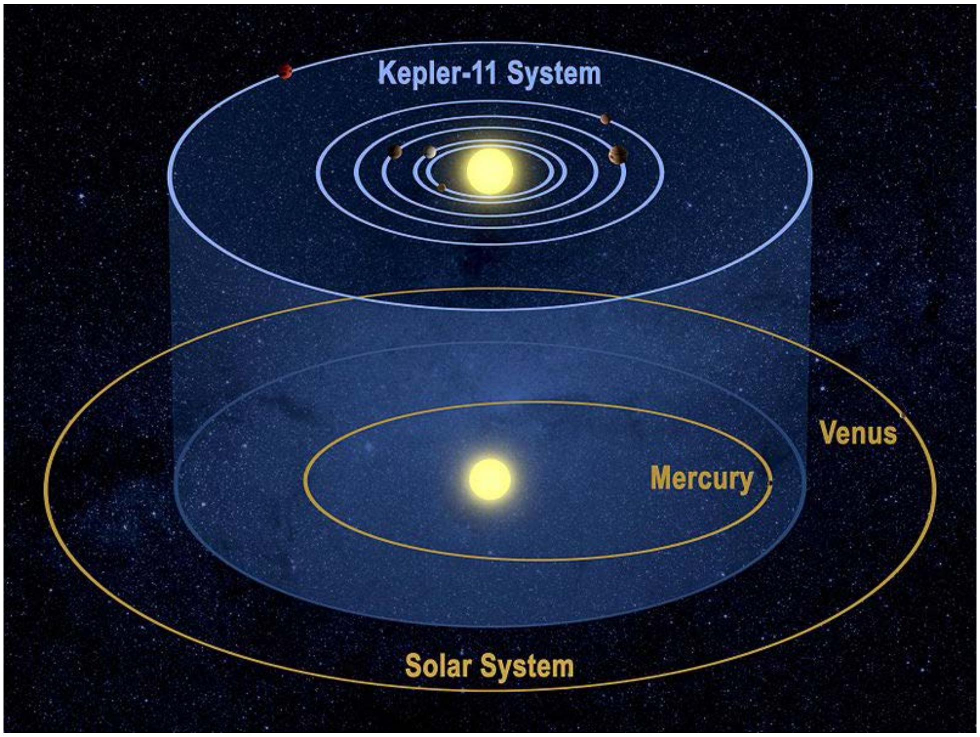
- T. I. M. van Werkhoven, M. Brogi, I. A. G. Snellen and C. U. Keller, Analysis and interpretation of 15 quarters of Kepler data of the disintegrating planet KIC 12557548b.

The picture above was made by a NASA artist and appears in the Wikipedia article on this planet and its star:

- Wikipedia, KIC 12557548.

October 25, 2015

A smaller system



Kepler-11 is a star 2000 light-years away that's very similar to our sun. It has at least 6 planets. But this solar system is small. All the planets would fit inside the orbit of Venus — and all but one fit inside the orbit of Mercury!

We used to think gas giants like Jupiter, Saturn and Neptune could only exist far from their host star. But that's when we just knew one solar system — our own. Now we know that there's a huge variety. Many have hot Jupiters or hot Neptunes — gas giants close to the star. We think they formed farther away and migrated in toward their stars when they got tired of the cold winters.

But beware: the easiest planets to detect are big ones close to the star! We're seeing the planets that are easy to see, not necessarily the 'typical' ones. There are probably lots of smaller planets we haven't seen yet.

Kepler-11 got its name because it's the 11th star where the Kepler spacecraft saw planets. Even better, they were found in 2011. Its planets have boring names: they're called b, c, d, e, f and g in order of increasing distance from their star. But they're pretty interesting. They have masses between those of Earth and Neptune. Their densities are all lower than Earth, so they're probably not rocky worlds. Planets d, e and f probably have a hydrogen atmosphere. Planets b and c seem to contain lots of ice.

Puzzle 1: how can you have a planet with lots of ice closer to a sun-like star than Venus is to the Sun?

Puzzle 2: why is there no planet a?

For more on Kepler-11, see:

- Wikipedia, Kepler-11.

For my November 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

home

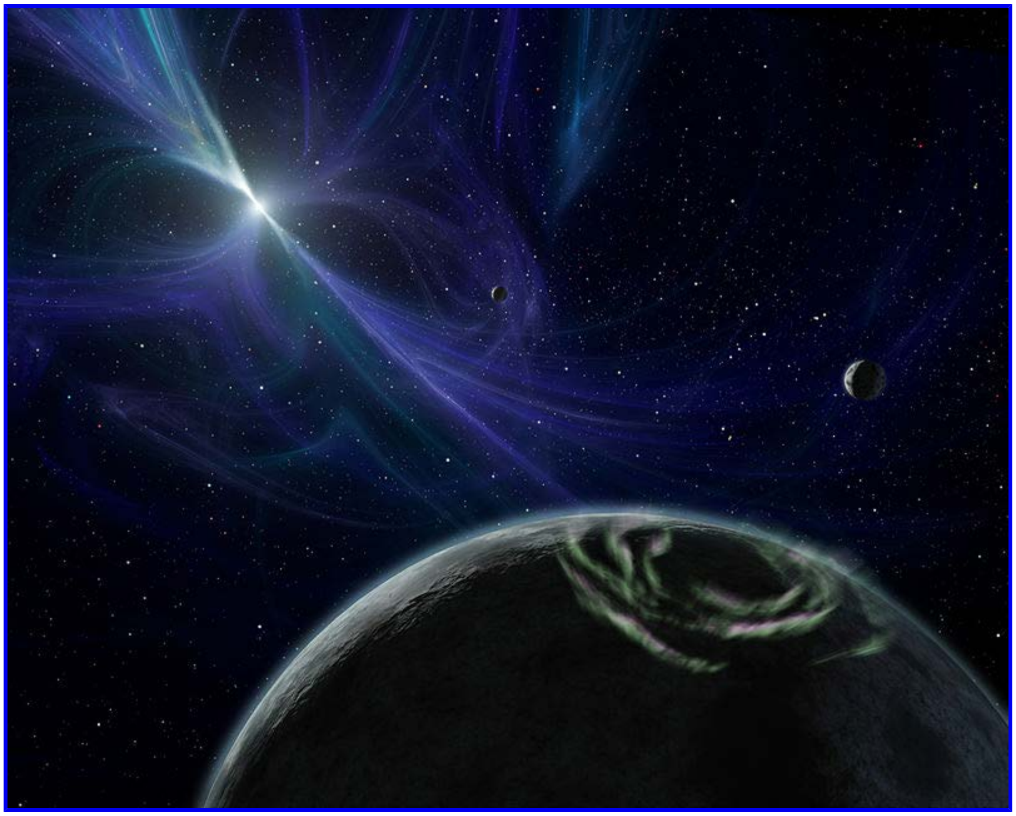
For my October 2015 diary, go here.

Diary - November 2015

John Baez

November 1, 2015

Pulsar planets



When a big star runs out of fuel, its core collapses and its outer layers explode into space. If it's not *too* big, its core becomes a big ball of neutrons. And if the resulting neutron star is spinning fast and emitting lots of radiation from its north and south magnetic poles, we call it a pulsar. It's called that because you see a pulse of radiation as it spins.

A pulsar is an amazing thing. Imagine something twice as heavy as our Sun, only 20 kilometers across, spinning around 1000 times a second, shooting out beams of radiation!

Now, imagine a planet near a pulsar: a dead world raked by intense radiation. It would be a strange, intensely alien place.

But in fact, the first planets to be discovered outside our solar system were pulsar planets! The reason is that pulsars keep time *very* accurately: the pulses are like the ticks of a clock. By detecting slight irregularities, we can tell if a pulsar is getting pulled back and forth by an orbiting planet.

In 1992, Aleksander Wolszczan and Dale Frail found the first planets outside our solar system: two planets orbiting a pulsar named PSR B1257+12. One is 4 times as heavy as the Earth, and it orbits the pulsar every 66 days. The other is just a bit heavier than the Earth, and it orbits every 99 days.

It seems these planets were formed from the debris of a companion star that used to orbit the pulsar. This star would have been destroyed by the huge explosion that formed the pulsar, called a supernova.

There's another pulsar planet that's very different. It's very close to a pulsar named PSR J1719-1438. It's so close that its orbit would fit inside our Sun, and it orbits the pulsar once every two hours.

It seems to have the mass of Jupiter, but a diameter just 4 times that of our Earth. If so, it must be extremely dense: a bit more dense than platinum!

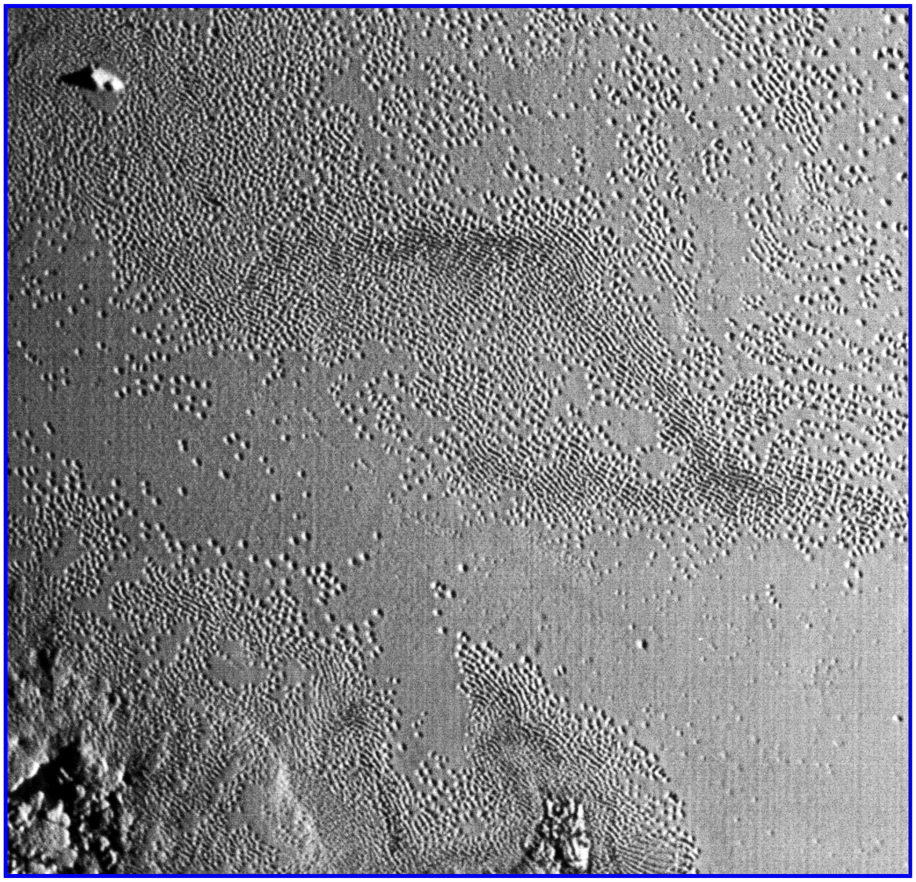
What could it be? People think it's the remains of a white dwarf star whose outer layers were blasted away by the supernova that formed the pulsar. If so, it could be made mostly of carbon and oxygen — leftover elements that the white dwarf wasn't able to burn.

In fact, it might even be similar to an enormous high-density diamond, so it's been nicknamed 'The Diamond Planet'. But nobody is sure, and these days people think it contains too much oxygen.

The picture here is from NASA, and it's an artist's impression of the planets orbiting PSR 1257+12.

November 3, 2015

The ice pits of Pluto



You are trapped on Pluto. Your only hope of survival is traveling a long distance to an old base where there is still a working rocket.

Your rover is insulated against the amazingly cold temperatures, and its huge corrugated metal wheels have no trouble driving over the ice — which is mostly made of frozen nitrogen.

But then you come to a large pit. It's about 10 meters deep and 100 meters across. Its walls are not very steep, so you can cross it, but it's a bit annoying.

Then you come to another pit. And another, and another.

You have entered Sputnik Planum, a huge field of ice pits in a plain named after the old Russian satellite Sputnik.

We don't know how these pits formed. They may be caused by sublimation where ice turns directly into gas as it warms up in the chilly Plutonian summer. They may start small and grow over time. But why are they here, and not all over Pluto?

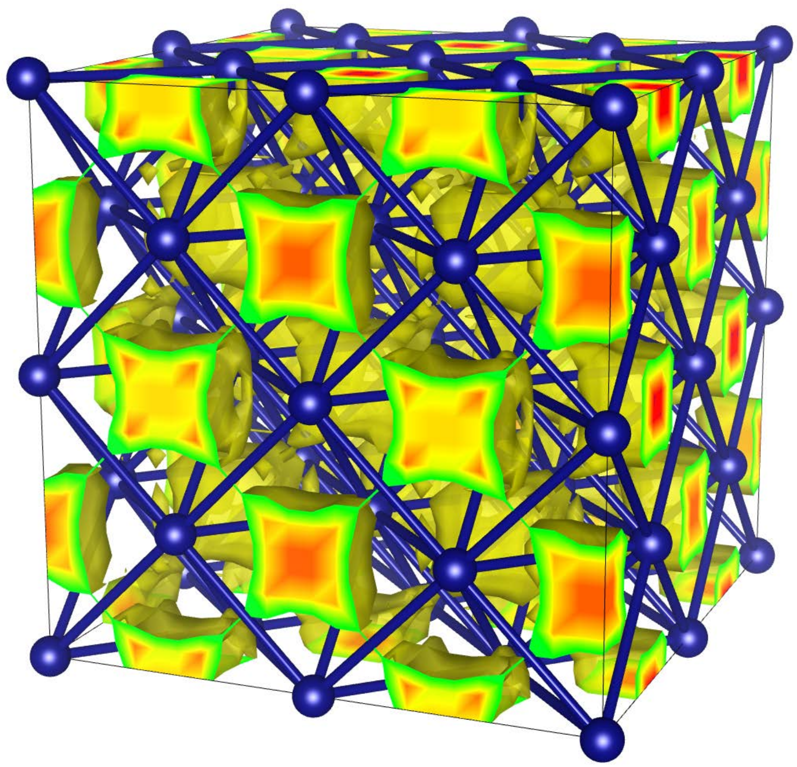
Many of these pits are connected, forming troughs that line up. Why? We don't know.

Good luck. Maybe you will find out. If you survive, maybe you can tell us.

For more, see:

- Phil Plait, Pluto is sublime. It's also the pits, *Bad Astronomy*, October 17, 2015.

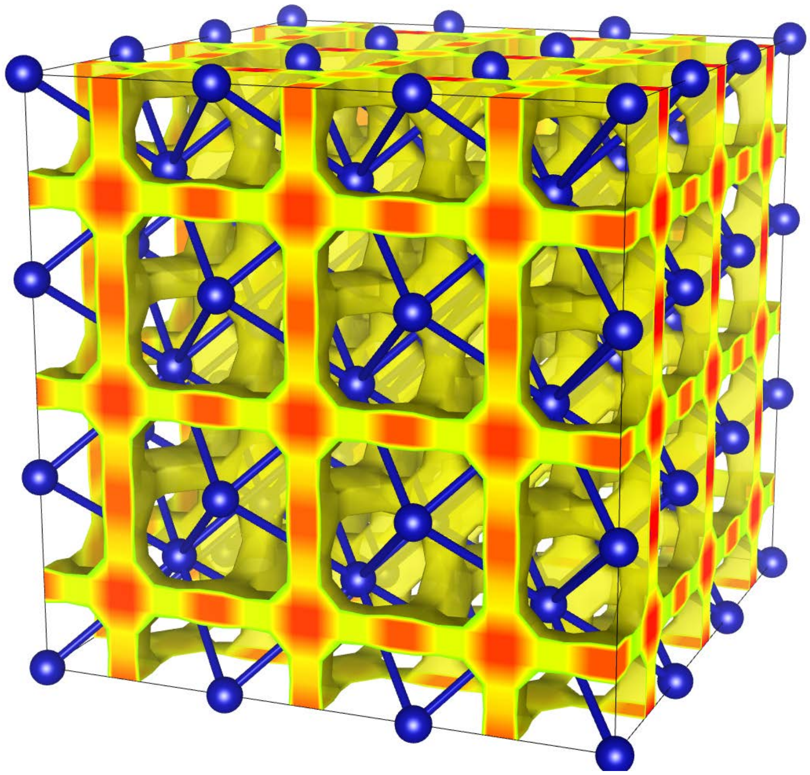
November 10, 2015



There are over 15 kinds of ice. Different kinds are stable at different pressures and temperatures. Some of the weirdest may exist inside 'ice giants': planets like Uranus and Neptune, which have also been found orbiting other stars. Most of what we know about these kinds of ice comes from computer simulations, since they only exist at very high pressures.

They're called 'superionic ices', because while the oxygen atoms get locked in a crystal structure, the hydrogen atoms become ionized, breaking apart into protons and electrons. The protons can then *move around like a liquid* between the oxygen atoms!

The first phase of superionic ice was predicted in 1999 by a group of Italian scientists. They predicted that this ice exists at pressures 500,000 times the atmospheric pressure here on Earth, and temperatures of a few thousand Kelvin. In this kind of ice, the oxygen atoms form a crystal called a body centered cubic:



In 2012, Hugh F. Wilson, Michael L. Wong, and Burkhard Militzer predicted the new phase shown at the top of this diary entry. This may show up above 1,000,000 times atmospheric pressure. The oxygen atoms, shown as blue spheres, form a pattern called a face centered cubic. The protons are likely to be found in the orange regions.

Hugh Wilson, who apparently drew the pictures of both forms of ice, said:

> Superionic water is a fairly exotic sort of substance. The phases of water we're familiar with all consist of water molecules in various arrangements, but superionic water is a non-molecular form of ice, where hydrogen atoms are shared between oxygens. It's somewhere between a solid and a liquid.the hydrogen atoms move around freely like in a liquid, while the oxygens stay rigidly fixed in place. It would probably flow more like a liquid, though, since the planes of oxygen atoms can slide quite freely against one another, lubricated by the hydrogens.

These simulations are hard, and newer papers are reporting different results. You can also try to make superionic ice in the lab, but that's even harder! In 2005 Laurence Fried tried to make it at the Lawrence Livermore National Laboratory in California. He smashed water molecules between diamond anvils while simultaneously zapping it with lasers. He seemed to find evidence for superionic ice.

Eventually theory and experiment will converge on the truth. Only then will we understand the hearts of the ice giants.

You can read more here:

- Lisa Zyga, New phase of water could dominate the interiors of Uranus and Neptune, *Phys.org*, April 25, 2013.

and for some even newer results, try this:

- Tien Nguyen, Scientists predict cool new phase of superionic ice, October 21, 2015.

Here's the paper on the first kind of superionic ice:

- C. Cavazzoni, G. L. Chiarotti, S. Scandolo, E. Tosatti, M. Bernasconi, and M. Parrinello, Superionic and metallic states of water and ammonia at giant planet conditions, *Science* **283** (1999), 44–46.

and here's the second kind:

- Hugh F. Wilson, Michael L. Wong, Burkhard and Militzer, Superionic to superionic phase change in water: consequences for the interiors of Uranus and Neptune.

November 11, 2015

Miniature atoms

In *The Incredible Shrinking Man*, a guy exposed to radiation becomes smaller and smaller. Eventually he realizes he'll shrink forever — even down to subatomic size. Of course that's impossible. But guess what: *we can now make miniature atoms!*

In fact we can make atoms almost like hydrogen, but 1/186 times as big across. Unfortunately they only last 2 microseconds. But that's still long enough for them to form molecules, and for us to do chemical experiments with them. Chemists have gotten really good at this stuff.

The trick is to take the electron in a hydrogen atom and replace it with a muon. This is a particle 207 times heavier than an electron, but otherwise very similar. Unfortunately a muon has a half-life of just 2 microseconds: then it decays into an electron and some other crud.

Why is an ordinary hydrogen atom the size it is, anyway? It's the uncertainty principle. The atom is making its energy as small as possible while remaining consistent with the uncertainty principle.

A hydrogen atom is made of an electron and a proton. If it were bigger, its potential energy would increase, because the electron would be further from the proton. So, the atom 'wants to be small'. And without quantum mechanics to save it, it would collapse down to a point: *The Incredible Shrinking Atom*.

But if the atom were smaller, you'd know the position of its particles more precisely &mash; so the uncertainty principle says you'd know their momentum less precisely. They'd be wiggling around more wildly and unpredictably So the kinetic energy would, on average, be higher.

So there's a tradeoff! Too big means lots of potential energy. Too small means lots of kinetic energy. Somewhere in the middle is the best — and you can use this to actually calculate how big a hydrogen atom is!

But what if you could change the mass of the electron? This would change the calculation. It turns out that making electrons heavier would make atoms smaller!

While we can't make electrons heavier, we can do the next best thing: use muons.

Muonic hydrogen is a muon orbiting a proton. It's like an atom, but much smaller than usual, so it does weirdly different things when it meets an ordinary atom. It's a whole new exotic playground for chemists.

And, you can do nuclear fusion more easily if you start with smaller atoms! It's called 'muon-catalyzed fusion', and people have really done it. The only problem is that it takes a whole lot of energy to make muons, and they don't last long. So, it's not practical — it doesn't pay off. At least not yet. Maybe we just need a few more brilliant ideas:

- Wikipedia, Muon-catalyzed fusion.

In my October 11th entry I talked about a version of hydrogen where we keep the electron and replace the proton by a positively charged antimuon. That's called muonium. Muonium is lighter than ordinary hydrogen but almost the same size, just a tiny bit bigger. It's chemically almost the same as hydrogen, except that it decays in 2 microseconds.

With muonic hydrogen it's the reverse: it's a lot smaller, but it's just a bit heavier. It's chemically very different from ordinary hydrogen.

If you do the calculation, you can show that the radius of a hydrogen-like atom is proportional to

$$mM/(m + M)$$

where m is the mass of the lighter particle and M is the mass of the heavier one. If we say an electron has mass 1, then a muon has mass 207 and a proton has mass 1836. You can use this formula to see that muonic hydrogen has a radius 1/186 as big as ordinary hydrogen, while muonium has a radius 1.004 times as big.

November 22, 2015

I've been staying at home for the last two days writing a paper about information and entropy in biological systems. My wife is away, and I'm trying to keep distractions to a bare minimum, trying to get into that state where I'm completely absorbed, there's always something to do, and it's lots of fun. That's what I love about writing. At first I feel stuck, frustrated. But gradually the ideas start falling into place - and once they do, I don't want to be anywhere else!

This state is called flow, and it's great. But life can't be all flow, it seems.

I like this chart. I like any chart that takes psychology and maps it down to a few axes in a reasonably plausible way. I don't have to 'believe in it' to enjoy a neat picture that pretends to tame the wild mess of the soul.

Apparently this chart goes back to Mihály Csikszentmihélyi's theory of flow. According to Wikipedia:

> In his seminal work, *Flow: The Psychology of Optimal Experience*, Csikszentmihályi outlines his theory that people are happiest when they are in a state of flow. a state of concentration or complete absorption with the activity at hand and the situation. It is a state in which people are so involved in an activity that nothing else seems to matter. The idea of flow is identical to the feeling of being in the zone or in the groove. The flow state is an optimal state of intrinsic motivation, where the person is fully immersed in what he is doing. This is a feeling everyone has at times, characterized by a feeling of great absorption, engagement, fulfillment, and skill.and during which temporal concerns (time, food, ego-self, etc.) are typically ignored.

> In an interview with *Wired* magazine, Csikszentmihályi described flow as "being completely involved in an activity for its own sake. The ego falls away. Time flies. Every action, movement, and thought follows inevitably from the previous one, like playing jazz. Your whole being is involved, and you're using your skills to the utmost."
>
> Csikszentmihályi characterized nine component states of achieving flow including "challenge-skill balance, merging of action and awareness, clarity of goals, immediate and unambiguous feedback, concentration on the task at hand, paradox of control, transformation of time, loss of self-consciousness, and autotelic experience".

What does autotelic mean? It seems to mean 'internally driven', as opposed to seeking external rewards. Csikszentmihályi says "An autotelic person needs few material possessions and little entertainment, comfort, power, or fame because so much of what he or she does is already rewarding." Anyway, back to the Wikipedia article:

> To achieve a flow state, a balance must be struck between the challenge of the task and the skill of the performer. If the task is too easy or too difficult, flow cannot occur. Both skill level and challenge level must be matched and high; if skill and challenge are low and matched, then apathy results.

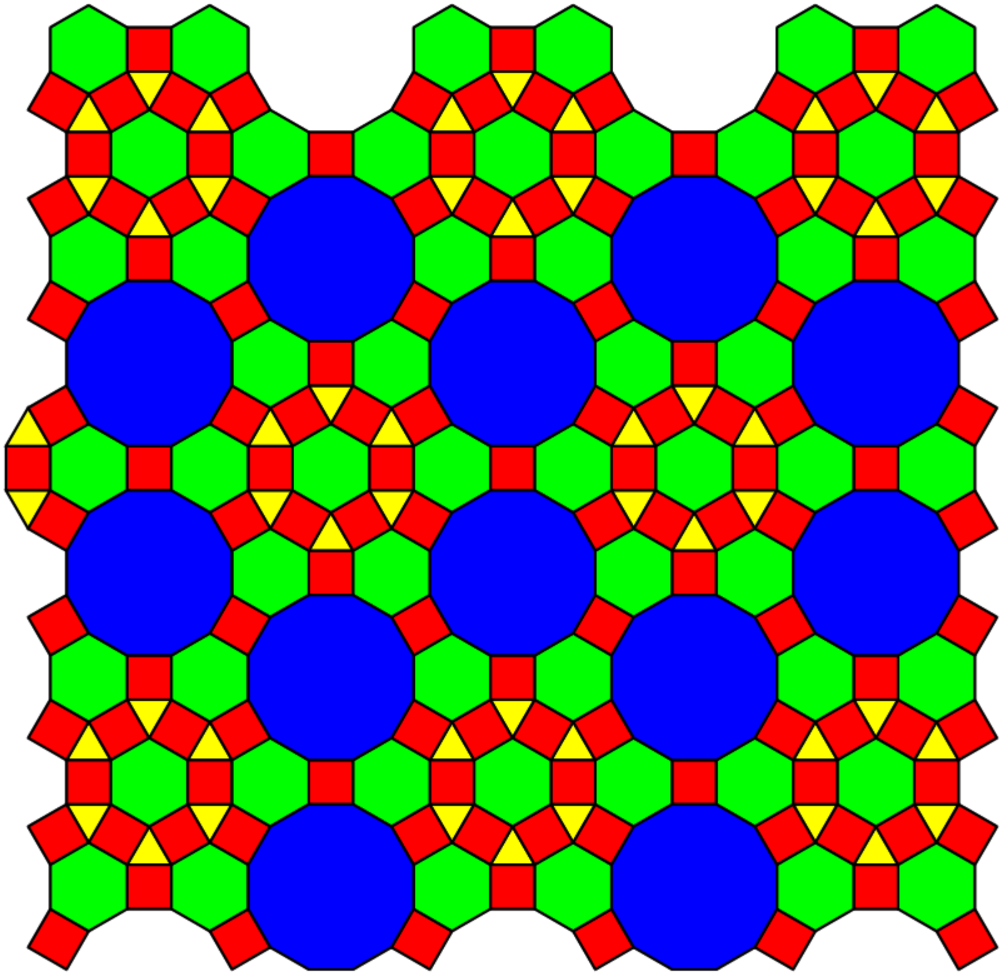
But in this chart, 'apathy' is just one of 8 options, the one diametrically opposite to 'flow'. I like the idea of how 'relaxation' is somewhere between flow and boredom, but I'm not sure it feels next to 'control'.

It's all very thought-provoking. We have these different modes, or moods, and we bounce between them without very much thought about what they're for and what's the overall structure of the space of these moods.

Moods seem like the opposite of mathematics and logic, but there's probably a science of moods which we haven't fully understood yet - in part because when we're in a mood, it dominates us and prevents us from thinking about it analytically.

November 30, 2015

The joy of tilings



Ever since I was a kid, I've loved the ways you could tile the plane with regular polygons. Some are used for floor tiles — pondering these is a great way to stay entertained while sitting in public restrooms. But unfortunately, a lot of the fancier ones have not come into wide use.

There are 3 regular tilings: you can use equal-sized regular triangles, squares or hexagons to tile the plane. If you let yourself use several kinds of regular polygons in the same tiling but demand that every vertex look alike, you get 8 more choices: the uniform tilings.

Only recently did I learn about the *k*-uniform tilings, where you relax a bit and let there be *k* different kinds of vertices.

The picture shows a 4-uniform tiling. There are 2 different kinds of vertices where

a blue dodecagon, a green hexagon and a red square meet

and 2 different kinds where

a red square, a green hexagon, a red square and a yellow triangle meet

They are different in this way: no symmetry of the whole tiling can carry the first to the second. So, while they look the same right at the vertex, further away they look different.

Puzzle 1: can you see the 2 kinds in both cases? Can you describe why they're different?

According to the experts, there are 20 2-uniform tilings. There are 61 3-uniform tilings. There are 151 4-uniform tilings. There are 332 5-uniform tilings. There are 673 6-uniform tilings. And I guess the list stops there only because people got tired!

This picture was drawn by Tom Ruen. You can find it, along with lots more, here:

- Wikipedia, Euclidean tilings by convex regular polytopes.

I think more of these should be deployed as bathroom tiles in public restrooms. We supposedly have this great, high-tech civilization, yet we're not taking full advantage of math in the decorative arts!

Puzzle 2: what uniform tiling is this 4-uniform tiling based on, and how?

For Tom Ruen's answer to Puzzle 2, see the conversation on my G+ post.

For my December 2015 diary, go here.

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baez@math.removethis.ucr.andthis.edu

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For my November 2015 diary, go here.

Diary - December 2015

John Baez

December 1, 2015

Too much happened this month for me to have time to write about it! For example, Lisa and I went on a great trip to Guanujuato. Maybe someday I'll put some pictures here.

For my January 2016 diary, go here.

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baez@math.removethis.ucr.andthis.edu

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