[For my December 2015 diary, go here.](#)

Diary - January 2016

John Baez

January 1, 2016

This is one of many impossible buildings imagined by the Barcelona-based artist and photographer Victor Enrich.

January 10, 2016

In December, the rover Curiosity reached some sand dunes on Mars, giving us the first views of these dunes taken from the ground instead of from above. It's impressive how the dune shoots up from the rocks here.

In fact this slope — the steep downwind slope of one of "Bagnold Dunes" along the northwestern flank of Mount Sharp — is just about 27 degrees. But mountaineers will confirm that slopes always looks steeper than they are.

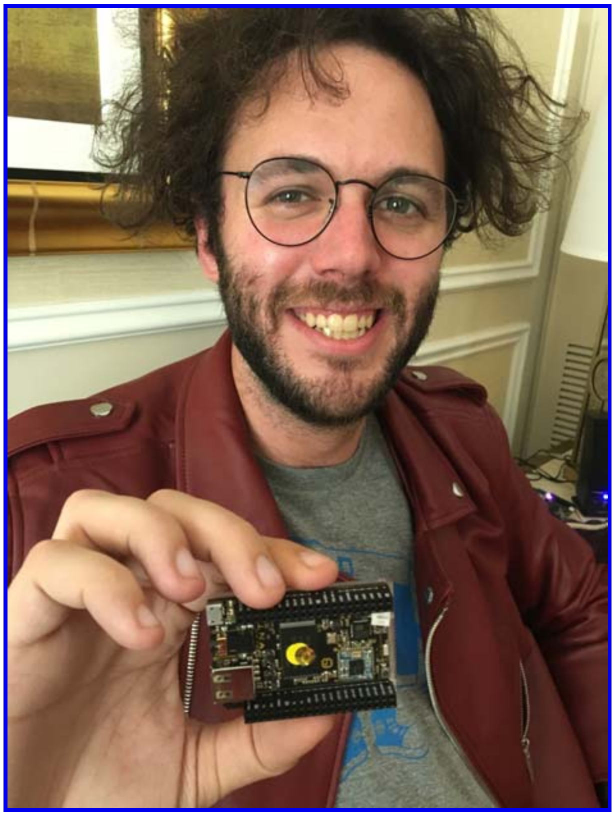
The wind makes this dune move about one meter per year.

For a much taller view, check out this:

- NASA JPL, Mastcom telephoto of a Martian dune's downwind face, January 4, 2016.

January 21, 2016

Someday computers will be free



Here is Dave Rauchwerk holding the computer his company sells. It has 4 gigabytes of storage. It does wifi and it has Bluetooth. It costs $9.

Keyboard and monitor not included — but still a good deal! Read more here:

- Laura Sydell, Can a $9 computer spark a new wave of tinkering and innovation?, *Morning Edition*, National Public Radio, January 21, 2016.

January 23, 2016

Not everyone likes the record-breaking snow storm that hit Washington DC today. But Tian Tian, the panda in the National Zoo, seems to love it! For more, watch him on YouTube.

For my February 2016 diary, go here.

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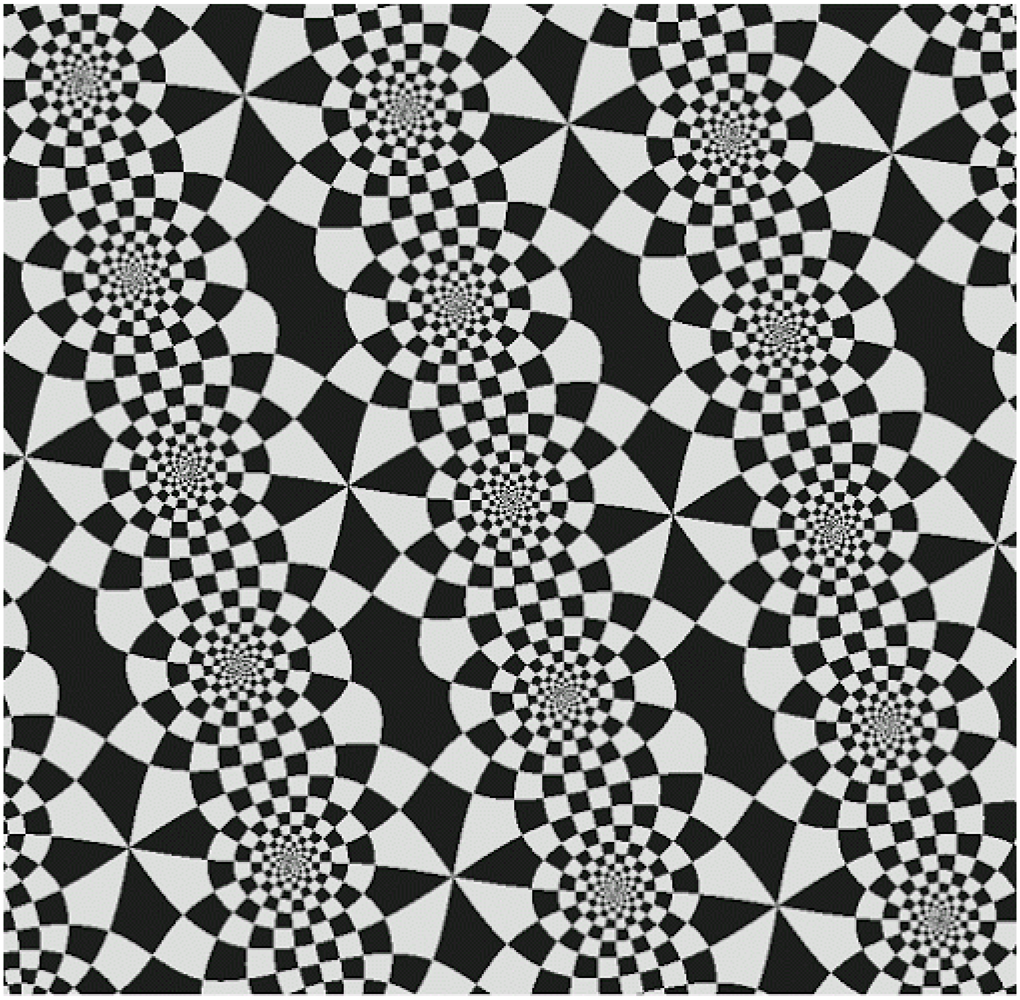
For my January 2016 diary, go here.

Diary - February 2016

John Baez

February 1, 2016



You probably know about the sine and cosine. These are the most basic functions that are periodic:

$$\sin(x + 2\pi) = \sin(x)$$

Elliptic functions are functions of two variables, x and y, that are periodic in two directions. For example, we can have:

$$f(x + 2\pi, y) = f(x, y)$$

and

$$f(x, y + 2\pi) = f(x, y)$$

This movie is a way of illustrating an elliptic function.

What makes elliptic functions so special is that you can think of them as functions of a single complex variable:

$$z = x + iy$$

and then they have a derivative in the special sense you learn about in a course on complex functions!

It's a lot harder for a complex function to have a derivative than an ordinary real function. A function like

$$f(x, y) = \sin(x)\sin(y)$$

is periodic in two directions, but it doesn't have a derivative df/dz. Mysterious as this may sound, this is the reason elliptic functions are so special.

In the late 1800s, all the best mathematicians thought about elliptic functions, so there are 'Jacobi elliptic functions' and 'Weierstrass elliptic functions' and many more. Now they're less popular, but they're still incredibly important. You need to think about them if you want to deeply understand how long the perimeter of an ellipse is. They're also important in physics, and fundamental to the proof of Fermat's Last Theorem.

An elliptic function actually has a derivative everywhere except at certain points where the function 'blows up' — that is, becomes infinite. These points are called poles. You can prove an elliptic function has to have poles unless it is constant (and thus too boring to talk about).

Because an elliptic function is periodic in two directions, its poles make a repeating pattern in the plane, which you can see in this movie. The poles are the points from which checkerboard pattern keeps expanding outward. The zeros of the elliptic function — the points where it's zero — are the points where the checkerboard keeps shrinking inward.

For more on elliptic functions, you could try this:

- Wikipedia, Elliptic function.

The animation here was created by Gerard Westendorp.

February 4, 2016

This is so cool I'm not sure I believe it. It's a photo of the night sky over a city in Finland. A rare atmospheric phenomenon called light pillars created a map of the city itself, in the sky!

Street lights were reflected back down by ice crystals in the air. This only happens when flat hexagonal crystals are floating horizontally in still air. Light bounces back down from the crystals.

This was taken on January 13, 2016, by Mia Heikkila in Eura, Finland. For more, read Phil Plait's article here:

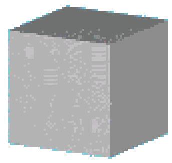
- Phil Plait, Optical phenonomenon draws a a map of a city in the sky, *Bad Astronomy*, January 16, 2016.

He's a smart guy. If he believes this is real, I guess I do too.

You can compare this picture to a city map here:

- John Metcalf, A city map spontaneously appeared in the sky over Finland, CityLab, January 15, 2016.

February 18, 2016



Inside every boring gray cube...

... there's a colorful dodecahedron yearning to unfold!

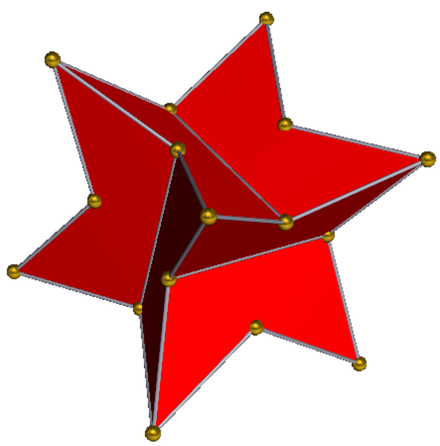
Puzzle. When we fold the dodecahedron back to a cube, does it fit together snugly, or is there some empty space left? What fraction of the cube is filled?

This animation was made by Hermann Serras, and I found it here:

- Simone Gregg, Two cubes, *Seek Echo*.

I will give away the answer to the puzzle, because it's so pretty: there is empty space left, and the fraction of the cube that's filled is $\Phi/2$, where $\Phi = \frac{\sqrt{5}+1}{2}$ is the 'big' golden ratio.

When you fold the dodecahedron into the cube, the shape of the empty space inside is called the concave pyritohedral dodecahedron or 'endo-docahedron':



For my March 2016 diary, go here.

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For my February 2016 diary, go here.

Diary - March 2016

John Baez

March 9, 2016

George Martin, 1926-2016

The Beatles' psychedelic music blew me away, but only later did I learn how much was due to the "fifth Beatle": George Martin. His idea of using the studio as an instrument was revolutionary and wonderful.

From the *New York Times* article by Allan Kozinn:

> Always intent on expanding the Beatles. horizons, Mr. Martin began chipping away at the group's resistance to using orchestral musicians on its recordings in early 1965. While recording the *Help!* album that year, he brought in flutists for the simple adornment that enlivens Lennon's "You've Got to Hide Your Love Away," and he convinced Mr. McCartney, against his initial resistance, that "Yesterday" should be accompanied by a string quartet.
>
> A year later, during the recording of the album *Revolver*, Mr. Martin no longer had to cajole: The Beatles prevailed on him to augment their recordings with arrangements for strings (on "Eleanor Rigby"), brass (on

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> ...arching band (on "Yellow Submarine") and solo French horn (on "For

No One"), as well as a tabla player for Harrison's Indian-influenced song "Love You To".

It was also at least partly through Mr. Martin's encouragement that the Beatles became increasingly interested in electronic sound. Noting their inquisitiveness about both the technical and musical sides of recording, Mr. Martin ignored the traditional barrier between performers and technicians and invited the group into the control room, where he showed them how the recording equipment at EMI's Abbey Road studios worked. He also introduced them to unorthodox recording techniques, including toying with tape speeds and playing tapes backward.

Mr. Martin had used some of these techniques in his comedy and novelty recordings, long before he began working with the Beatles.

"When I joined EMI," he told The New York Times in 2003, "the criterion by which recordings were judged was their faithfulness to the original. If you made a recording that was so good that you couldn't tell the difference between the recording and the actual performance, that was the acme. And I questioned that. I thought, O.K., we're all taking photographs of an existing event. But we don't have to make a photograph; we can paint. And that prompted me to experiment."

Soon the Beatles themselves became intent on searching for new sounds, and Mr. Martin created another that the group adopted in 1966 (followed by many others). During the sessions for "Rain", Mr. Martin took part of Lennon's lead vocal and overlaid it, running backward, over the song's coda.

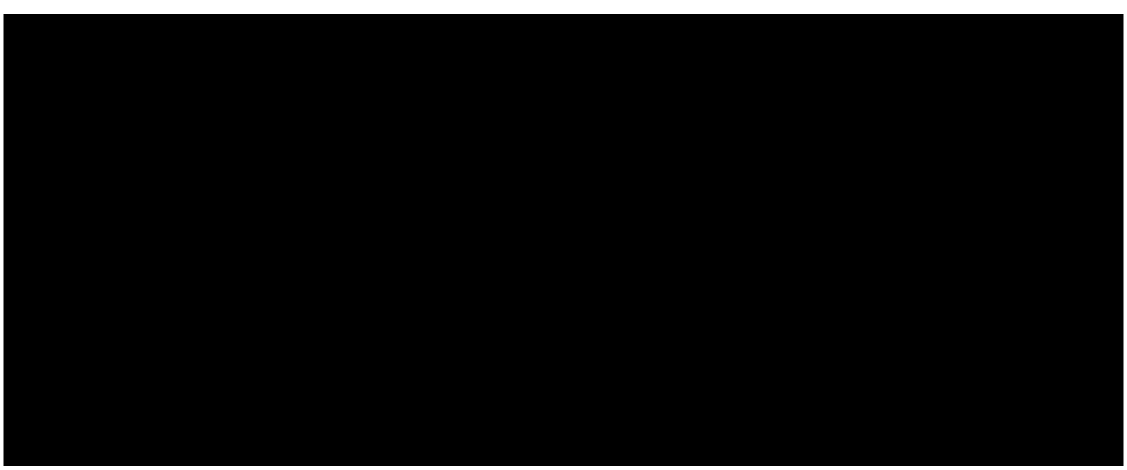
"From that moment," Mr. Martin said, "they wanted to do everything backwards. They wanted guitars backwards and drums backwards, and everything backwards, and it became a bore." The technique did, however, benefit "I'm Only Sleeping (with backward guitars) and "Strawberry Fields Forever (with backward drums).

Mr. Martin was never particularly trendy, and when the Beatles adopted the flowery fashions of psychedelia in 1966 and 1967 he continued to attend sessions in a white shirt and tie, his hair combed back in a schoolmasterly pre-Beatles style. Musically, though, he was fully in step with them. When Lennon wanted a circus sound for his "Being for the Benefit of Mr. Kite," Mr. Martin recorded a barrel organ and, following the example of John Cage, cut the tape into small pieces and reassembled them at random. His avant-garde orchestration and spacey production techniques made "A Day in the Life" into a monumental finale for the kaleidoscopic album *Sgt. Pepper's Lonely Hearts Club Band*.

For more, see:

- Allan Kozinn, George Martin, redefining producer who guided the Beatles, dies at 90, *New York Times*, March 9, 2016.

March 12, 2016



The computer program AlphaGo just won its third game against the excellent Korean player Lee Sedol.

But what would it feel like to watch one of these games, if you're good at go? David Ormerod explains:

> It was the first time we'd seen AlphaGo forced to manage a weak group within its opponent's sphere of influence. Perhaps this would prove to be a weakness?
>
> This, however, was where things began to get scary.
>
> Usually developing a large sphere of influence and enticing your opponent to invade it is a good strategy, because it creates a situation where you have numerical advantage and can attack severely.
>
> In military texts, this is sometimes referred to as "force ratio".
>
> The intention in Go though is not to kill, but to consolidate territory and gain advantages elsewhere while the opponent struggles to defend themselves.
>
> Lee appeared to be off to a good start with this plan, pressuring White's invading group from all directions and forcing it to squirm uncomfortably.
>
> But as the battle progressed, White gradually turned the tables — compounding small efficiencies here and there.
>
> Lee seemed to be playing well, but somehow the computer was playing even better.
>
> In forcing AlphaGo to withstand a very severe, one-sided attack, Lee revealed its hitherto undetected power.
>
> Move after move was exchanged and it became apparent that Lee wasn't gaining enough profit from his attack.
>
> By move 32, it was unclear who was attacking whom, and by 48 Lee was desperately fending off White's powerful counter-attack.
>
> I can only speak for myself here, but as I watched the game unfold and the realization of what was happening dawned on me, I felt physically unwell.
>
> Generally I avoid this sort of personal commentary, but this game was just so disquieting. I say this as someone who is quite interested in AI and who has been looking forward to the match since it was announced.
>
> One of the game's greatest virtuosos of the middle game had just been upstaged in black and white clarity.

March 13, 2016

After losing the first three, Lee Sedol won his 4th game against the program AlphaGo!

Lee was playing white, which for go means taking the second move. So, he was on the defensive at first, unlike the previous game, where he played black.

After the first two hours of play, commenter Michael Redmond called the contest "a very dangerous fight". Lee Sedol likes aggressive play, and he seemed to be in a better position than last time.

But after another 20 minutes, Redmond felt that AlphaGo had the edge. Even worse, Lee Sedol had been taking a long time on his moves, so had only about 25 minutes left on his play clock, nearly an hour less than AlphaGo. Once your clock runs out, you need to make each move in less than a minute!

At this point, AlphaGo started to play less aggressively. Maybe it thought it was bound to win: it tries to maximize its probability of winning, so when it thinks it's winning it becomes more conservative. Commenter Chris Garlock said "This was AlphaGo saying: 'I think I'm ahead. I'm going to wrap this stuff up'. And Lee Sedol needs to do something special, even if it doesn't work. Otherwise, it's just not going to be enough".

On his 78th move, Lee did something startling.

He put a white stone directly between two of his opponent's stones, with no other white stone next to it. You can see it marked in red above. This is usually a weak type of move, since a stone that's surrounded is "dead".

I'm not good enough to understand precisely how strange this move was, or why it was actually good. At first all the commenters were baffled. And it seems to have confused AlphaGo. In the 87th move, AlphaGo placed a stone in a strange position which commentators said was "difficult to understand."

"AlphaGo yielded its own territory more while allowing its opponent to expand his own," said commentator Song Tae-gon, a Korean nine-dan professional go player. "This could be the starting point of AlphaGo's self-destruction."

Later AlphaGo placed a stone in the bottom left corner without reinforcing its territory in the center. Afterwards it seemed to recover, which Song said would be difficult for human players under such pressure. But Lee remained calm and blocked AlphaGo's attacks. The machine resigned on the 180th move.

Lee was ecstatic. "This win cannot be more joyful, because it came after three consecutive defeats. It is the single priceless win that I will not exchange for anything."

"AlphaGo seemed to feel more difficulties playing with black than white," he said. "It also revealed some kind of bug when it faced unexpected positions."

Lee has already lost the match, since AlphaGo won 3 out of the 5 games. But Lee wants to play black next time, and see if he can win that way.

You can play through the whole game here:

- Eidogo, Google Deepmind challenge match #4, March 13, 2016.

Even if you don't understand go, it has a certain charm.

March 14, 2016

What percent of primes end in a 7? I mean when you write them out in base ten.

Well, if you look at the first hundred million primes, the answer is 25.000401%. That's very close to 1/4. And that makes sense, because there are just 4 digits that a prime can end in, unless it's really small: 1, 3, 7 and 9.

So, you might think the endings of prime numbers are random, or very close to it. But 3 days ago two mathematicians shocked the world with a paper that asked some other questions, like this:

If you have a prime that ends in a 7, what's the probability that the next prime ends in a 7?

I would have expected the answer to be close to 25%. But these mathematicians, Robert Oliver and Kannan Soundarajan, actually looked. And they found that among the first hundred million primes, the answer is just 17.757%.

So if a prime ends in a 7, it seems to somehow tell the next prime *"I rather you wouldn't end in a 7. I just did that."*

This initially struck me as weird. And apparently it's not just because I don't know enough number theory. Ken Ono is a real expert on number theory, and when he learned about this, he said:

> I was floored. I thought, "For sure, your program's not working".

Needless to say, it's not magic. There is an explanation. In fact, Oliver and Soundarajan have conjectured a formula that says exactly how much of a discrepancy to expect — and they've checked it, and it seems to work. It works in every base, not just base ten. There's nothing special about base ten here. But we still need a proof that the formula really works.

By the way, their formula says the discrepancy gets smaller and smaller when we look at more and more primes. If we look at primes less than N, the discrepancy is on the order of

$$\frac{\log(\log(N))}{\log(N)}$$

This goes to zero as $N \to \infty$. But this discrepancy is *huge* compared to the discrepancy for the simpler question, "what percentage of primes ends in a given digit?" For that, the discrepancy, called the Chebyshev bias, is on the order of

$$\frac{1}{\log(N)\sqrt{N}}$$

Of course, what's really surprising is not this huge correlation between the last digits of consecutive primes, but that number theorists hadn't thought to look for it until now!

Any amateur with decent programming skills could have spotted this and won everlasting fame, if they'd thought to look. What other patterns are hiding in the primes?

For a good nontechnical summary, read this:

- Erica Klarreich, Mathematicians discover prime conspiracy, *Quanta*, March 13, 2016.

For a more technical explanation of what's going on, this is very good:

- Terry Tao, Biases between consecutive primes, *What's New*, March 14, 2016.

and of course there's the actual paper:

- Robert J. Lemke Oliver and Kannan Soundararajan, Unexpected biases in the distribution of consecutive primes, March 11, 2016.

Their work involves a variant of the Hardy–Littlewood k-tuple conjecture, which is a conjectured formula for the density of 'constellations' of primes of a given 'shape' — that is, k-tuples of primes that are of the form

$$(a_1 + n, \dots, a_k + n)$$

for some given 'shape' $(a_1, \dots, a_k)$.

I just noticed something funny. It seems that the Hardy–Littlewood k-tuple conjecture is also called the 'first Hardy–Littlewood conjecture'. The 'second Hardy–Littlewood conjecture' says that

$$\pi(M + N) \le \pi(M) + \pi(N)$$

whenever $M, N \ge 2$, where $\pi(N)$ is the number of primes $\le N$.

What's funny is what Wikipedia says about the second Hardy–Littlewood conjecture! It says:

> This is probably false in general as it is inconsistent with the more likely first Hardy–Littlewood conjecture on prime k-tuples, but the first violation is likely to occur for very large values of M.

Is this true? If so, did Hardy and Littlewood *notice* that their two conjectures contradicted each other? Isn't there some rule against this? Otherwise you could just conjecture P and also $not(P)$, disguising $not(P)$ in some very different language, and be sure that *one* of your conjectures was true!

(Unless, of course, you're an intuitionist.)

March 15, 2016

Scared of big numbers? Don't read this!

People love twin primes — primes separated by two, like 11 and 13. Nobody knows if there are infinitely many. There probably are. There are certainly lots.

But a while back, a computer search showed that among numbers less than a trillion, most common distance between successive primes is 6.

It seems that this trend goes on for quite a while longer.

... but in 1999, three mathematicians discovered that at some point, the number 6 ceases to be the most common gap between successive primes!

When does this change happen? It seems to happen around here:

$$17,427,000,000,000,000,000,000,000,000,000$$

At about this point, the most common gap between consecutive primes switches from 6 to 30.

They didn't prove this, but they gave a sophisticated heuristic argument for their claim. They also checked the basic idea using Maple's 'probable prime' function. It takes work to check if a number is prime, but there's a much faster way to check if it's *probably* prime in a certain sense. Using this, they worked out the gaps between probable primes from 10^{30} and $10^{30} + 10^7$. They found that there are 5278 gaps of size 6 and just 5060 of size 30. They also worked out the gaps between probable primes from 10^{40} and $10^{40} + 10^7$. There were 3120 of size 6 and 3209 of size 30.

So, it seems that somewhere between 10^{30} and 10^{40}, the number 30 replaces 6 as the most probable gap between successive primes!

This is a nice example of how you may need to explore very large numbers to understand the true behavior of primes.

Using the same heuristic argument, they argued that somewhere around 10^{450}, the number 30 ceases to be the most probable gap. The number 210 replaces 30 as the champion — and reigns for an even longer time.

Furthermore, they argue that this pattern continues forever, with the main champions being the primorials:

$$2 \quad 3 = 6$$

$$2 \quad 3 \quad 5 = 30$$

$$2 \quad 3 \quad 5 \quad 7 = 210$$

$$2 \quad 3 \quad 5 \quad 7 \quad 11 = 2310$$

etc.

Their paper is here:

- Andrew Odlyzko, Michael Rubinstein, and Marek Wolf, Jumping champions, *Experimental Mathematics* **8** (1999), 107–118.

They say the number

$$2 \quad 3 \quad 5 = 30$$

starts becoming more common as a gap between primes than

$$2 \quad 3 = 6$$

roughly when we reach

$$\exp(2 \quad 3 \quad 4 \quad 3) = e^{72} \approx 1.8 \quad 10^{31}$$

That's pretty rough, since as I mentioned, they say the actual turnover occurs around $1.7427 \quad 10^{35}$. But you probably can't see the pattern yet, so let me go on!

The number

$$2 \quad 3 \quad 5 \quad 7 = 210$$

starts becoming more common as a gap between primes than

$$2 \quad 3 \quad 5 = 30$$

roughly when we reach

$$\exp(2 \quad 3 \quad 5 \quad 6 \quad 5) = e^{900} \approx 10^{390}$$

Again, this is pretty rough: they must have a more accurate formula that they use elsewhere in the paper. But they mention this rough one early on.

I bet you still can't see the pattern in that exponential, so let me do a couple more examples! The number

$$2 \quad 3 \quad 5 \quad 7 \quad 11 = 2310$$

starts becoming more common as a gap between primes than

$$2 \quad 3 \quad 5 \quad 7 = 210$$

roughly when we reach

$$\exp(2 \quad 3 \quad 5 \quad 7 \quad 10 \quad 9) = e^{18900}$$

The number

$$2 \quad 3 \quad 5 \quad 7 \quad 11 \quad 13 = 30030$$

starts becoming more common as a gap between primes than

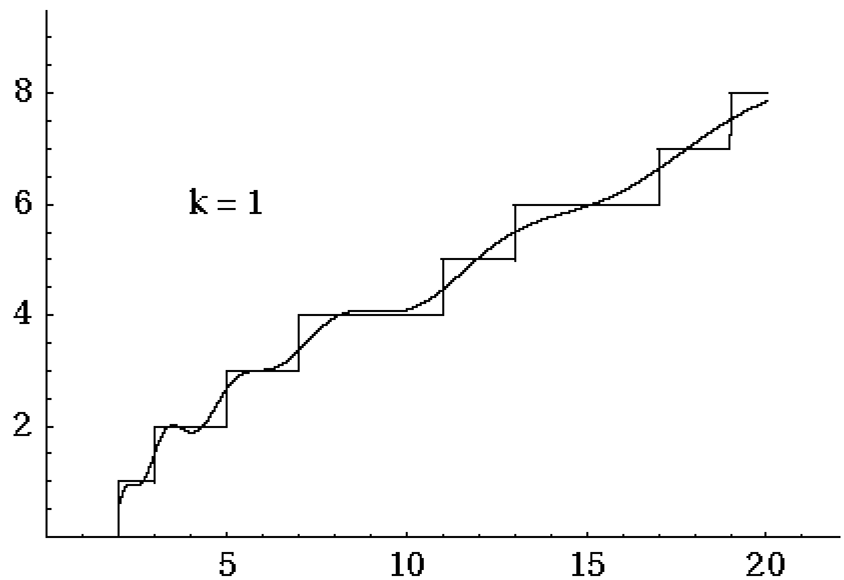
$$2 \quad 3 \quad 5 \quad 7 \quad 11 = 2310$$

roughly when we reach

$$\exp(2 \quad 3 \quad 5 \quad 7 \quad 11 \quad 12 \quad 11) = e^{304920}$$

Get the pattern? If not, read their paper.

March 16, 2016



I often hear there's no formula for prime numbers. But Riemann came up with something just as good: a formula for the prime counting function.

This function, called $\pi(x)$, counts how many prime numbers there are less than x, where x is any number you want. It keeps climbing like a staircase, and it has a step at each prime. You can see it above.

Riemann's formula is complicated, but it lets us compute the prime counting function using a sum of oscillating functions. These functions oscillate at different frequencies. Poetically, you could say they reveal the secret music of the primes.

The frequencies of these oscillating functions depend on where the Riemann zeta function equals zero.

So, Riemann's formula turns the problem of counting primes less than some number into another problem: finding the zeros of the Riemann zeta function!

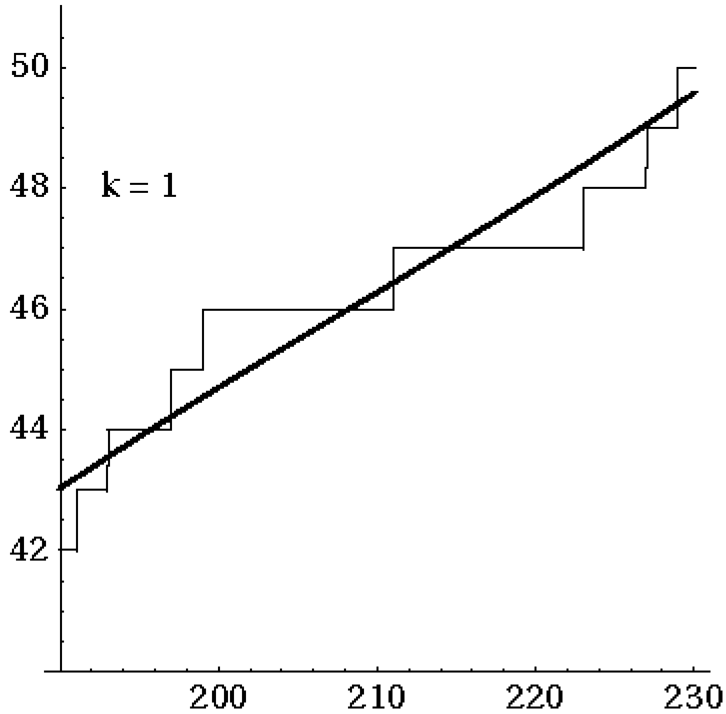
This doesn't make the problem easier... but, it unlocks a whole new battery of tricks for understanding prime numbers! Many of the amazing things we now understand about primes are based on Riemann's idea.

It also opens up new puzzles, like the Riemann Hypothesis: a guess about where the Riemann zeta function can be zero.

If someone could prove this, we'd know a lot more about prime numbers!

The animated gif here shows how the prime counting function is approximated by adding up oscillating functions, one for each of the first 500 zeros of the Riemann zeta function. So when you see something like "k = 317", you're getting an approximation that uses the first 317 zeros.

Here's a view of these approximations of the prime counting function $\pi(x)$ between $x = 190$ and $x = 230$:



I got these gifs here:

- J. Laurie Snell, Bill Peterson, Jeanne Albert and Charles Grinstead, [Chance in the primes](#).

and Maximilian S. and Kram Einsnulldreizwei prepared looped versions.

and here you can see Riemann's formula. You'll see that some other functions, related to the prime counting function, have simpler formulas.

And by the way: when I'm talking about zeros of the Riemann zeta function, I only mean zeros in the critical strip, where the real part is between 0 and 1. The Riemann Hypothesis says that for all of these, the real part is exactly 1/2. This has been checked for the first 10,000,000,000,000 zeros.

That sounds pretty convincing, but it shouldn't be. After all, the number 6 is the most common gap between consecutive primes if we look at numbers less than something like 17,427,000,000,000,000,000,000,000,000,000... but then that pattern stops!

So, you shouldn't look at a measly few examples and jump to big conclusions when it comes to primes.

March 17, 2016

Greg Bernhardt runs a website called for discussing physics, math and other topics. He recently did a two-part interview of me, and you can see it there or all in one place on my website.

March 18, 2016

This album cover contains a math puzzle. i is the square root of minus one. It takes a bit of work to wrap ones head around i to the ith power.

Here's how you figure it out. i is e to the power of $i\pi/2$, since multiplying by i implements a quarter turn rotation, that is, a rotation by $\pi/2$. So,

$$i^i = (e^{i\pi/2})^i = e^{i \quad i\pi/2} = e^{-\pi/2} = 0.20787957...$$

Sorta strange. But now:

Puzzle. Can you solve the equation on the album cover for ≡?

$$i^i = \equiv^{\overline{-\frac{\equiv}{2}}}$$

If you get stuck, see Greg Egan's answer in the comments to my [Google+ post](). But my real puzzle is about the album cover this equation appears on!

Is there any way to get a good electronic copy of this album for less than \$50? There's one CD of it for sale on Amazon for \$50.

It's called *This Crazy Paradise* and it's by Pyewackett. It's a cool album! It was made in 1986. It's an unusual blend of the cutting-edge electronic rock of that day and traditional folk music. The singer, Rosie Cross, has a voice that reminds me of Maddy Prior of Steeleye Span.

When it first came out I liked the electronic aspects, but not the folk. Now I like both — and it bothers me that this unique album seems almost lost to the world!

My wife Lisa has a tape of it. She transferred the tape to mp3 using Audacity but the result was fairly bad... a lot of distortion. Part of the problem ws a bad cassette tape deck — I've got a much better one, and I should try it. But I'm afraid another part of the problem is that without a special sound card, using the 'line in' on your laptop produces crappy recordings. I'll see.

Here's a description of the band from Last.fm:

> The English folkrock group Pyewackett was founded at the end of the 1970s by Ian Blake and Bill Martin. They were a resident band and the London University college folk club.
>
> Pyewackett played traditional folk music by the motto "pop music from the last five centuries": 15th century Italian dances, a capella harmonies, traditional songs in systems/minimalist settings, 1920's ballads, etc.. The distinctive sound was characterised by the unusual combination of woodwinds, strings and keyboards. The voices were also a strong trademark. All these features of the Pyewackett sound are shown best on the second album the band released in 1984, The Man in the Moon Drinks Claret. It is this album that has been re-released in Music & Words. Folk Classics series. At the time of this recording the band was formed by Ian Blake, Bill Martin, Mark Emerson, Rosie Cross and guest drummer Micky Barker. The album has been co-produced by Andrew Cronshaw.

Here's a review by Craig Harris:

> One of the lesser-known of the British folk bands, Pyewackett is remembered for updating 18th century songs with modern harmonies and inventive instrumentation. While none of their four albums are easy to find, the search is worth it. The group's sense of fun and reverence for musical traditions allowed them to bring ancient tunes to life. Pyewackett took their name from an imp that a 17th century Essex woman

> claimed possessed her. According to legendary witch-hunter Matthew Hopkins, it was a name that "no mortal could invent."

If you want to hear a bit of Pyewackett, try this:

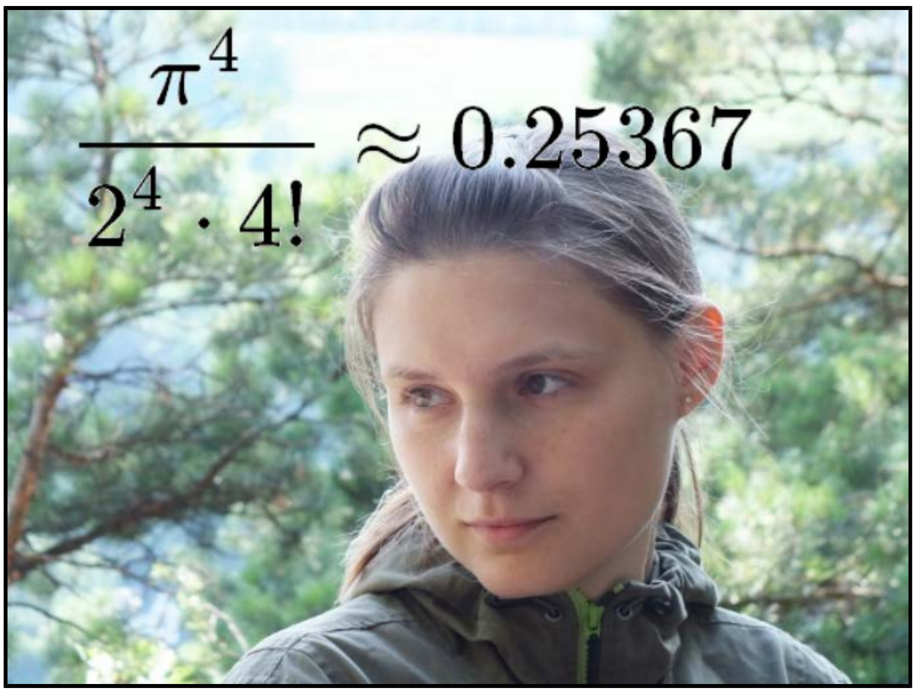
It's less electronic, but still a good bass line spices up this rendition of the traditional "Tam Lin". For the lyrics, see this:

- Tam Lin: Pyewackett, *Tam Lin Balladry*.

According to Wikipedia, Tam Lin is:

> [...] a character in a legendary ballad originating from the Scottish Borders. It is also associated with a reel of the same name, also known as Glasgow Reel. The story revolves around the rescue of Tam Lin by his true love from the Queen of the Fairies. While this ballad is specific to Scotland, the motif of capturing a person by holding him through all forms of transformation is found throughout Europe in folktales. The story has been adapted into various stories, songs and films.

March 24, 2016



Ten days ago, the Ukranian mathematician Maryna Viazovska showed how to pack spheres in 8 dimensions as tightly as possible. In this arrangement the spheres occupy about 25.367% of the space. That looks like a strange number — but it's actually a wonderful number, as shown here.

People had guessed the answer to this problem for a long time. If you try to get as many equal-sized spheres to touch a sphere in 8 dimensions, there's exactly one way to do it — unlike in 3 dimensions, where there's a lot of wiggle room! And if you keep doing this, on and on, you're forced into a unique arrangement, called the E_8 lattice. So this pattern is an obvious candidate for the densest sphere packing in 8 dimensions. But none of this *proves* it's the best!

In 2001, Henry Cohn and Noam Elkies showed that no sphere packing in 8 dimensions could be more than 1.000001 times as dense than E_8. Close... but no cigar.

Now Maryna Viazovska has used the same technique, but pushed it further. Now we know: *nothing can beat E_8 in 8 dimensions!*

Viazovska is an expert on the math of modular forms, and that's what she used to crack this problem. But when she's not working on modular forms, she writes papers on physics! Serious stuff, like "Symmetry and disorder of the vitreous vortex lattice in an overdoped $BaFe_{2-x}Co_x As_2$ superconductor."

After coming up with her new ideas, Viaskovska teamed up with other experts including Henry Cohn and proved that another lattice, the Leech lattice, gives the densest sphere packing in 24 dimensions.

Different dimensions have very different personalities. Dimensions 8 and 24 are special. You may have heard that string theory works best in 10 and 26 dimensions — two more than 8 and 24. That's not a coincidence.

The densest sphere packings of spheres are only known in dimensions 0, 1, 2, 3, and now 8 and 24. Good candidates are known in many other low dimensions: the problem is *proving* things — and in particular, ruling out the huge unruly mob of non-lattice packings.

For example, in 3 dimensions there are uncountably many non-periodic packings of spheres that are just as dense as the densest lattice packing!

In fact, the sphere packing problem is harder in 3 dimensions than 8. It was only solved earlier because it was more famous, and one man — Thomas Hales — had the nearly insane persistence required to crack it.

His original proof was 250 pages long, together with 3 gigabytes of computer programs, data and results. He subsequently verified it using a computerized proof assistant, in a project that required 12 years and many people.

By contrast, Viazovska's proof is extremely elegant. It boils down to finding a function whose Fourier transform has a simple and surprising property! For details on that, try my blog article:

- John Baez, E_8 is the best, *The n-Category Café*, 24 March 2016.

For my April 2016 diary, go here.

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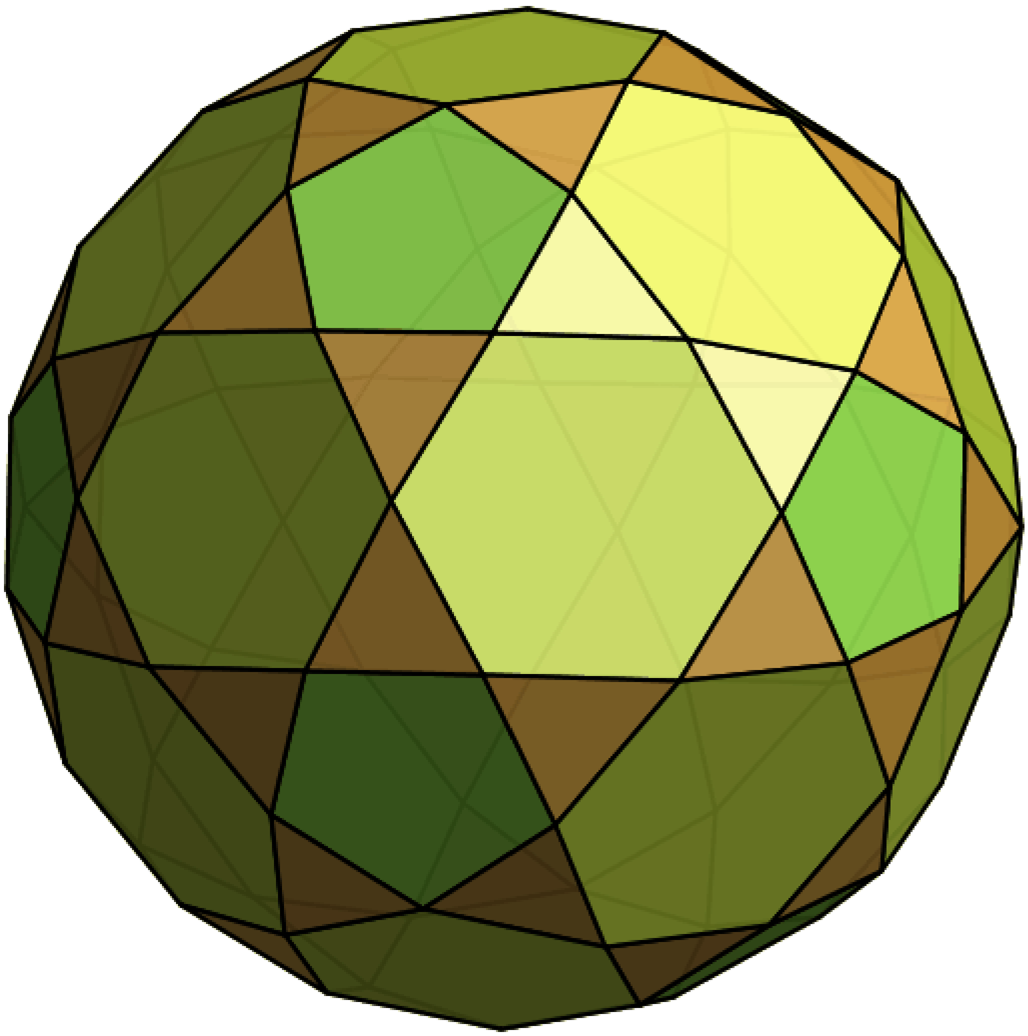
home

[For my March 2016 diary, go here.]

Diary - April 2016

John Baez

April 1, 2016



The 'rectified truncated icosahedron' is a surprising new polyhedron discovered by Craig Kaplan. It has a total of 60 triangles, 12 pentagons and 20 hexagons as faces.

It came as a shock because it's a brand-new Johnson solid — a convex polyhedron whose faces are all regular polygons.

Johnson solids are named after Norman Johnson, who in 1966 published a list of 92 such solids. He conjectured that this list was complete, but did not prove it.

In 1969, Victor Zalgaller proved that Johnson's list was complete, using the fact that there are only 92 elements in the periodic table.

It thus came as a huge shock to the mathematical community when Craig Kaplan, a computer scientist at the University of Waterloo, discovered an additional Johnson solid!

At the time, he was compiling a collection of 'near misses': polyhedra that come very close to being Johnson solids. In an interview with the *New York Times*, he said:

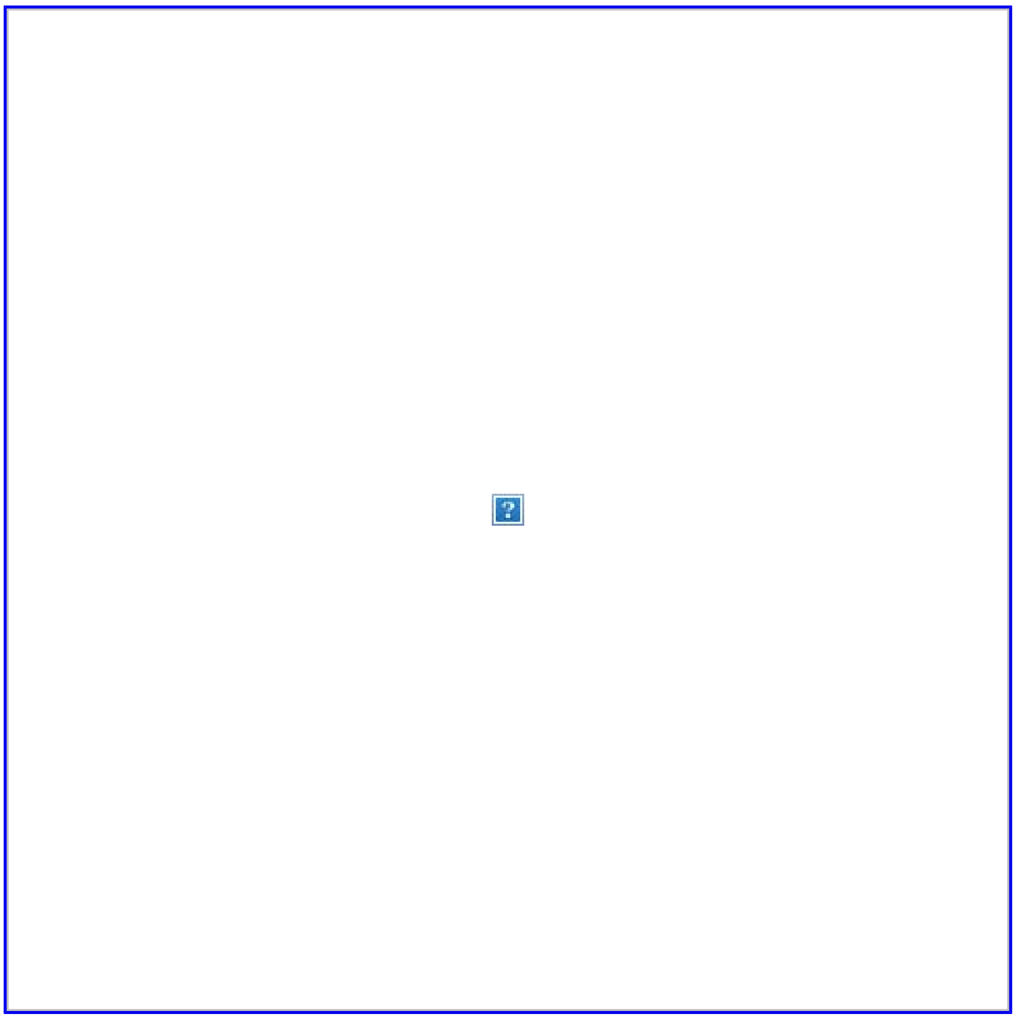
> When I found this one, I was impressed at how close it came to being a Johnson solid. But then I did some calculations, and I was utterly flabbergasted to discover that the faces are exactly regular! I don.t know how people overlooked it.

It turned out there was a subtle error in Zalgaller's lengthy proof.

Or maybe not; for details see:

- John Baez [Rectified truncated icosahedron](), *Visual Insight*, April 1, 2016.

April 3, 2016



As you know, a lot of conservatives in the US support the right to bear arms. It's in the Bill of Rights, after all:

> "A well regulated militia being necessary to the security of a free state, the right of the people to keep and bear arms shall not be infringed."

The idea is basically that if enough of us good guys are armed, criminals and the government won't dare mess with us.

In this they are in complete agreement with the Black Panthers, a revolutionary black separatist organization founded in the 1960s by Huey P. Newton. Later it became less active, but in 1989 the New Black Panther Party was formed in South Dallas, a predominantly black part of Dallas, Texas. They helped set up the Huey P. Long Gun Club, "uniting five local black and brown paramilitary organizations under a single banner."

Above you see some of their members marching in a perfectly legal manner down the streets of South Dallas. They started doing this after the killing of Michael Brown by a policeman in Ferguson.

From last year:

> On a warm fall day in South Dallas, ten revolutionaries dressed in kaffiyehs and ski masks jog the perimeter of Dr. Martin Luther King Jr. Park bellowing "No more pigs in our community!" Military discipline is in full effect as the joggers respond to two former Army Rangers in desert-camo brimmed hats with cries of "Sir, yes, sir!" The Huey P. Newton Gun Club is holding its regular Saturday fitness-training and self-defense class. Men in Che fatigues run with weight bags and roll around on the grass, knife-fighting one another with dull machetes." I used to salute the fucking flag!" the cadets chant. "Now I use it for a rag!"

You'd think that white conservatives would applaud this "well-regulated militia", since they too are suspicious of the powers of the government. Unfortunately they have some differences of opinion.

For one thing, there's that white versus black business, and the right-wing versus left-wing business. To add to the friction, the Black Panthers are connected to the Nation of Islam, a black Muslim group, while the white conservatives tend to be Christian.

It was thus not completely surprising when a gun-toting right-wing group decided to visit a Nation of Islam mosque in South Dallas. This group has an amusingly bland name: The Bureau of American Islamic Relations. They wrote:

> We cannot stand by while all these different Anti American, Arab radical Islamists team up with Nation of Islam/Black Panthers and White anti American Anarchist groups, joining together in the goal of destroying our Country and killing innocent people to gain Dominance through fear!

So, yesterday, the so-called Bureau showed up at the Nation of Islam mosque in South Dallas. They were openly carrying guns.

But the Huey P. Newton Gun Club expected this. So they showed up in larger numbers, carrying more guns.

Things became tense. People stood around holding guns, holding signs, yelling at each other, exercising all their constitutional freedoms like good Americans: the right of free speech, the right of assembly, the right to bear arms.

In the end, no shots were fired. The outgunned Bureau went home.

One of the co-founders of the Huey P. Newton Gun Club was interviewed while this was going on. He said:

> Those banditos are *out of their minds* if they think they're going to come to South Dallas like this.

See? This is how the 2nd Amendment works.

For more, see:

- Bethania Palma Markus, Armed hate group met at Texas mosque protest by gun-toting worshipers, *Raw Story*, April 2, 2016.

April 4, 2016

Last month the logician Joel David Hamkins proved a surprising result: you can compute uncomputable functions!

Of course there's a catch, but it's still interesting.

Alan Turing showed that a simple kind of computer, now called a Turing machine, can calculate a lot of functions. In fact we believe Turing machines can calculate anything you can calculate with any fancier sort of computer. So we say a function is computable if you can calculate it with some Turing machine.

Some functions are computable, others aren't. That's a fundamental fact.

But there's a loophole.

We think we know what the natural numbers are:

$$0, 1, 2, 3, \ldots$$

and how to add and multiply them. We know a bunch of axioms that describe this sort of arithmetic: the Peano axioms. But these axioms don't completely capture our intuitions! There are facts about natural numbers that most mathematicians would agree are true, but can't be proved from the Peano axioms.

Besides the natural numbers you think you know — but do you really? — there are lots of other models of arithmetic. They all obey the Peano axioms, but they're different. Whenever there's a question you can't settle using the Peano axioms, it's true in some model of arithmetic and false in some other model.

There's no way to decide which model of arithmetic is the right one — the so-called 'standard' natural numbers.

Hamkins showed there's a Turing machine that does something amazing. It can compute any function from the natural numbers to the natural numbers, depending on which model of arithmetic we use.

In particular, it can compute the uncomputable... but only in some weird 'alternative universe' where the natural numbers aren't what we think they are.

These other universes have 'nonstandard' natural numbers that are bigger than the ones you understand. A Turing machine can compute an uncomputable function... but it takes a nonstandard number of steps to do so.

So: computing the computable takes a 'standard' number of steps. Computing the uncomputable takes a little longer.

This is not a practical result. But it shows how strange simple things like logic and the natural numbers really are.

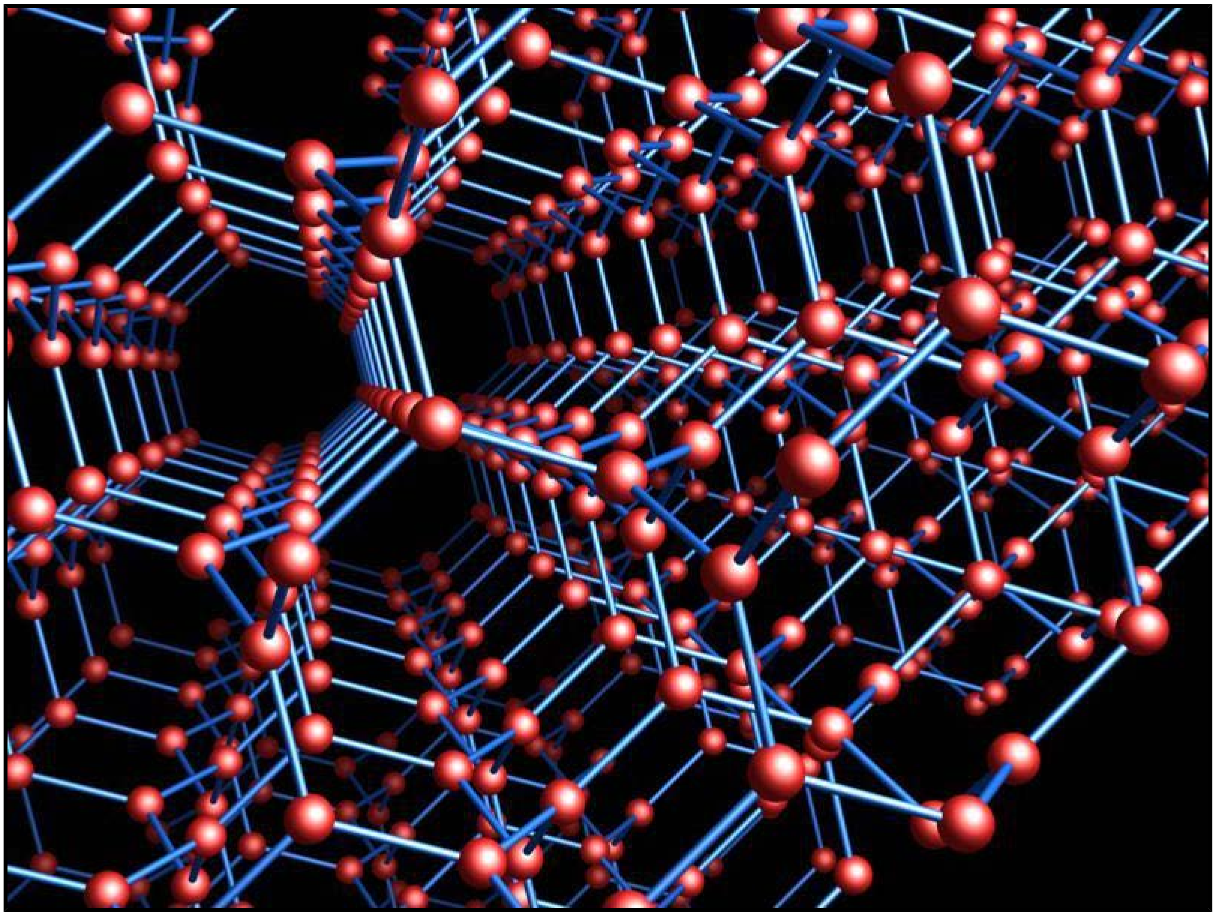
For a better explanation, read my blog post:

- John Baez, Computing the uncomputable, *Azimuth*, April 2, 2016.

And for the actual proof, go on from there to the blog article by Joel David Hamkins.

April 12, 2016

The crystal that nature forgot: the triamond



Carbon can form diamonds, and the geometry of the diamond crystal is amazingly beautiful. But there's another crystal, called the 'triamond', that is just as beautiful. It was discovered by mathematicians, but it doesn't seem to exist in nature.

In a triamond, each carbon atom would be bonded to three others at 120° angles, with one double bond and two single bonds. Its bonds lie in a plane, so we get a plane for each atom.

But here's the tricky part: for any two neighboring atoms, these planes are *different*. And if we draw these bond planes for all the atoms in the triamond, they come in four kinds, parallel to the faces of a regular tetrahedron!

The triamond is extremely symmetrical. But it comes in left- and right-handed forms, unlike a diamond.

In a diamond, the smallest rings of carbon atoms have 6 atoms. A rather surprising thing about the triamond is that the smallest rings have 10 atoms! Each atom lies in 15 of these 10-sided rings.

When I heard about the triamond, I had to figure out how it works. So I wrote this:

- John Baez, Diamonds and triamonds, *Azimuth*, April 11, 2016.

The thing that got me excited in the first place was a description of the 'triamond graph' — the graph with carbon atoms as vertices and bonds as edges. It's a covering space of the complete graph with 4 vertices. It's not the universal cover, but it's the 'universal abelian cover'.

I guess you need to know a fair amount of math to find that exciting. But fear not — I lead up to this slowly: it's just a terse way to say a lot of fun stuff.

And while the triamond isn't found in nature (yet), the mathematical pattern of the triamond is found in some butterfly wings.

April 14, 2016

In math there are infinite numbers called cardinals, which say how big sets are. Some are small. Some are big. Some are infinite. Some are so infinitely big that they're inaccessible — very roughly, you can't reach them using operations you can define in terms of smaller cardinals.

An inaccessible cardinal is so big that if it exists, we can't prove that using the standard axioms of set theory!

The reason why is pretty interesting. Assume there's an inaccessible cardinal κ. If we restrict attention to sets that we can build up using fewer than κ operations, we get a whole lot of sets. Indeed, we get a set of sets that does not contain every set, but which is big enough that it's 'just as good' for all practical purposes.

We call such a set a Grothendieck universe. It's not *the* universe — we reserve that name for the collection of *all* sets, which is too big to be a set. But all the usual axioms of set theory apply if we restrict attention to sets in a Grothendieck universe.

In fact, if we assume that an inaccessible cardinal exists, we can use the resulting Grothendieck universe to prove that the usual axioms of set theory are consistent! The reason is that the Grothendieck universe gives a 'model' of the axioms — it obeys the axioms, so the axioms must be consistent.

However, Gödel's first incompleteness theorem says we can't use the axioms of set theory to prove themselves consistent... unless they're inconsistent, in which case all bets are off.

The upshot is that we probably can't use the usual axioms of set theory to prove that it's consistent to assume there's an inaccessible cardinal. If we could, set theory would be inconsistent!

Nonetheless, bold set theorists are fascinated by inaccessible cardinals, and even much bigger cardinals. For starters, they love the infinite and its mysteries. But also, if we assume these huge infinities exist, we can prove things about arithmetic that we can't prove using the standard axioms of set theory!

I gave a very rough definition of inaccessible cardinals. It's not hard to be precise. A cardinal κ is **inaccessible** if you can't write it as a sum of fewer than κ cardinals that are all less than κ, and if α is any cardinal less than κ, then 2^α is also less than κ.

Well, not quite. According to this definition, 0 would be inaccessible — and so would the [very smallest infinite cardinal](), $\aleph_0$. Neither of these can be reached 'from below'. But we don't count these two cardinals as inaccessible.

April 23, 2016

Math tells us three of the saddest love stories:

- of parallel lines, who will never meet;
- of tangent lines, who were together once, and then parted forever;
- and of asymptotes, who come closer and closer, but can never truly be together.

But mathematicians invented projective geometry to provide a happy ending to the first story. In this kind of geometry, parallel lines do meet — not in ordinary space, but at new points, called 'points at infinity'.

The Barth sextic is an amazing surface with 65 points that look like the place where two cones meet — the most possible for a surface described using polynomials of degree 6. But in the usual picture of this surface, which emphasizes its symmetry, 15 of these points lie at infinity.

In this picture by Abdelaziz Nait Merzouk, the Barth sextic has been rotated to bring some of these points into view! It's also been sliced so you can see inside.

You can learn more about the Barth sextic here:

- John Baez, [Barth sextic](), *Visual Insight*, April 15, 2016.

April 25, 2016

There are lots of flights that go near the North Pole. When you fly from California to Europe, for example, that's an efficient route! Are there flights that go near the South Pole? If not, why not?

A friend of mine asked this question, and I promised I'd try to get an answer. When she flew from Argentina to New Zealand she took a very long route. Why, she wondered, don't airplanes take a southerly route? Is the weather too bad?

My guess is that maybe there's not enough demand to fly from South America to New Zealand for there to be direct flights. Or from South America to South Africa, or Madagascar.

But I haven't even checked! Maybe there are such flights!

April 30, 2016


(Image courtesy of Mike Zolensky, NASA JSC)

On January 18, 2000, at 8:43 in the morning, a meteor hit the Earth's atmosphere over Canada and exploded with the energy of a 1.7 kiloton bomb. Luckily this happened over a sparsely populated part of British Columbia.

It was over 50 tons in mass when it hit the air, but 97% of it vaporized. Just about a ton reached the Earth. It landed on Tagish Lake, which was frozen at the time. Local inhabitants said the air smelled like sulfur.

Only about 10 kilograms was found and collected. Except for a gray crust, the pieces look like charcoal briquettes.

And here is where things get interesting.

Analysis of the Tagish Lake fragments show they're very primitive. They contain dust granules that may be from the original cloud of material that created our Solar System and Sun! They also have a lot of of organic chemicals, including amino acids.

It seems this rock was formed about 4.55 billion years ago.

Scientists tried to figure out where it came from. They reconstructed its direction of motion and compared its properties with the spectra of various asteroids. In the end, they guessed that it most likely came from 773 Irmintraud.

773 Irmintraud is a dark, reddish asteroid from the outer region of the asteroid belt. It's about 92 kilometers in diameter. It's just 0.034 AU away from a chaotic zone associated with one of the gaps in the asteroid belt created by a resonance with Jupiter. So, if a chunk got knocked off, it could wind up moving chaotically and make it to Earth!

And here's what really intrigues me. 773 Irmintraud is a D-type asteroid — a very dark and rather rare sort. One model of Solar System formation says these asteroids got dragged in from very far out in the Solar System: the Kuiper Belt, out beyond Pluto. (Some scientists think Mars' moon Phobos is also a D-type asteroid.)

So, this chunk of rock here may have been made out in the Kuiper Belt, over 4.5 billion years ago!

For more, see:

- Wikipedia, Tagish Lake meteorite.

For my May 2016 diary, go here.

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home

For my April 2016 diary, go here.

Diary - May 2016

John Baez

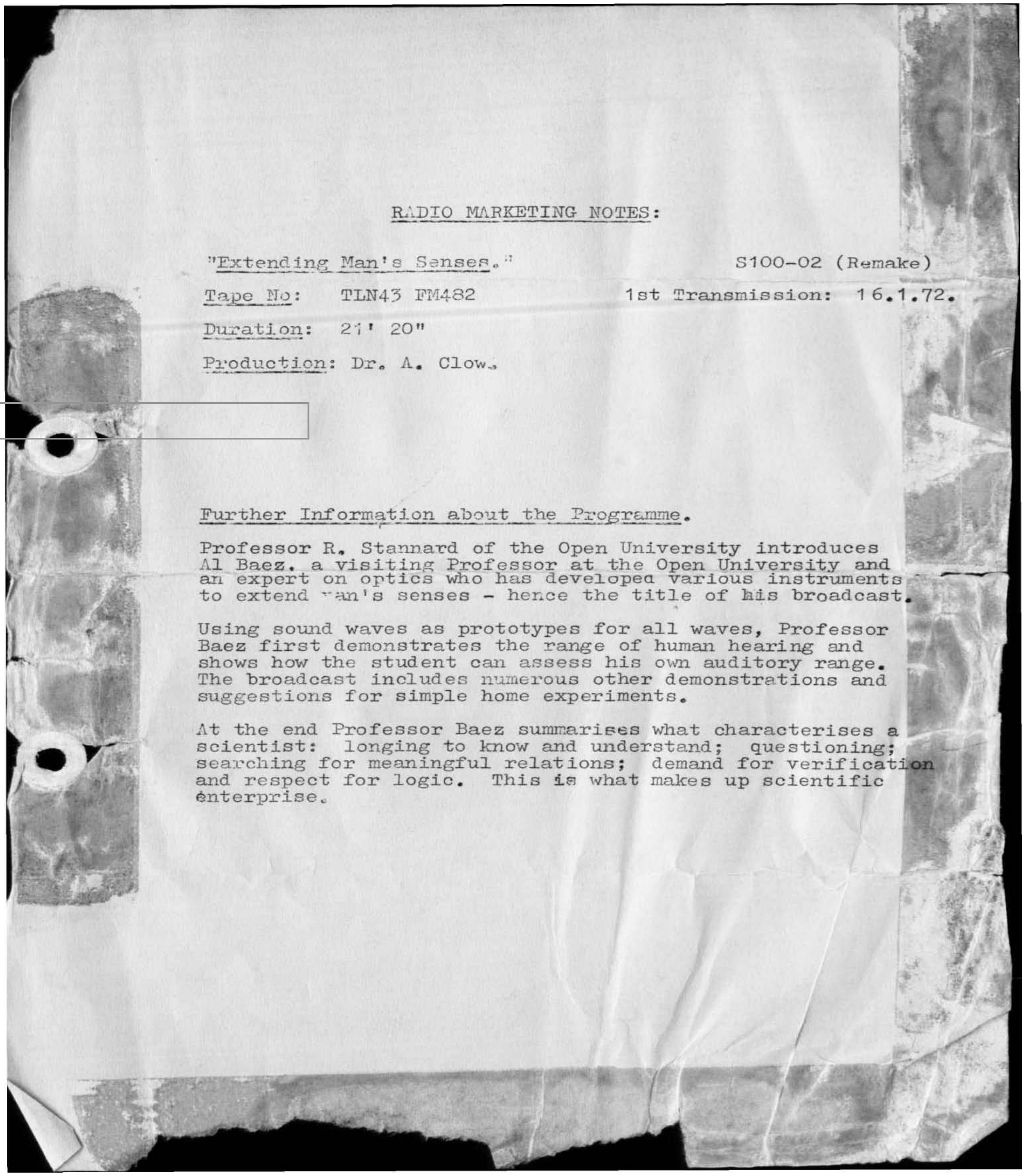
May 1, 2016

Life is but a dream

A sea otter and her sleeping pup float downstream.

For the whole adorable video, taken by Connie Levenhagen in the 'Great Tide Pool' at the Monterey Bay Aquarium, see YouTube.

May 2, 2016



RADIO MARKETING NOTES:

"Extending Man's Senses." S100-02 (Remake)

Tape No: TLN43 FM482 1st Transmission: 16.1.72.

Duration: 21' 20"

Production: Dr. A. Clow.

Further Information about the Programme.

Professor R. Stannard of the Open University introduces Al Baez, a visiting Professor at the Open University and an expert on optics who has developed various instruments to extend man's senses - hence the title of his broadcast.

Using sound waves as prototypes for all waves, Professor Baez first demonstrates the range of human hearing and shows how the student can assess his own auditory range. The broadcast includes numerous other demonstrations and suggestions for simple home experiments.

At the end Professor Baez summarises what characterises a scientist: longing to know and understand; questioning; searching for meaningful relations; demand for verification and respect for logic. This is what makes up scientific enterprise.

David Broadhurst is a particle physicist at Open University who is famous for his calculations of complicated Feynman diagrams revealing fascinating connections to number theory. Open University, in England, is the world's first successful distance learning university. They used to give courses on TV and radio. Maybe they still do, but now the internet is the big thing.

Today David Broadhurst emailed me a link to this old announcement. As you can see here, my uncle Albert Baez was a visiting professor at Open Universsity and gave a physics lecture there. I like this quick philosophy of science:

> At the end Professor Baez summarises what characterises a scientist: longing to know and understand; questioning; searching for meaningful relations; demand for verification and respect for logic.

May 10, 2016

The Earth is flat — and accelerating upwards

It's fun to read the frequently asked questions on the Flat Earth Society wiki. First question:

> **Is this site a joke?**

Answer: no, we're just diagonally parked in a parallel universe.

Sorry — that's my answer, not theirs. This site claims not to be a joke. But it's sure funny.

> **How do you explain day/night cycles and seasons?**
>
> Day and night cycles are easily explained on a flat earth. The sun moves in circles around the North Pole. When it is over your head, it's day. When it's not, it's night. The sun acts like a spotlight and shines downward as it moves. The picture below illustrates how the sun moves and also how seasons work on a flat earth. The apparent effect of the sun rising and setting is usually explained as a perspective effect.

And all that stuff about the Moon's phases, and lunar and solar eclipses, was apparently set up just to fool us into thinking the Earth, Moon and Sun are round objects, with the Earth able to come between the Sun and Moon, and the Moon able to come between the Earth and Sun.

But what I really like is the explanation of gravity. Wouldn't gravity pull the Earth into a round ball? No:

> The earth is constantly accelerating up at a rate of 32 feet per second squared (or 9.8 meters per second squared). This constant acceleration causes what you think of as gravity. Imagine sitting in a car that never stops speeding up. You will be forever pushed into your seat.

That's brilliant! But wait a minute...

Objects cannot exceed the speed of light. Doesn't this mean that the Earth can't accelerate forever?

They've got an answer to that too:

> Due to special relativity, this is not the case. At this point, many readers will question the validity of any answer which uses advanced, intimidating-sounding physics terms to explain a position. However, it is true. The velocity can never reach the speed of light, regardless of how long one accelerates for and the rate of the acceleration.

Fantastic!

What I like about this is that people can understand special relativity, yet not believe the Earth is round. I had never encountered that combination. I know more people who go the other way.

Of course there's the problem of what's powering this eternal acceleration. But they have an answer to that too: it's the Universal Accelerator, also known as dark energy or the "aetheric wind".

Here's the site:

- Flat Earth Wiki, Frequently asked questions.

Do not be angry. Enjoy!

Puzzle. Assume the Earth is 6000 years old and has been accelerating at 9.8 meters per second per second throughout this time, as measured in its own instantaneous rest frame. If it started at rest, how fast is it going now? Use the formula for Lorentz contractions to compute the thickness of the Earth today as measured by an observer at rest.

I'd guess it's much thinner than the diameter of a proton!

Some hints for anyone who has a calculator and is willing to use it:

If an object has constant acceleration a in its own rest frame and starts at rest, after time t its velocity will be

$$v = c \tanh(at/c)$$

where c is the speed of light and tanh is the hyperbolic tangent function. Also, if an object is moving at a velocity v, the Lorentz contraction will multiply its thickness by

$$1/\sqrt{1 - v^2/c^2}$$

On the long discussion about this flat Earth theory on G+, Asbjørn Held provided the following answer:

> As they say, go big or go home... I used Boost's Multiprecision library to compute the inverse gamma factor with 10000 decimal digits precision (maybe I could have used less, but 1000 was not enough). This was assuming the earth was 6000 years.
>
> The thickness of the earth relative to an observer at rest would be about 3.69×10^{-2684} meters... I'd consider that flat!
>
> For the record, at that speed β (that is v/c, the fraction of speed of light) equals
>
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Just in case I made some errors, here's the code.

I had tried this computation myself but I soon realized that the Earth would be very flattened after accelerating at 1 gee for 6000 years.

I believe its speed would be

$$\tanh(\mathrm{year}/\mathrm{second} \times 6000\ \mathrm{year} \times 9.8\ \mathrm{meter}/\mathrm{second}^2/c) =$$

$$\tanh(31536000 \times 6000 \times 9.8/299792458) \approx$$

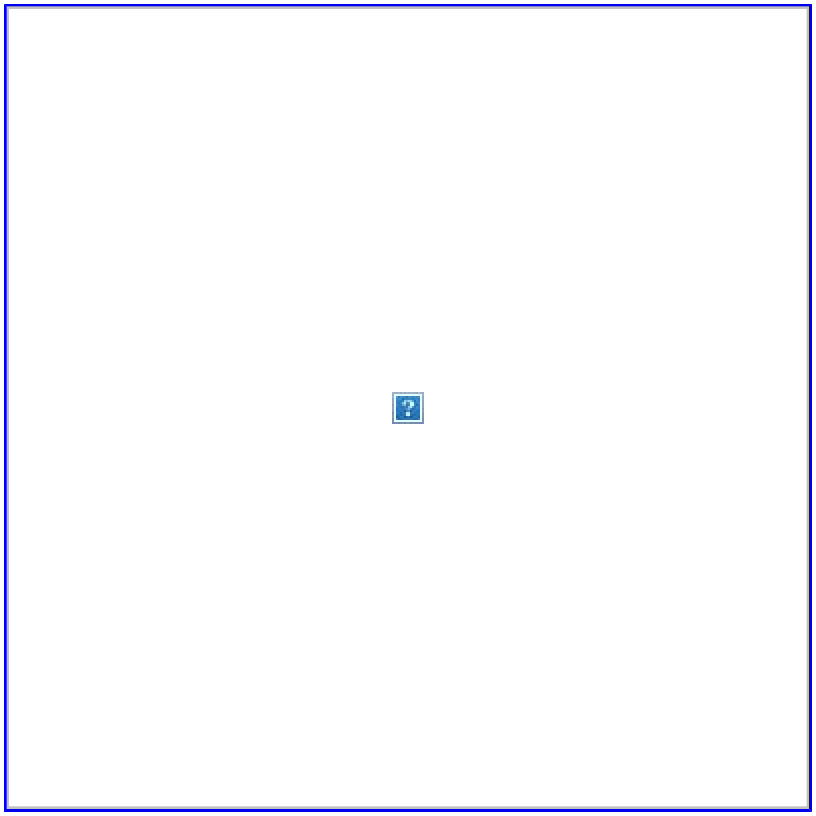
$$\tanh(6185.33505603) \approx$$

$$1 - 2\exp(-2 \times 6185.33505603) \approx$$

$$1 - 2\exp(-12370.6701121)$$

times the speed of light. In short: absurdly close to the speed of light! So, yes, it will be very flattened.

May 12, 2016



Kevin Kelly has claimed that "tools never die": that any tool ever made is still being made somewhere. There are interesting arguments about this online. You can find videos on how to make stone hand axes. You can find instructions on how make a calcium oxide light — the old-fashioned 'limelight' used in theaters until it was replaced by electric arc lamp in the 1890s.

And you can certainly buy a longsword. That's a sword with a long double-edged blade and a cross-shaped handle, as shown here. They reached their height of popularity from 1350 to 1550. But people still fight with them — mainly for fun.

In fact, this weekend on Staten Island there's a course for women who want to fight with longswords! And there's a tournament, too! It's called Fecht Yeah. Bring our weapon.

It's part of the Historical European Martial Arts movement, or HEMA. Here's the ad:

> A weekend of training, learning, and collaboration for women who study HEMA and other sword arts.
>
> This is an event for women of all skill levels with varied interests to come together and develop their skills. Workshops for beginners will be available. Free from tournament pressure and the constraints of classes, we have the ability to workshop teaching methods, rulesets, and learning strategies with other dedicated practitioners.
>
> We will have laurel tournaments in longsword, sword and buckler, rapier, and saber. Prizes will be modest. Attend to learn, not win.

I'm an absurdly nonviolent guy, who will pick up a spider and take it outside rather than squash it. But I admire skills like sword-fighting, and I'm glad people are keeping those skills alive. Why? I'm not completely sure. I could theorize about it, but never mind.

Check out this video of German longsword fighting:

As you might expect, female swordfighters get flack from some male ones. There's a nice article about Fecht Yeah here, and it gets into that a bit:

- Ann Babe, [Fecht club: New York's women warriors kick ass](), *Village Voice*, May 11, 2016.

The woman in the picture above is Laura McBride, photographed by Brad Trent. Tiby Kantorowitz, one of the women running Fecht Yeah, treats swordfighting as a spiritual exercise:

> It's the flip side to yoga. It's easy to Zen out with twinkly music, incense, and soft light. But can I maintain the same equanimity when there's some six-foot guy" — she's four-ten — "with a sword who's trying to brain me?"

For Kevin Kelly's claim, try this:

- Robert Krulwich, Tools never die. Waddaya mean, never?, *Krulwich Wonders*, National Public Radio, February 11, 2011.

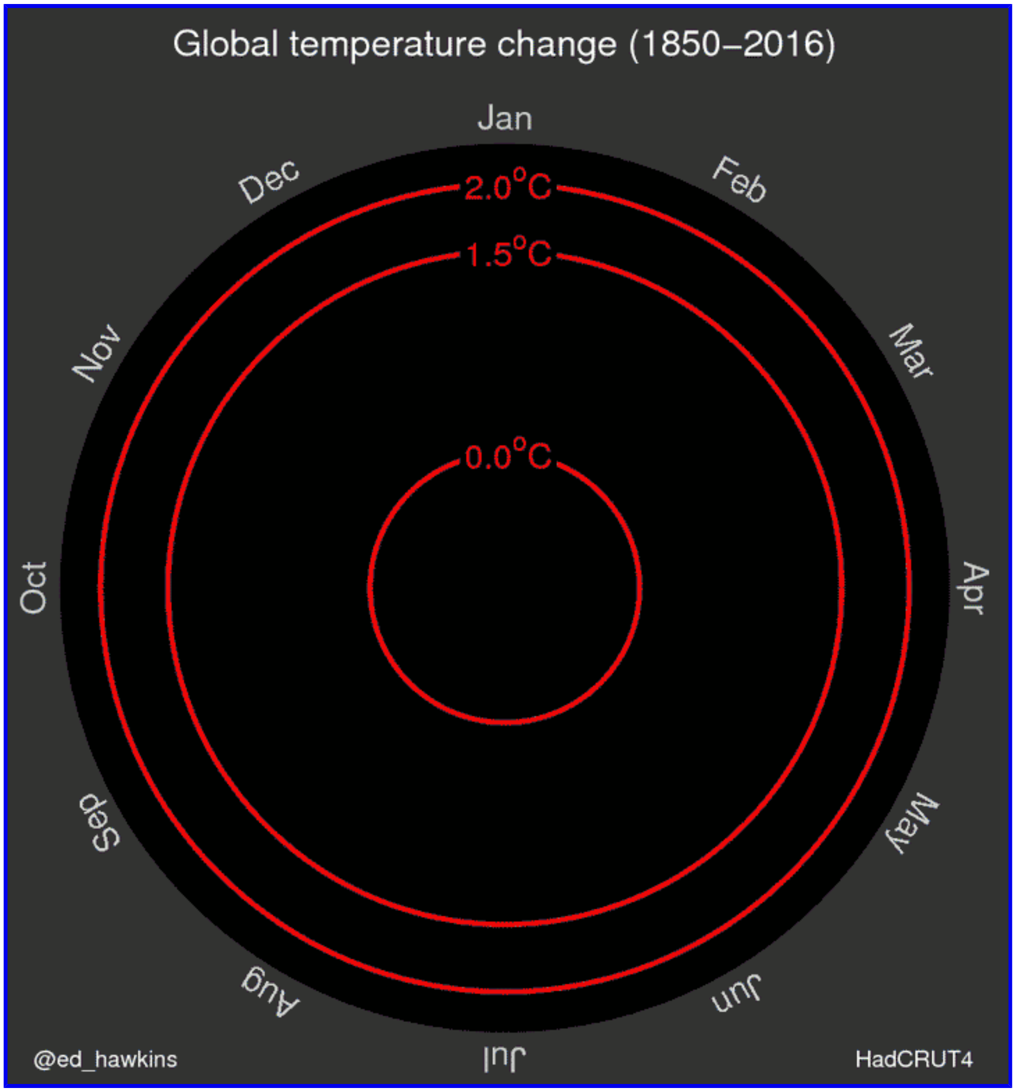
Here's a snippet:

> **Krulwich**: And then he made this ridiculous bet. He said: I bet you can't find any tool, any machine — go back to any century you like — that still isn't being made and made new today. So all I have to do is find a single tool that's not being made anymore, and I win.
>
> (Soundbite of laughter)
>
> **Kelly**: Yes, that's right.
>
> **Krulwich**: You're so going to lose this.

And then the show explores this...

- Robert Krulwich, Tools never die, the finale, *Krulwich Wonders*, National Public Radio, February 24, 2011.

May 14, 2016



The Earth is heating up rapidly. This great image was made by Ed Hawkins, a climate scientist at the University of Reading in the United Kingdom.

He points out some features:

> 1877-78: strong El Niño event warms global temperatures
>
> 1880s-1910: small cooling, partially due to volcanic eruptions
>
> 1910-1940s: warming, partially due to recovery from volcanic eruptions, small increase in solar ouput and natural variability
>
> 1950s-1970s: fairly flat temperatures as cooling sulphate aerosols mask the greenhouse gas warming
>
> 1980-now: strong warming, with temperatures pushed higher in 1998 and 2016 due to strong El Niño events

He used temperature data from January 1850 to March 2016. The numbers give the temperature above the average of 1850-1900. The temperatures are from a British data set called HadCRUT4.4. You can get that data here:

- Met Office Hadley Centre Observations Datasets, HadCRUT4

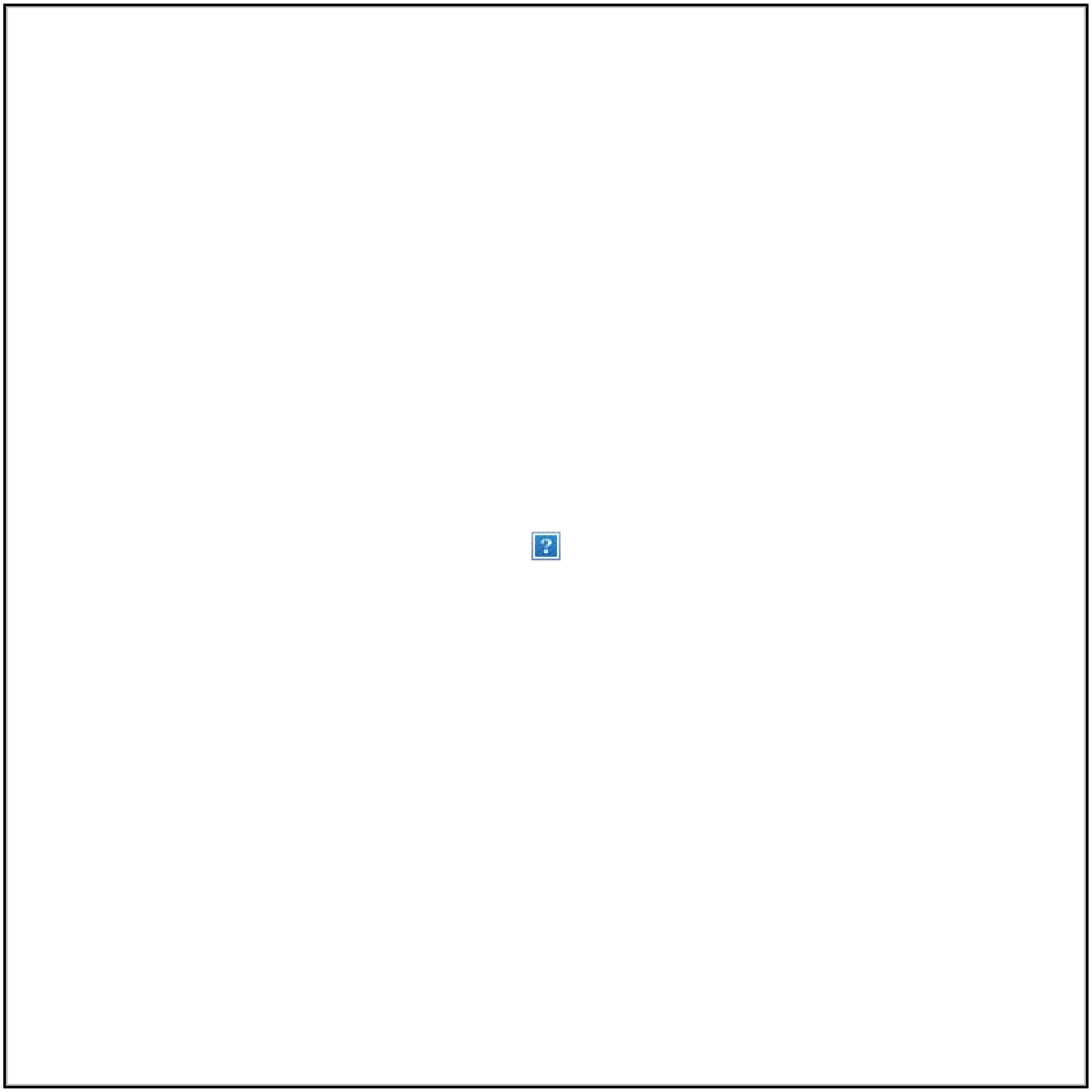
For more details, read this article on *The Guardian*:

- Andrea Thompson, See Earth's temperature spiral toward 2C rise, *The Guardian*, May 10, 2016.

and Hawkin's blog:

- Ed Hawkins, Spiralling global temperatures, *Climate Lab Book*, May 9, 2016.

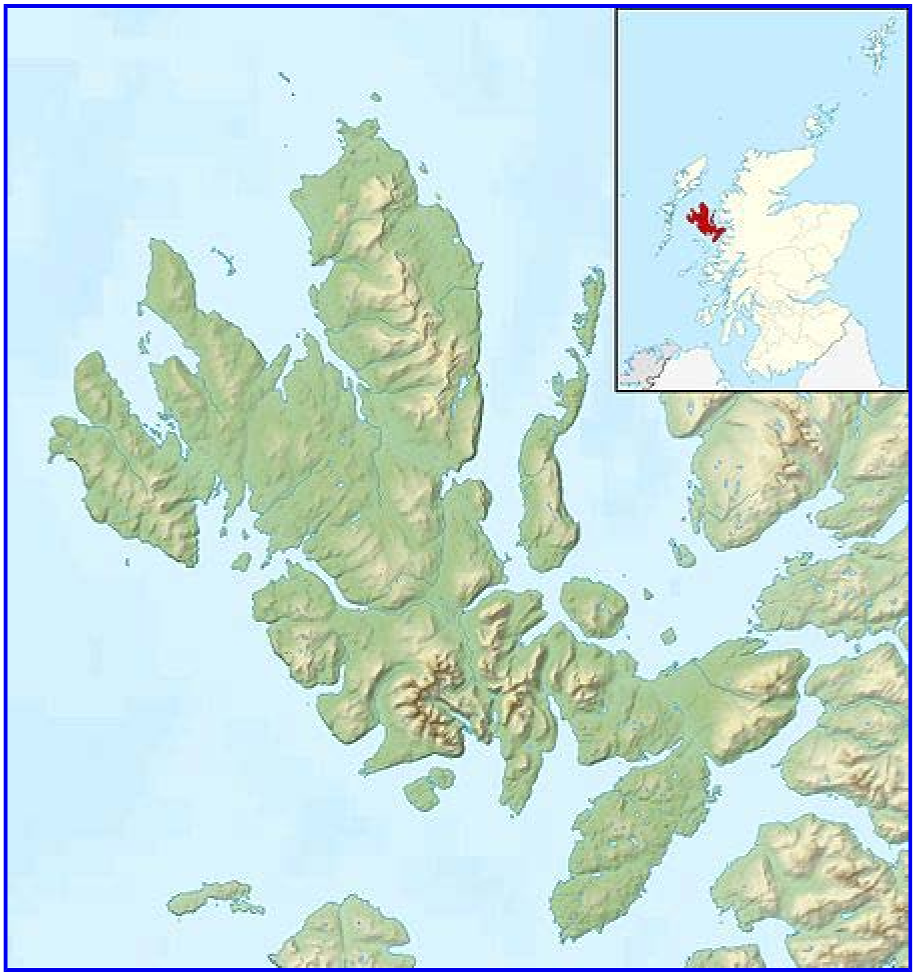
May 22, 2016



This is Danny MacAskill on the Inaccessible Pinnacle on the Isle of Skye.

He is a great mountain biker, but he had to carry the bike up the last part of this scary peak.

The Isle of Skye is an island off the west coast of Scotland. It's the largest of the Inner Hebrides, and the most northerly of the large islands in this group. In the center of this island is a mountain range called the Cuillin, and the Inaccessible Pinnacle sits among these.



Skye has been occupied since Mesolithic times, and it appears in Norse poetry, for example in this romantic line:

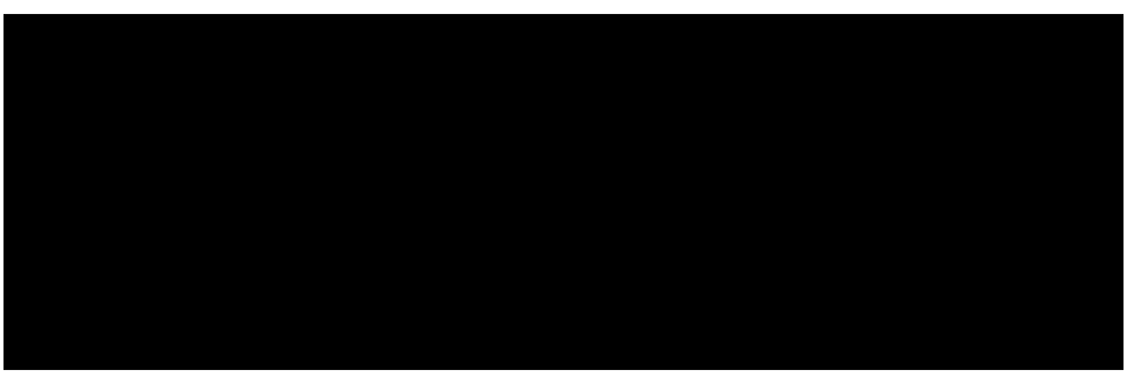
The hunger battle-birds were filled in Skye with blood of foemen killed.

Almost a third of the inhabitants still speak Gaelic, and apart from a few bigger towns, the population lives in crofting townships scattered around the coastline. "Crofting"? Yeah, a croft is a small farm with a wall around it.

The only distillery on the Isle of Skye is the Talisker Distillery, which makes a rather famous single malt Scotch whisky. It's in a village on the south shore.

I've always been fascinated by the Inner Hebrides and the even more exotic-sounding Outer Hebrides. I'm annoyed at how all my visits to the British Isles have only taken me to the lofty centers of academe, not places like this. I don't know much about them, but anything remote appeals to me: inaccessible pinnacles, inaccessible cardinals, the Taklamakan desert, the underground oceans of Europa....

Danny MacAskill is actually from the Isle of Skye! You can see his whole journey along the Cuillin Ridgeline here:



Pretty impressive! Beautiful scenery, too!

May 24, 2016

Logic hacking

In mathematics, unlike ordinary life, the boundary between the knowable and the unknowable is a precisely defined thing. But finding it isn't easy. Its exact location could itself be unknowable. But we don't even know that!

This month, a bunch of 'logic hackers' have stepped up to the plate and made a lot of progress. They've sharpened our estimate of where this boundary lies. How? By writing shorter and shorter computer programs for which it's unknowable whether these programs run forever, or stop.

A 'Turing machine' is a simple kind of computer whose inner workings have N different states, for some number N = 1,2,3,...

The 'Busy Beaver Game' is to look for the Turing machine with N states that runs as long as possible before stopping.

Machines that never stop are not allowed in this game.

We know the winner of the Busy Beaver Game for N = 1,2,3 and 4. Already for N = 5, the winner is unknown. The best known contestant is a machine that runs for 47,176,870 steps before stopping. There are 43 machines that might or might not stop — we don't know.

When N is large enough, the winner of the Busy Beaver Game is unknowable.

More precisely, if you use the ordinary axioms of mathematics, it's impossible to prove that any particular machine with N states is the winner of the Busy Beaver Game... as long as those axioms are consistent.

How big must N be, before we hit this wall?

We don't know.

But earlier this month, Adam Yedidia and Scott Aaronson showed that it's 7910 or less.

And by now, thanks to a group of logic hackers like Stefan O'Rear, we know it's 1919 or less.

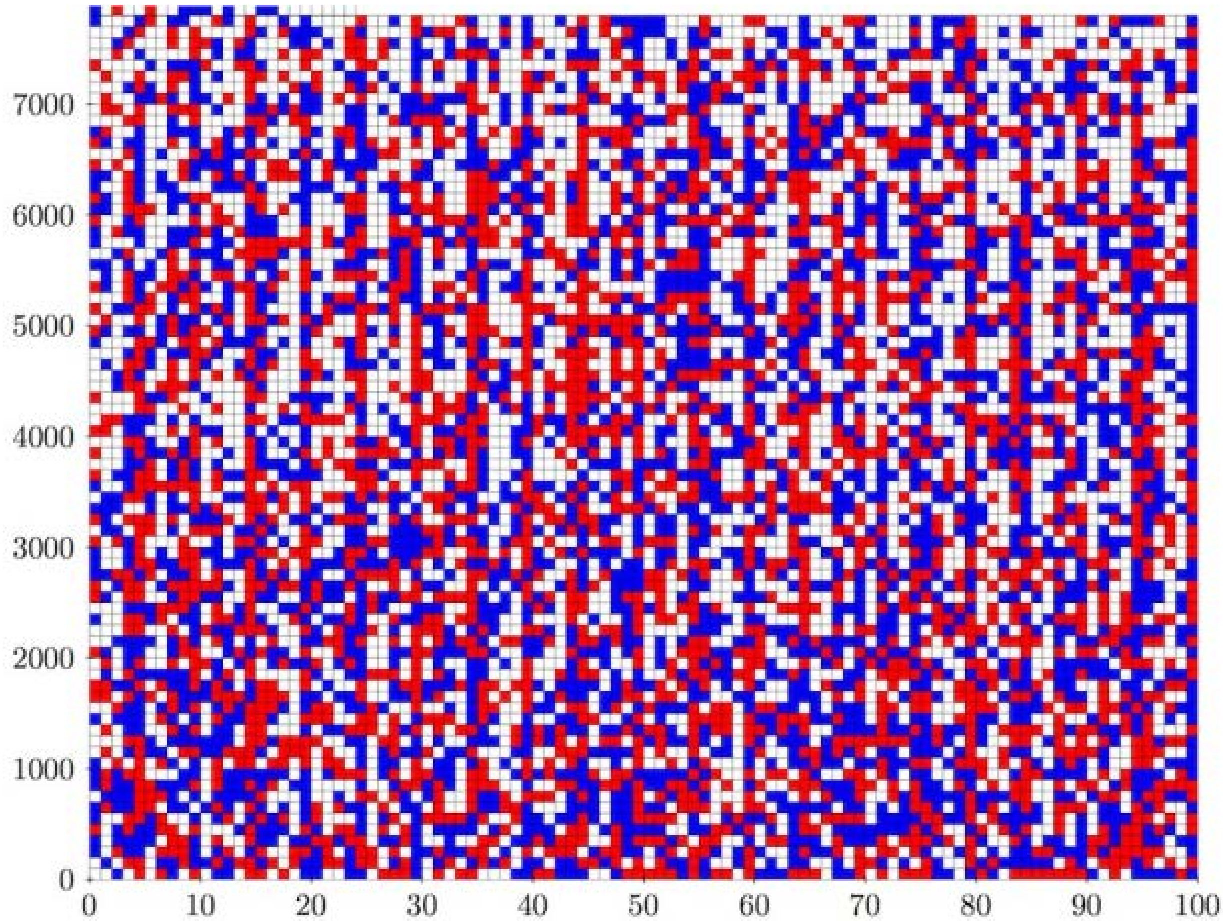
So, the unknowable kicks in — the winner of the Busy Beaver Game for N-state Turing machines becomes unknowable using ordinary math - somewhere between N = 5 and N = 1919.

The story of how we got here is is fascinating, and you can read about it on my blog post:

- John Baez, The busy beaver game, *Azimuth*, May 21, 2016.

Anything that I didn't make clear here, should be explained there.

May 27, 2016



In the 1980s, the mathematician Ronald Graham asked if it's possible to color each positive integer either red or blue, so that no triple of integers *a*, *b* and *c* obeying Pythagoras' famous equation:

$$a^2 + b^2 = c^2$$

all have the same color. He offered a prize of $100.

Now it's been solved! The answer is *no*. You can do it for numbers up to 7824, and a solution is shown in this picture. But you can't do it for numbers up to 7825.

To prove this, you could try all the ways of coloring these numbers and show that nothing works. Unfortunately that would require trying

> 3 628 407 622 680 653 855 043 364 707 128 616 108 257 615 873 380 491 654 672 530 751 098 578 199
> 115 261 452 571 373 352 277 580 182 512 704 196 704 700 964 418 214 007 274 963 650 268 320 833
> 348 358 055 727 804 748 748 967 798 143 944 388 089 113 386 055 677 702 185 975 201 206 538 492
> 976 737 189 116 792 750 750 283 863 541 981 894 609 646 155 018 176 099 812 920 819 928 564 304
> 241 881 419 294 737 371 051 103 347 331 571 936 595 489 437 811 657 956 513 586 177 418 898 046
> 973 204 724 260 409 472 142 274 035 658 308 994 441 030 207 341 876 595 402 406 132 471 499 889
> 421 272 469 466 743 202 089 120 267 254 720 539 682 163 304 267 299 158 378 822 985 523 936 240
> 090 542 261 895 398 063 218 866 065 556 920 106 107 895 261 677 168 544 299 103 259 221 237 129
> 781 775 846 127 529 160 382 322 984 799 874 720 389 723 262 131 960 763 480 055 015 082 441 821
> 085 319 372 482 391 253 730 679 304 024 117 656 777 104 250 811 316 994 036 885 016 048 251 200
> 639 797 871 184 847 323 365 327 890 924 193 402 500 160 273 667 451 747 479 728 733 677 070 215
> 164 678 820 411 258 921 014 893 185 210 250 670 250 411 512 184 115 164 962 089 724 089 514 186
> 480 233 860 912 060 039 568 930 065 326 456 428 286 693 446 250 498 886 166 303 662 106 974 996
> 363 841 314 102 740 092 468 317 856 149 533 746 611 128 406 657 663 556 901 416 145 644 927 496

655 933 158 468 143 482 484 006 372 447 906 612 292 829 541 260 496 970 290 197 465 492 579 693
769 880 105 128 657 628 937 735 039 288 299 048 235 836 690 797 324 513 502 829 134 531 163 352
342 497 313 541 253 617 660 116 325 236 428 177 219 201 276 485 618 928 152 536 082 354 773 892
775 152 956 930 865 700 141 446 169 861 011 718 781 238 307 958 494 122 828 500 438 409 758 341
331 326 359 243 206 743 136 842 911 727 359 310 997 123 441 791 745 020 539 221 575 643 687 646
417 117 456 946 996 365 628 976 457 655 208 423 130 822 936 961 822 716 117 367 694 165 267 852
307 626 092 080 279 836 122 376 918 659 101 107 919 099 514 855 113 769 846 184 593 342 248 535
927 407 152 514 690 465 246 338 232 121 308 958 440 135 194 441 048 499 639 516 303 692 332 532
864 631 075 547 542 841 539 848 320 583 307 785 982 596 093 517 564 724 398 774 449 380 877 817
714 717 298 596 139 689 573 570 820 356 836 562 548 742 103 826 628 952 649 445 195 215 299 968
571 218 175 989 131 452 226 726 280 771 962 970 811 426 993 797 429 280 745 007 389 078 784 134
703 325 573 686 508 850 839 302 112 856 558 329 106 490 855 990 906 295 808 952 377 118 908 425
653 871 786 066 073 831 252 442 345 238 678 271 662 351 535 236 004 206 289 778 489 301 259 384
752 840 495 042 455 478 916 057 156 112 873 606 371 350 264 102 687 648 074 992 121 706 972 612
854 704 154 657 041 404 145 923 642 777 084 367 960 280 878 796 437 947 008 894 044 010 821 287
362 106 232 574 741 311 032 906 880 293 520 619 953 280 544 651 789 897 413 312 253 724 012 410
831 696 803 510 617 000 147 747 294 278 502 175 823 823 024 255 652 077 422 574 922 776 413 427
073 317 197 412 284 579 070 292 042 084 295 513 948 442 461 828 389 757 279 712 121 164 692 705
105 851 647 684 562 196 098 398 773 163 469 604 125 793 092 370 432

possibilities. But recently, three mathematicians cleverly figured out how to eliminate most of the options. That left fewer than a trillion to check!

So they spent 2 days on a supercomputer, running 800 processors in parallel, and checked all the options. None worked. They verified their solution on another computer.

This is the world's biggest proof: it's 200 terabytes long! That's about equal to all the digitized text held by the US Library of Congress. There's also a 68-gigabyte digital signature - sort of a proof that a proof exists - if you want to skim it.

It's interesting that these 200 terabytes were used to solve a yes-or-no question, whose answer takes a single bit to state: *no*.

I'm not sure breaking the world's record for the longest proof is something to be proud of. Mathematicians prize *short*, *elegant* proofs. I bet a shorter proof of this result will eventually be found.

Still, it's fun that we can do such things. Here's a story about the proof:

- Evelyn Lamb, Two-hundred-terabyte maths proof is largest ever, *Nature*, May 26, 2016.

and here's the actual paper:

- Marijn J. H. Heule, Oliver Kullmann and Victor W. Marek, Solving and verifying the Boolean Pythagorean triples problem via cube-and-conquer.

The 'cube-and-conquer' paradigm is a "hybrid SAT method for hard problems, employing both look-ahead and CDCL solvers"... whatever that means. It would be interesting to learn about this. But it's time for breakfast!

By the way, despite the title of the *Nature* article, in the comments to my G+ post about this, Michael Nielsen pointed out a longer proof by Chris Jefferson, who wrote:

> Darn, I had no idea one could get into the media with this kind of stuff.
>
> I had a much larger "proof", where we didn't bother storing all the details, in which we enumerated 718,981,858,383,872 semigroups, towards counting the semigroups of size 10.

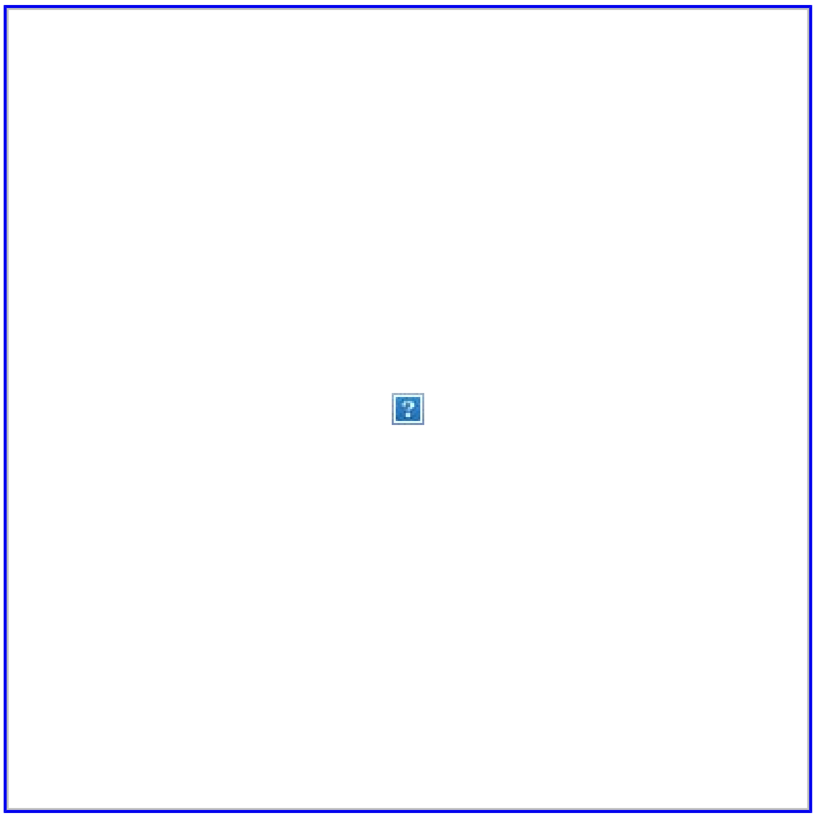
> Uncompressed, it would have been about 63,000 terabytes just for the semigroups, and about a thousand times that to store the "proof", which is just the tree of the search.
>
> Of course, it would have compressed extremely well, but also I'm not sure it would have had any value, you could rebuild the search tree much faster than you could read it from a disc, and if anyone re-verified our calculation I would prefer they did it by a slightly different search, which would give us much better guarantees of correctness.

His team found a total of 12,418,001,077,381,302,684 semigroups of size 10. They only had to find 718,981,858,383,872 by a brute force search, which is 0.006% of the total:

- Andreas Distler, Chris Jefferson, Tom Kelsey, and Lars Kottho, The semigroups of order 10, in *Principles and Practice of Constraint Programming*, Springer Lecture Notes in Computer Science **7514**, Springer, Berlin, pp. 883–899.

May 30, 2016

Let us read what we paid for!



Imagine a business like this: you get highly trained experts to give you their research for free... and then you *sell it back to them*. Of course these experts need equipment, and they need to earn a living... so you get taxpayers to foot the bill.

And if the taxpayers want to actually read the papers they paid for? Then you *charge them a big fee!*

It's not surprising that with this business model, big publishers are getting rich while libraries go broke. Reed-Elsevier has a 37% profit margin!

But people are starting to fight back — from governments to energetic students like Alexandra Elbakyan here.

On Friday, the Competitiveness Council — a gathering of European ministers of science, innovation, trade, and industry

— said that *by 2020, all publicly funded scientific papers published in Europe should be made immediately free for everyone to read*.

This will start a big fight, and it may take longer than 2020. But Alexandra Elbakyan isn't waiting around.

In 2011, as a computer science grad student in Kazakhstan, she got sick of paying big fees to read science papers. She set up SciHub, a pirate website that steals papers from the publishers and sets them free.

SciHub now has 51,000,000 papers in its database. In October 2015, Elsevier sued them. In November, their domain name was shut down. But they popped up somewhere else. By February, people were downloading 200,000 papers per day. Even scientists with paid access to the publisher's databases are starting to use SciHub, because it's *easier to use*.

Clearly piracy is the not the ultimate solution. Elbakyan now lives in an undisclosed location, to avoid being extradited. But she gave the world a much-needed kick in the butt. The old business model of *get smart people to work for free and sell the product back to them* is on its way out.

For more, read:

- John Bohannon, Who's downloading pirated papers? Everyone, *Science*, April 28, 2016.

and the SciHub Twitter feed. Also read this:

- Martin Enserink, In dramatic statement, European leaders call for 'immediate' open access to all scientific papers by 2020, *Science*, May 27, 2016.

The key word here is *immediate* — right now the US lets the journals sit on publicly funded papers for a year. The Dutch government is really pushing this! Congratulations to them!

For my June 2016 diary, go here.

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home

[For my May 2016 diary, go here.]

Diary - June 2016

John Baez

June 1, 2016

Especially before the fall of the USSR, the best Russian mathematicians would often meet and discuss their work at seminars.

Gelfand's seminar in Moscow was especially famous, since he would stop speakers any time they said something unclear. In fact, sometimes he'd appoint an audience member to play the role of arbiter: if this guy in the audience doesn't understand it, the speaker has to explain it better!

As a result, the seminar would often go on until late at night, even after the building was locked up. But everyone learned a lot of math.

With such exhaustive seminars, publishing proofs sometimes became a mere afterthought. You'll often see short papers from this era making important claims with just a tiny sketch of an argument to back them up.

That annoyed Western mathematicians. And I've bumped into a few mysteries that I'm having trouble with, thanks to these short Russian papers without clear proofs. Here is one.

This image by Greg Egan shows the set of points (a, b, c) for which the quintic

$$x^5 + ax^4 + bx^2 + c$$

has repeated roots... with the plane $c = 0$ removed. You'll notice this surface crosses over itself in a cool way, creating lines of sharp cusps.

Vladimir Arnol'd, who ran one of these famous seminars, says that one O. V. Lyashko studied this surface in 1982 with the help of a computer - a very primitive computer by our standards, I'm sure. And he says Lyashko proved this surface looks the same as another surface defined using the icosahedron.

Arnol'd doesn't mention removing the plane $c = 0$, so his claim is technically wrong. But if you remove that plane, it looks right! So I'd like to see a proof that these surfaces are the same after a smooth change of coordinates. The icosahedron and the quintic equation are connected in many ways, so there should be a nice explanation. But I don't know it!

For more details on this surface, see this blog post:

- John Baez, Discriminant of restricted quintic, *Visual Insight*, June 1, 2016.

You'll also see the other surface, defined using the icosahedron. And you can read a full explanation of that other surface here:

- John Baez, Discriminant of the icosahedral group, May 15, 2016.

As I explain, the same surface shows up in yet another disguise — but again, I don't know a proof! If you make progress on these mysteries, let me know!

The icosahedron is connected to some of the most fascinating symmetrical structures in the mathematical universe, such as E_8 and the Golay code. I'm trying to get to the bottom of this, so every clue helps.

Here is a longer description of Gelfand's seminar, as told by Simon Gindikin:

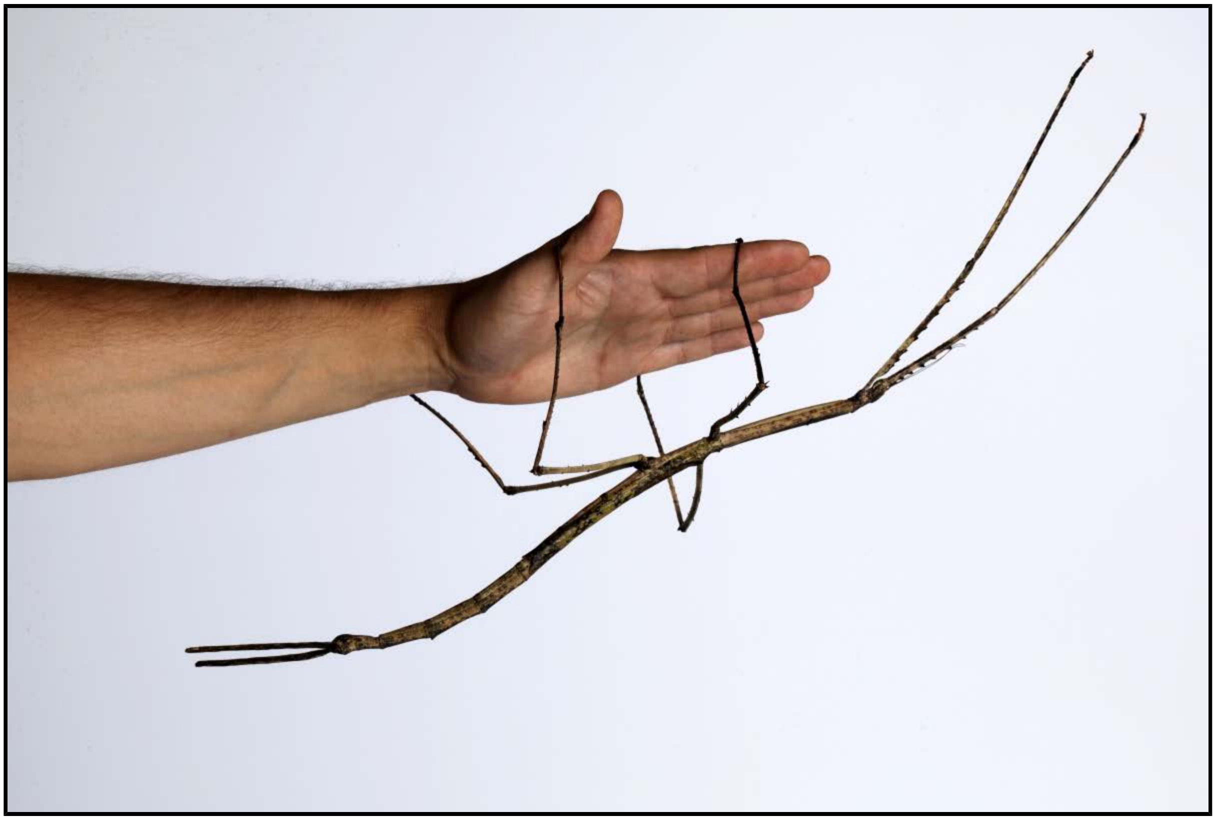
> The Gelfand seminar was always an important event in the very vivid mathematical life in Moscow, and, doubtless, one of its leading centers. A considerable number of the best Moscow mathematicians participated in it at one time or another. Mathematicians from other cities used all possible pretexts to visit it. I recall how a group of Leningrad students agreed to take turns to come to Moscow on Mondays (the day of the seminar, to which other events were linked), and then would retell their friends what they had heard there. There were several excellent and very popular seminars in Moscow, but nevertheless the Gelfand seminar was always an event.
>
> I would like to point out that, on the other hand, the seminar was very important in Gelfand's own personal mathematical life. Many of us witnessed how strongly his activities were focused on the seminar. When, in the early fifties, at the peak of antisemitism, Gelfand was chased out of Moscow University, he applied all his efforts to seminar. The absence of Gelfand at the seminar, even because of illness, was always something out of the ordinary.
>
> One cannot avoid mentioning that the general attitude to the seminar was far from unanimous. Criticism mainly concerned its style, which was rather unusual for a scientific seminar. It was a kind of a theater with

a unique stage director playing the leading role in the performance and organizing the supporting cast, most of whom had the highest qualifications. I use this metaphor with the utmost seriousness, without any intention to mean that the seminar was some sort of a spectacle. Gelfand had chosen the hardest and most dangerous genre: to demonstrate in public how he understood mathematics. It was an open lesson in the grasping of mathematics by one of the most amazing mathematicians of our time. This role could be only be played under the most favorable conditions: the genre dictates the rules of the game, which are not always very convenient for the listeners. This means, for example, that the leader follows only his own intuition in the final choice of the topics of the talks, interrupts them with comments and questions (a privilege not granted to other participants) [....] All this is done with extraordinary generosity, a true passion for mathematics.

Let me recall some of the stage director's strategems. An important feature were improvisations of various kinds. The course of the seminar could change dramatically at any moment. Another important mise en scene involved the "trial listener" game, in which one of the participants (this could be a student as well as a professor) was instructed to keep informing the seminar of his understanding of the talk, and whenever that information was negative, that part of the report would be repeated. A well-qualified trial listener could usually feel when the head of the seminar wanted an occasion for such a repetition. Also, Gelfand himself had the faculty of being "unable to understand" in situations when everyone around was sure that everything is clear. What extraordinary vistas were opened to the listeners, and sometimes even to the mathematician giving the talk, by this ability not to understand. Gelfand liked that old story of the professor complaining about his students: "Fantastically stupid students - five times I repeat proof, already I understand it myself, and still they don't get it."

It has remained beyond my understanding how Gelfand could manage all that physically for so many hours. Formally the seminar was supposed to begin at 6 pm, but usually started with an hour's delays. I am convinced that the free conversations before the actual beginning of the seminar were part of the scenario. The seminar would continue without any break until 10 or 10:30 (I have heard that before my time it was even later). The end of the seminar was in constant conflict with the rules and regulations of Moscow State University. Usually at 10 pm the cleaning woman would make her appearance, wishing to close the proceedings to do her job. After the seminar, people wishing to talk to Gelfand would hang around. The elevator would be turned off, and one would have to find the right staircase, so as not to find oneself stuck in front of a locked door, which meant walking back up to the 14th (where else but in Russia is the locking of doors so popular!). The next riddle was to find the only open exit from the building. Then the last problem (of different levels of difficulty for different participants) - how to get home on public transportation, at that time in the process of closing up. Seeing Gelfand home, the last mathematical conversations would conclude the seminar's ritual. Moscow at night was still safe and life seemed so unbelievably beautiful!

June 2, 2016

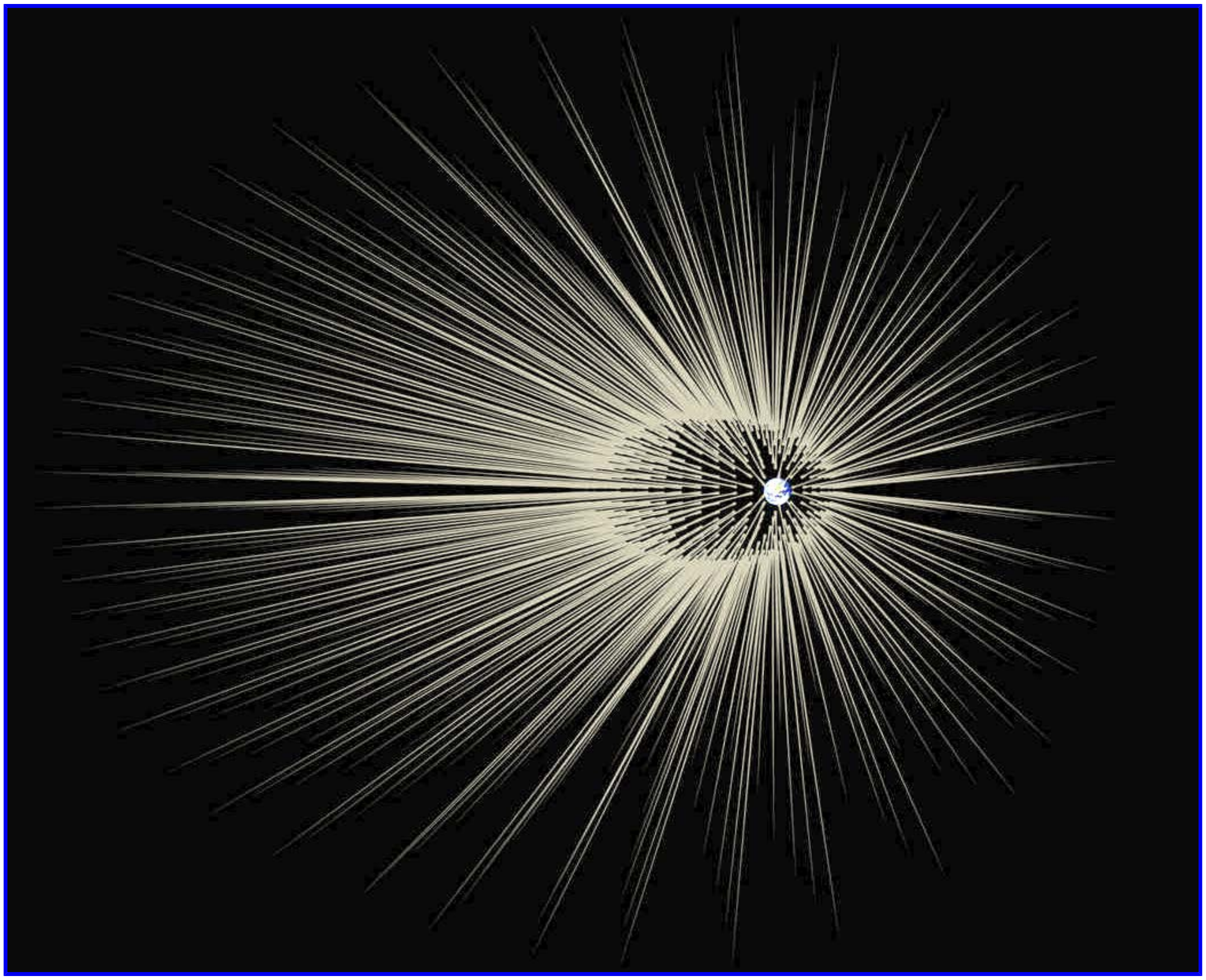


One of the world's largest insects lives in Australia. It looks like a stick and it's called *Ctenomorpha gargantua*. It's very hard to find, because it lives in the highest parts of the rainforests in Queensland, and it's only active at night!

In 2014 one fell down. Scientists found it hanging on a bush. They took it to the Museum Victoria, in Melbourne. They named it 'Lady Gaga-ntuan'. And now it has a daughter that's 0.56 meters long — that is, 22.2 inches long!

June 7, 2016

Does dark matter have dark hair?



By now there's a lot of evidence that dark matter exists, but not so much about what it is. The most popular theories say it's some kind of particles that don't interact much with ordinary matter, except through gravity. These particles would need to be fairly massive — as elementary particles go — so that despite having been hot and energetic shortly after the Big Bang, they'd move slow enough to bunch up thanks to gravity. Indeed, the bunching up of dark matter seems necessary to explain the formation of the visible galaxies!

Searches for dark matter particles have not found much. The DAMA experiment, a kilometer underground in Italy, seemed to detect them. Even better, it saw more of them in the summer, when the Earth is moving faster relative to the Milky Way, than in the winter. That's just what you'd expect! But other similar experiments haven't seen anything. So most physicists doubt the DAMA results.

Maybe dark matter is not made of massive weakly interacting particles. Maybe it's a superfluid made of light but strongly interacting particles. Maybe there are lot more 25-solar-mass black holes than most people think! There are lots of theories, and I don't have time to talk about them all.

I just want to tell you about a cool idea which assumes that dark matter is made of massive weakly interacting particles. It's still the most popular theory, so we should take it seriously and ask: if they exist, what would these particles do?

In the early Universe they'd attract each other by gravity. They'd bunch up, helping seed the formation of galaxies. But after stars and planets formed, they'd pull at the dark matter, making it thicker in some places, thinner in others.

And this is something we can simulate using computers! After all, the relevant physics is well-understood: just Newton's law of gravity, applied to stars, planets and zillions of tiny dark matter particles.

Gary Prezeau of NASA's Jet Propulsion Laboratory did these simulations and discovered something amazing.

When dark matter flows past the Earth, it gets deflected and focused by the Earth's gravity. Like light passing through a lens, it gets intensely concentrated at certain locations!

This creates long thin 'hairs' where the density of dark matter is enhanced by a factor of 10 million. Each hair is densest at its 'root'. At the root, the density of dark matter is about a billion times greater than average!

The hairs in this picture are not to scale: the Earth is drawn too big. The roots of the hairs would be about a million kilometers from Earth, while the Earth's radius is only 6,400 kilometers.

Of course we don't know dark matter particles exist. What's cool is that *if* they exist, it forms such beautiful structures! And if we could do a dark matter search in space, near one of these possible roots, we might have a better chance of finding something.

Let me paraphrase Prezeau, because the real beauty is in the details. From his abstract:

> It is shown that compact bodies form strands of concentrated dark matter filaments henceforth simply called 'hairs'. These hairs are a consequence of the fine-grained stream structure of dark matter halos surrounding galaxies, and as such they constitute a new physical prediction of the standard model of cosmology. Using both an analytical model of planetary density and numerical simulations (a fast way of computing geodesics) with realistic planetary density inputs, dark matter streams moving through a compact body are shown to produce hugely magnified dark matter densities along the stream velocity axis going through the center of the body. Typical hair density enhancements are 10^7 for Earth and 10^8 for Jupiter. The largest enhancements occur for particles streaming through the core of the body that mostly focus at a single point called the root of the hair. For the Earth, the root is located at about 10^6 kilometers from the planetary center with a density enhancement of around 10^9 while for a gas giant like Jupiter, the root is located at around 10^5 kilometers with a enhancement of around 10^{11}. Beyond the root, the hair density precisely reflects the density layers of the body providing a direct probe of planetary interiors.

The mathematicians and physicists among you may enjoy even more detail. Again, I'll paraphrase:

> According to the standard model of cosmology, the velocity dispersion of cold dark matter (CDM) is expected to be greatly suppressed as the universe expands and the CDM collisionless gas cools. In particular, for a weakly interacting mass particle with mass of 100 GeV that decoupled from normal matter when the Universe cooled to an energy of 10 MeV per particle, the velocity dispersion is only about 0.0003 meters per second.

As the Universe cools and the nonlinear effects of gravity become more prominent and galactic halos grow, the dispersion of velocities will increase somewhat, but 10 kilometers per second is an upper limit on the velocity dispersion of the resulting dark matter streams.

Dark matter starts out having a very low spread in velocities, but its location can be anywhere. So, it forms a 3-dimensional sheet in the 6-dimensional space of position-velocity pairs, called 'phase space'.

As time passes this sheets gets bent, but it can never be broken. When this sheet gets folded enough, we get a 'caustic where lots of different dark matter particles have almost the same position, though different velocities. You can see a caustic by shining light into a reflective coffee cup, or shining light through a magnifying glass. The same math applies here:

> A phase-space perspective sheds additional light on the processes affecting the CDM under the influence of gravity. When the CDM decouples from normal matter, the CDM occupies a 3-dimensional sheet in the 6-dimensional phase space since it has a tiny velocity dispersions. The process of galactic halo formation cannot tear this hypersurface, thanks to generalization of Liouville's theorem. Under the influence of gravity, a particular phase space volume of the hypersurface is stretched and folded with each orbit of the CDM creating layers of fine-grained dark matter streams, each with a vanishingly small velocity dispersion.

> These stretches and folds also produce caustics: regions with very high CDM densities that are inversely proportional to the square root of the velocity dispersion.

Here are some more pictures:

- Elizabeth Landau, Earth might have hairy dark matter, NASA, November 23, 2015.

and here's the paper:

- Gary Prezeau, Dense dark matter hairs spreading out from Earth, Jupiter and other compact bodies.

June 21, 2016

During the primaries, Trump claimed he was rich and couldn't be bought. He said he wouldn't have a super-PAC. Now he has a lot of super-PACs - all fighting each other. But his campaign has very little cash!

In May he tweeted:

> Good news is that my campaign has perhaps more cash than any campaign in the history of politics.

But this was a lie. By the end of May his campaign had less than $1.3 million. At least, that's what he reported to the Federal Election Commission.

That may sound like a lot if you don't know US politics. But Clinton, by comparison, had $42 million. Even Ben Carson - remember that guy, the nutty candidate who claimed the pyramids were built for storing grain? - had $1.7 million when he quit back in March.

So, by US standards, Trump's campaign is broke.

And he keeps putting campaign money back into his own pocket!

Throughout his campaign, up to the end of May, he has given $6.2 million of campaign funds to companies he owns. That's roughly 10% of his campaign spending so far. And in May this rose to almost 20%: he spent $6.7 million on his campaign, but over $1 million of that went to his own companies.

According to the Huffington Post:

> The most striking expenditure in the new filings was $423,372, paid by the Trump campaign for rentals and catering at Trump.s 126-room Palm Beach, Florida, mansion, Mar-A-Lago, which Trump operates as a private club.
>
> Other Trump-owned recipients of campaign funds include Trump Restaurants, which raked in $125,080 in rent and utilities; Trump Tower Commercial, which charged $72,800 in rent and utilities in the building that houses Trump.s campaign headquarters; the Trump National Golf Club, in Jupiter, Florida, which collected $35,845 for facilities rental and catering; and the Trump International Golf Club in West Palm Beach, Florida, which billed the campaign for $29,715, for facilities rentals and catering.

So, Trump has given a whole new meaning to the term 'self-funding'. In 2000, he said:

> It's very possible that I could be the first presidential candidate to run and make money on it.

It seems that Trump plans to let the Republican National Committee pay for most of his campaign. They've got some money: they started June with $20 million in cash. But four years ago at this time, they had more than $60 million. Their big donors are shying away from Trump.

I would love to get money out of US politics. I hadn't expected Trump to take the lead.

Here is his May report to the Federal Election Commission:

- Donald Trump, Federal Election Commission report of receipts and disbursements, filed June 20, 2016.

Here is the *Huffington Post* article:

- Christina Wilkie, Donald Trump's campaign paid Trump companies more than $1 million in May, *The Huffington Post*, June 21, 2016.

Here is an article on Trump's super-PACs:

- Russ Choma, Donald Trump has a super-PAC problem, *Mother Jones*, June 20, 2016.

For Trump's boast that his might be the first presidential campaign to make money, read this:

- David A. Graham, The lie of Trump's 'self-funding' campaign, *The Atlantic*, May 13, 2016.

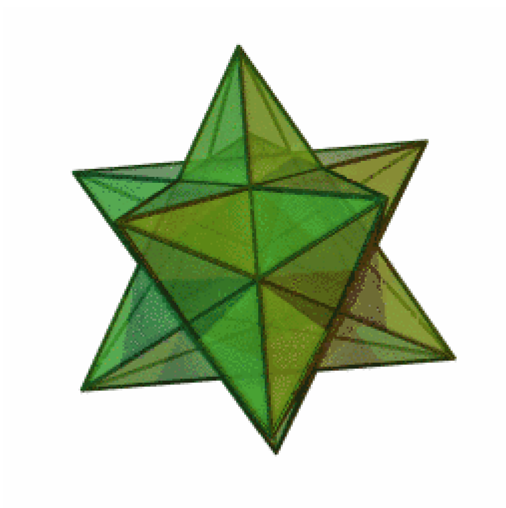
I got other figures from here:

- Rebecca Ballhaus, Stark gap in fundraising between presumptive nominees, *The Wall Street Journal*, June 21, 2016.

- Steve Benen, In more ways than one, the Trump campaign is broke, *The MaddowBlog*, June 21, 2016.

June 23, 2016

Superstar



This is the small stellated dodecahedron. It's like a star made of stars. It has 12 pentagrams, 5-pointed stars, as faces. These stars cross over each other. Five meet at each sharp corner.

But here's the really cool part: you should think of each pentagram as a pentagon that's been mapped into space in a very distorted way, with a 'branch point of order 2' at its center.

What does that mean?

Stand at the center of a pentagon! Measure the angle you see between two corners that are connected by an edge. You'll get $2\pi/5$. But now stand at the center of a *pentagram*. Measure the angle you see between two corners that are connected by an edge. You get $4\pi/5$. Twice as big!

So, to map a pentagon into space in a way that makes it look like a pentagram, you need to *wrap it twice around its central point*. That's what a branch point of order 2 is all about.

That's the cool way to think of this shape you see spinning before you. It's a surface made of 12 pentagons, each wrapped twice around its center, with 5 meeting at each sharp corner.

There's another way to think about this surface! Any equation of this sort

$$z^5 + pz + q = 0$$

has 5 solutions, or 'roots'. To make this true we need to bend the rules a bit. First, we let the solutions be complex numbers... so let p and q be complex too. Second, we must allow for the possibility of 'repeated roots': when you factor $z^4 + pz + q$, the same root may show up twice.

Now here's the cool part: the small stellated dodecahedron is the set of all lists of 5 numbers that are roots of some equation of this form:

$$z^5 + pz + q = 0$$

So it's not just a pretty star-shaped thing. It's a serious mathematical entity! It's actually a Riemann surface, the most symmetrical Riemann surface with 4 holes! You can build it starting from a tiling of the hyperbolic plane by pentagons. In this tiling 5 pentagons meet at each corner — just like 4 squares meet at each corner in a square tiling of the ordinary plane.

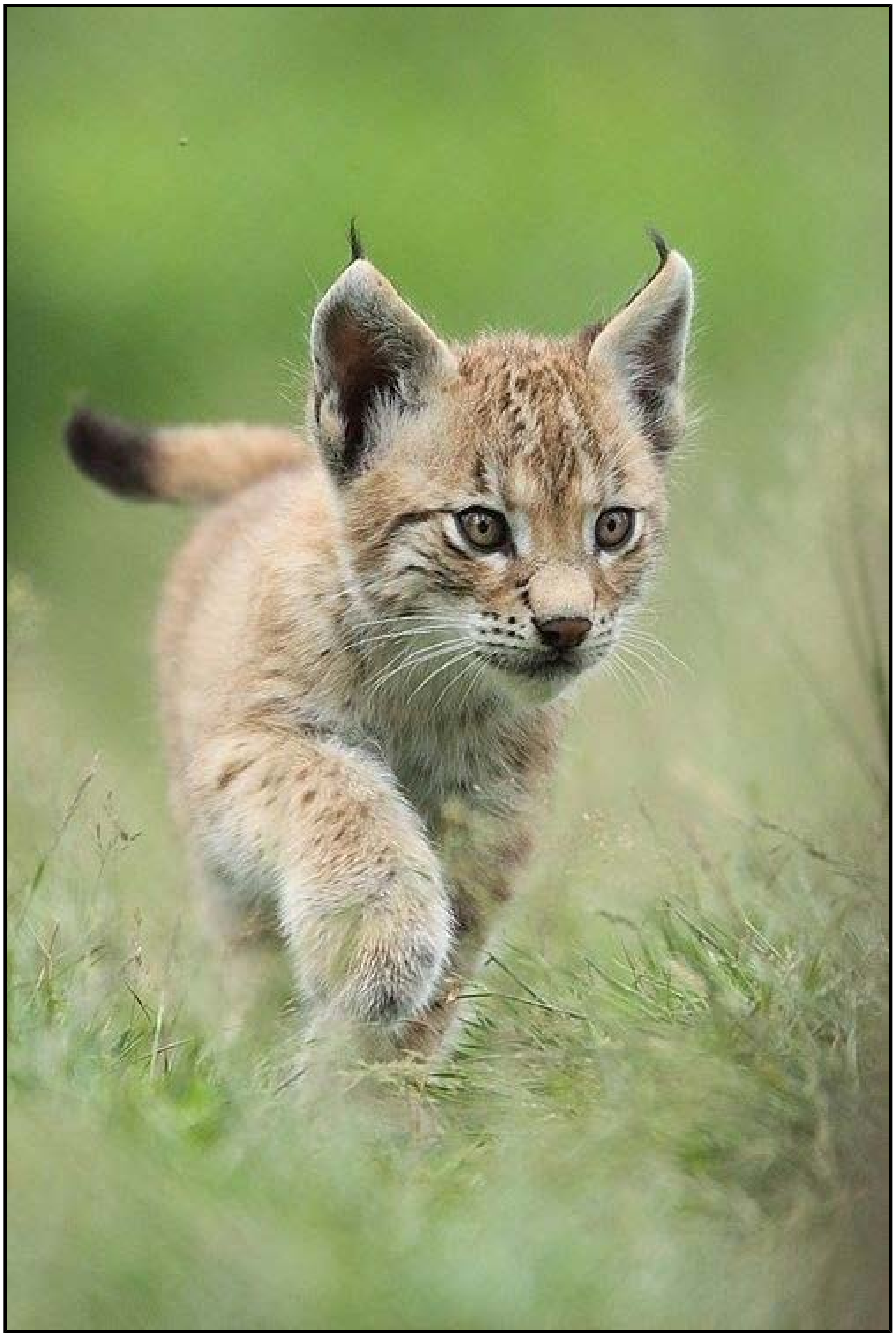
It's all about the number 5, which has a lot of star power. To understand more, read my blog article:

- John Baez, Small stellated dodecahedron, *Visual Insight*, June 15, 2016.

Most of this was discovered by Felix Klein in 1877. He discovered lots of cool facts like this. It's almost annoying. I keep learning cool things about Riemann surfaces and the hyperbolic plane... and it keeps turning out they were discovered by Klein. He found more than his fair share.

The image above was created by someone named 'Cyp' and placed on Wikicommons.

June 24, 2016



A lynx kitten bounds forward, confident and focused.

I need this picture today, to cheer myself up. I don't like the Brexit. The very best possible interpretation I can put on it is that it's ordinary folks poking a stick in the eye of the elite, demanding more local control of government, more democracy. Maybe the elite will wake up, stop trying to hog all the wealth, and realize that in the long run it pays to help the downtrodden.

Maybe London will become less dominated by corrupt financiers. Maybe Scotland will become independent and join the EU.

I can imagine a wave of decentralization and localization being a good thing.... if it's balanced by the right larger-scale structures, allowing plenty of free trade, free movement of people, and so on. But I don't get any sense that the Brexiters have a constructive vision for the future.

Back to the theme of youth:

The young are generally bolder, less careful, less fearful. It's got pros and cons.

75% of British people between ages 18 and 24 said they voted for Britain to stay in the EU. For people 25-49 it was 56%. For people 50-64 it was 44%. For people above 65, just 39%.

So this is an interesting case. Perhaps the old are more fearful — of refugees, of Polish plumbers, of EU bureaucrats — but in this case they were more eager to do something rash. It's quite amazing how little is known about what will happen next! About all we be sure about is that it will create a big mess.

Good luck, Britain! Good luck, EU!

June 27, 2016

Why bees are fuzzy

The fuzz on bees helps them collect pollen. But it may also help them detect electric fields!

The surprising part — to me — is that flowers have electric fields. And different kinds of flowers have noticeably different fields.

Gregory Sutton, a biomechanical engineer who is studying this, says that flower petals tend to accumulate electric charge. So, they produce an electric field just like when you rub a balloon on a woolly sweater — but smaller:

> "It's a very small electrical field, which is why we're quite astounded that bees can actually detect it," Sutton says. "And there is different charge distribution at different locations on the petals of different species of flowers. So two flowers of the same species will have a similar electric field, whereas two flowers of a different species will have different electric fields."

> Together with biophysics researcher Erica Morley and some other scientists, Sutton did experiments to test the theory that bees use electric fields to help find food.
>
> They built 10 flowers with the same shape, size and smell. They put sugar water on some of the flowers and then added small static electric fields to those flowers. On the rest of the flowers, they put bitter water and no electric field. They let the bees loose among the flowers and kept moving the flowers around so the bees couldn.t learn the location of the sugar water.
>
> "As they forage, they start to go to the flowers with the sugar water 80 percent of the time," Sutton says. "So you know they've figured out the difference between the two sets of flowers. The last step is you just turn off the voltage and then check to see if they can continue telling the difference. And when we turned off the voltage, they were unable to tell the difference. And that's how we knew it was the voltage itself that they were using to tell the difference between the flowers."

It's good that they did this last step, because otherwise I'd be unconvinced. They also studied the mechanism that bees use to detect electric fields. Basically, bee hairs get pulled by an electric field, and the bee can feel it:

> "What we found in bees is that they're using a mechanic receptor," Morley says. "It's not a direct coupling of this electrical signal to the sensory system. They're using mechanical movement of hair in a very non-conductive medium. Air doesn't conduct electricity very well — it's very resistive. So these hairs have moved in response to the field, which then causes the nerve impulses from the cells at the bottom of the hair."

I love results like this, which show the world is bigger and more interesting than I thought. But I'm a bit suspicious too, so I hope more scientists try to replicate these experiments or poke holes in them.

The paper is open-access, so if you have questions you can read it yourself!

- Gregory P. Sutton, Dominic Clarke, Erica L. Morley and Daniel Robert, [Mechanosensory hairs in bumblebees (*Bombus terrestris*) detect weak electric fields], *Proc. Nat. Acad. Sci.* **113** (2016), 7261–7265.

I got my quotes from here:

- Elizabeth Shockman, [Flowers give off electrical signals to bees], *Science Friday*, PRI, June 26, 2016.

June 29, 2016

[Metaculus] is a website where you can ask about future events and predict their probabilities. The "wisdom of crowds" says that this is a pretty reasonable way to divine the future. But some people are better predictors than others, and this skill can be learned.

Metaculus was set up by two professors at U.C. Santa Cruz. Anthony Aguirre, a physicist, is a co-founder of the Foundational Questions Institute, which tries to catalyze breakthrough research in fundamental physics, and the Future of Life Institute, which studies disruptive technologies like AI. Greg Laughlin, an astrophysicist, is an expert at predictions from the millisecond predictions relevant to high-frequency trading to the ultra-long-term stability of the solar system.

I've asked and answered a few questions there. It's fun, and it will get more fun as more people take it seriously! Here's some stuff from their [latest report]:

> Dear Metaculus Users,
>
> We recently logged our 10,000th prediction. Not quite Big Data (which will take lots more growth), but

> we're making progress! With this milestone passed, it seems like a good time to share an overview of our results.
>
> First, the big picture. This can be summarized with a single histogram that shows the distribution of the first 10,042 predictions on our first 146 questions. Unambiguously, the three most popular predictions are 1%, 50% and 99%, with spikes of varying strength at each multiple of 5%. There.s a definite overall skew toward lower percentages. This phenomenon stems in part from the fact that the subset of provocative low-probability questions is most naturally worded in a way that the default outcome is negative, e.g., Question: Will we confirm evidence for megastructures orbiting the star KIC 8462852? (Answer: No.) The histogram also makes the point that while 99% confidence — the equivalent of complete confidence — is very common, it's very rare that anyone is ever 98% sure about anything. One takeaway from the pileup at 1% and 99% is that we could use more possible values there, so we plan to introduce an expanded range, from 0.1% to 99.9% soon — but as cautioned below, be careful in using it. Excluding the 1% and 99% spikes and smoothing a bit, the prediction distribution turns out to be a pretty nice gaussian, illustrating the ubiquitous effect of the law of large numbers.
>
> The wheels of Metaculus are grinding slowly, but they grind very fine. Almost 80% of the questions that have been posed on site are still either active (open), or closed (pending resolution) We are starting, however, to get meaningful statistics on questions that have resolved to date — a collection that spans a wide range of topics (from Alpha Go to LIGO and from VIX to SpaceX). We've been looking at different metrics to evaluate collective predictive success. A simple approach is to chart the fraction of outcomes that actually occurred, after aggregating over all of the predictions in each percentage bin. In the limit of a very large number of optimally calibrated predictions on a very large number of questions, the result would be the straight line shown in gold on Figure 2 below. It's clear that the optimal result compares quite well to the aggregation produced by the Metaculus user base. Error bars are 25% and 75% confidence intervals, based on bootstrap resampling of the questions. The only marginally significant departure from the optimal result comes at the low end: as a whole, the user base has been slightly biased toward pessimism, assigning a modest overabundance of low probabilities to events that actually wound up happening. In particular, the big spike in the 1% bin in Figure 1 isn't fully warranted. (This is also somewhat true at 99%: these predictions have come true 90% of the time.) Take-away: if you're inclined to pull the slider all the way to the left or even right, give it a second thought...
>
> It has been demonstrated that the art of successful prediction is a skill that can be learned. Predictors get better over time, and so it's interesting to look at the performance of the top predictors on Metaculus, as defined by users with a current score greater than 500. The histogram of predictions for the subset of top users shows some subtle differences with the histogram of all the predictions. The top predictors tend to be more equivocal. The 50% bin is still highly prominent, whereas the popularity of 1% votes is quite strongly diminished.

I recently predicted — not on Metaculus — that Hillary Clinton has a 99% chance of getting the Democratic nomination. Maybe I should have said 98%. But I definitely should put my prediction on Metaculus! This could develop into a useful resource.

If you want to become a "super-forecaster", you need to learn about the Good Judgment Project. Start here:

- Alix Spiegel, So you think you're smarter than a CIA agent, *Morning Edition*, National Public Radio, April 2, 2014.

A little taste:

> For the past three years, Rich and 3,000 other average people have been quietly making probability estimates about everything from Venezuelan gas subsidies to North Korean politics as part of the Good Judgment Project, an experiment put together by three well-known psychologists and some people inside the intelligence community.

> According to one report, the predictions made by the Good Judgment Project are often better even than intelligence analysts with access to classified information, and many of the people involved in the project have been astonished by its success at making accurate predictions.

Then read Philip Tetlock's books *Expert Political Judgment* and *Superforecasting: The Art and Science of Prediction*. I haven't! But I would like to become a super-forecaster.

For a nice discussion of Metaculus and related issues, check out the comments on my G+ post.

For my July 2016 diary, go here.

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home

[For my June 2016 diary, go here.](#)

Diary - July 2016

John Baez

July 1, 2016

On the 4th of July, a NASA spacecraft named [Juno](#) will try to start orbiting Jupiter. It has traveled for 5 years and 2.8 billion kilometers to get there. This is going to be exciting!

Juno will try to aim its main engine towards the Sun, turn it on for 35 minutes, and slow down to 58 kilometers per second, so it can be captured by Jupiter's gravitational field. Says the lead scientist:

> There's a mixture of tension and anxiety because this is such a critical maneuver and everything is riding on it. We have to get into orbit. The rocket motor has to burn at the right time, in the right direction, for just the right amount of time.

With luck, Juno will enter a highly eccentric polar orbit, and make 37 orbits lasting 14 days each. Each time it will dive down to just 4000 kilometers above Jupiter's cloud tops, closer than we've ever come! Each time it will shoot back up to a height of 2.7 million kilometers. It will map Jupiter using many instruments. The first dive is scheduled for August.

Juno will gradually be damaged by Jupiter's intense radiation, even though the main computer is encased in a 200-kilogram titanium box. After its last orbit, it will deliberately plunge to its death — so that it has no chance of contaminating the oceans of Europa.

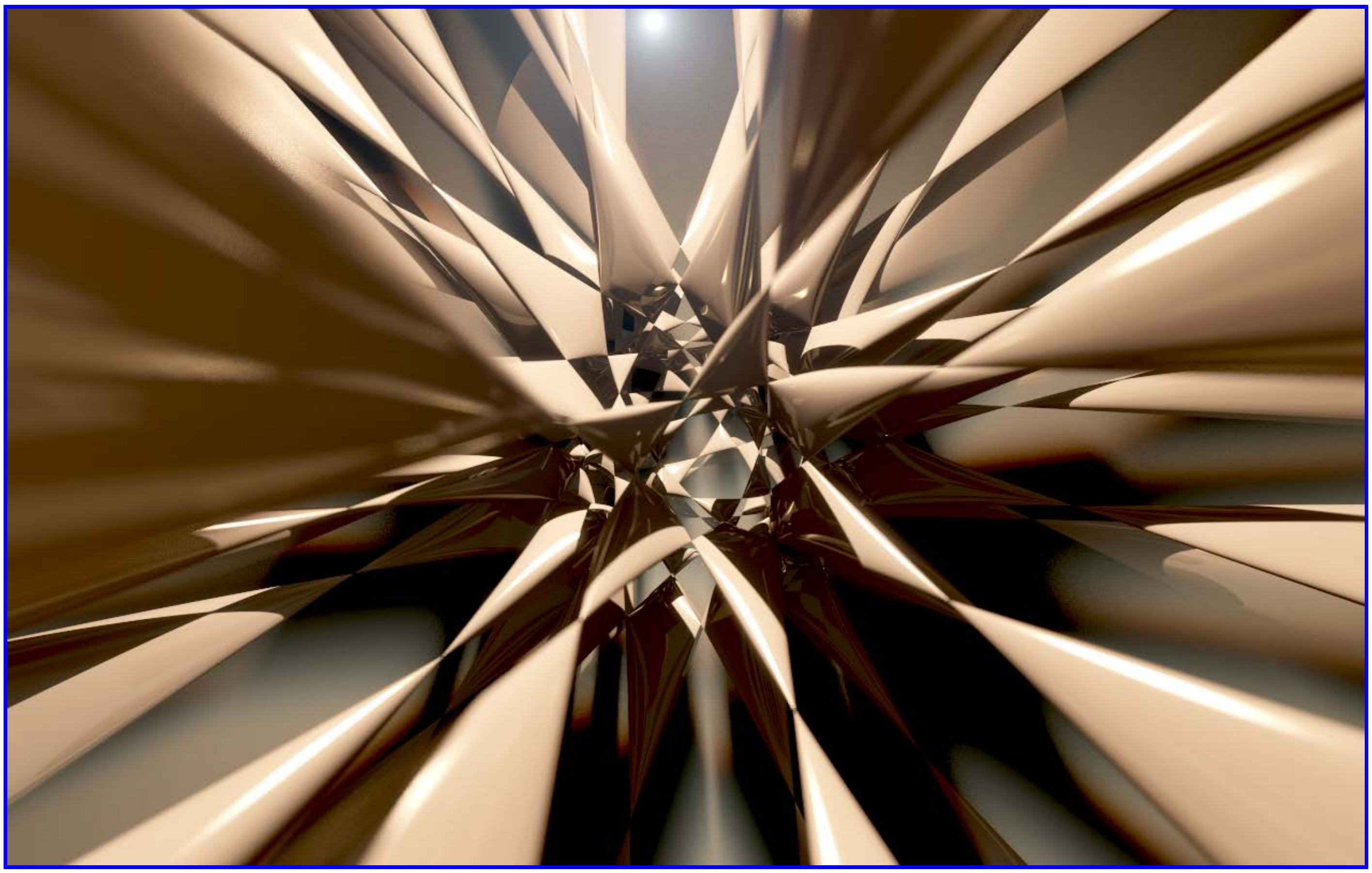
Juno has already entered Jupiter's magnetosphere - the region of space dominated by Jupiter's powerful magnetic field. You can hear it here:

For details of Juno's trajectory, go here:

- Juno mission and trajectory design, *Spaceflight 101*.

The Jupiter orbit insertion should begin at 03:18 July 5th UTC, which is 20:18 on the 4th of July in California.

July 3, 2016



This is a view of Barth's decic surface drawn by Abdelaziz Nait Merzouk. It's a frightening shape with 345 cone-shaped singularities — the most possible for a surface described by a polynomial of degree 10.

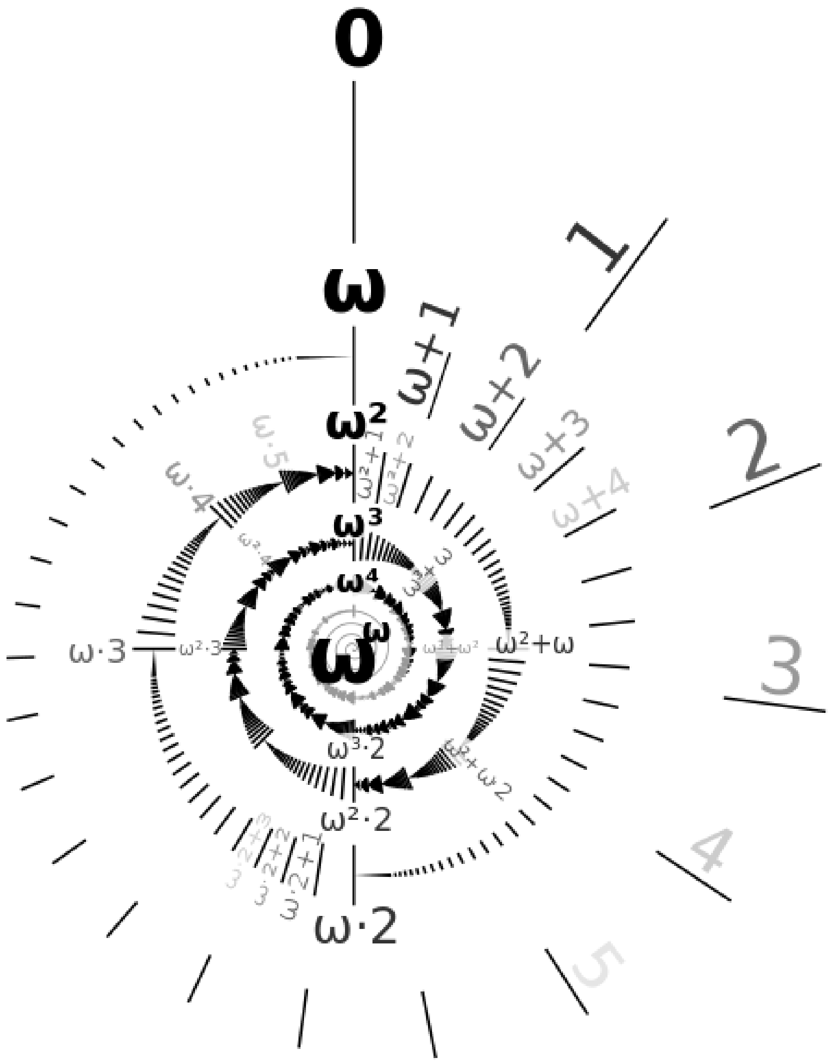
And yet, despite its nightmarish complexity, this surface is highly symmetrical. It has the same symmetries as a regular icosahedron!

For more views of this surface, go here:

- John Baez, Barth decic, *Visual Insight*, July 15, 2016.

I have no idea how Wolf Barth dreamt up this surface, along with the closely related 'Barth sextic', back in 1994. The equations describing them feature the golden ratio... but they're complicated. I bet there's a more conceptual way to get your hands on these surfaces. If you know it, please tell me!

July 4, 2016



A while back I wrote a story about infinity on Google+. It featured a character who was recruited by the US government to fight in the War on Chaos. His mission was to explore larger and larger infinities.

You can see that story in my bigness collection on G+: lots of posts, each one its own little chapter.

But I keep wanting to talk about infinity — it's endlessly interesting! I keep learning more about it. Some posts here by +Refurio Anachro re-ignited my desire to write about it, and now I have. Here's the first of three articles:

- John Baez, Large countable ordinals (part 1), *Azimuth*, June 29, 2016.

If you read this, you'll learn about the two basic kinds of infinities discovered by Cantor: cardinals and ordinals. Then we'll go on a road trip through larger and larger ordinals.

The picture above shows some of the first ones we'll meet on our trip. Omega, written ω, is the first infinite ordinal:

$$\omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\}$$

Each turn of the spiral here takes you to a higher power of omega, and if you go around infinitely many times, you reach omega to the omegath power. There are many ways to visualize this ordinal, and I explain a few.

But my road trip will take you much further than that!

In this first episode, we reach an ordinal called 'epsilon nought', first discovered by Cantor. In the second episode we'll go up the Feferman–Schütte ordinal. In the third we'll reach the small Veblen ordinal and even catch a glimpse of the large Veblen ordinal.

All these are *countable* ordinals, and you can write computer programs to calculate with them, so I consider them just as concrete as the square root of 2. And yet, they're quite mind-blowing.

July 5, 2016

The 'Jupiter orbit insertion' went flawlessly, and I had fun [watching it live on NASA TV](). The most risky moments occurred from 11:18 to 11:53 am, here in Singapore. Success wasn't guaranteed, and indeed NASA cleverly played up the dangers, to get people interested, and to have an excuse if things didn't work.

- Kenneth Chang, [What to expect during Juno's mission to Jupiter](), *New York Times*, July 4, 2016.

> ## What could possibly go wrong?
>
> Lots.
>
> "I'm confident it's going to work," Scott Bolton, Juno's principal investigator, said before the announcement Monday night that the spacecraft had arrived, "but I'll be happy when it's over and we're in orbit."

Some of the ways that this could turn into a bad day:

Juno blows up. In August 1993, NASA's instrument-packed Mars Observer spacecraft vanished. An inquiry concluded that a fuel leak caused the spacecraft to spin quickly and fall out of communication. While Juno's setup is different, there is always a chance of an explosion with rocket fuel.

The engine doesn't fire at all. The Japanese probe Akatsuki was all set to arrive at Venus in December 2010, but its engine didn't fire, and Akatsuki sailed right past Venus. Last year, Akatsuki crossed paths with Venus again, and this time, using smaller thrusters, it was able to enter orbit.

It crashes into something. Jupiter does not possess the majestic rings of Saturn, but it does have a thin of ring of debris orbiting it. Juno will pass through a region that appears clear, but that does not mean it actually is. Even a dust particle could cause significant damage, as Juno will be moving at a speed of 132,000 miles per hour relative to Jupiter.

It flies too close to Jupiter and is ripped to pieces. In one of NASA's most embarrassing failures, the Mars Climate Orbiter spacecraft, was lost in 1999 because of a mix-up between English and metric units. Climate Orbiter went far deeper into Mars. atmosphere than planned. On its first orbit, Juno is to pass within 2,900 miles of Jupiter's cloud tops, so a miscalculation could be catastrophic.

The computer crashes. On July 4 last year, the mission controllers of the New Horizons spacecraft that was about to fly by Pluto experienced some nervous moments when the spacecraft stopped talking to them. The computer on New Horizons crashed while trying to interpret some new commands and compressing some images it had taken, the electronic equivalent of walking while chewing gum.

The controllers put New Horizons back in working order within a few days, and the flyby occurred without a hitch. For Juno, the scientific instruments have been turned off for its arrival at Jupiter. "We turn off everything that is not necessary for making the event work," said Dr. Levin, the project scientist. "This is very important to get right, so you don't do anything extra."

The intense barrage of radiation at Jupiter could knock out Juno's computer, even though it is shielded in a titanium vault. Usually, when there is a glitch, a spacecraft goes into "safe mode" to await new instructions from Earth, but in this case, that would be too late to save Juno. The spacecraft has been programmed to automatically restart the engine to allow it to enter orbit.

"If that doesn't go just right, we fly past Jupiter, and of course, that's not desirable," Dr. Bolton said.

July 6, 2016

♥ ♥ ♥ I love infinity ♥ ♥ ♥

Some infinities are countable, like the number of integers. Others are uncountable, like the number of points on a line.

Uncountable infinities are hard to fully comprehend. For example, even if you think an infinity is uncountable, someone else may consider it countable! That's roughly what the Löwenheim–Skolem theorem says.

How is this possible?

Ultimately, it's because there are only a countable number of sentences in any language with finitely many letters. So, no matter how much you talk, you can never convince me that you're talking about something uncountable!

Now, if we take a really hard-ass attitude, we have to admit we can never actually write infinitely many sentences. So even countable infinities remain outside our grasp. However, we come "as close as we want", in the sense that we can keep counting

$$0, 1, 2, 3, 4, \ldots$$

and nothing seems to stop us. So, while we never actually reach the countably infinite, it's pretty easy to imagine and work with.

Thus, my favorite infinities are the countable ordinals — in particular, the computable ones. You can learn to do arithmetic with them. You can learn to visualize them just as vividly as the set of all natural numbers, which is the first countable ordinal:

$$\omega = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\}$$

For example,

$$\omega + 1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots, \omega\}$$

But as you keep trying to understand larger and larger countable ordinals, strange things happen. You discover that you're fighting your own mind.

As soon as you see a systematic way to generate a sequence of larger and larger countable ordinals, you know this sequence has a limit that's larger then all of those! And this opens the door to even larger ones....

So, this journey feels a bit like trying to outrace your car's own shadow as you drive away from the sunset: the faster you drive, the faster it shoots ahead of you. You'll never win.

On the other hand, you never need to lose. You only lose when you get tired.

And that's what I love: it becomes so obvious that the struggle to understand the infinite is a kind of mind game. But it's a game that allows clear rules and well-defined outcomes, not a disorganized mess.

In this post:

- John Baez, Large countable ordinals (part 2), *Azimuth*, July 4, 2016.

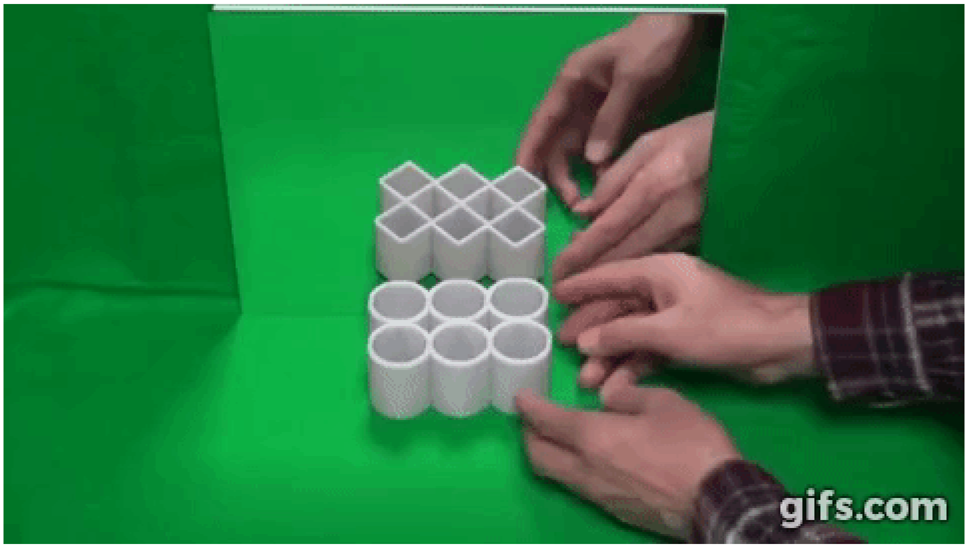
I'll take you on a tour of countable ordinals up to the Feferman–Schütte ordinal. Hop in and take a ride!

And if you don't know the Löwenheim–Skolem theorem, you've gotta learn about it. It's one of the big surprises of early 20th-century logic:

- Wikipedia, Löwenheim–Skolem theorem.

The pink and the hearts, by the way, are there to see if you become uncomfortable. They are 'girly'.

July 7, 2016



Wow! These plastic cylinders look round — but in the mirror they look diamond-shaped. If you turn them around, they look diamond-shaped - but in the mirror they look round!

This video was made by Kokichi Sugihara, an engineer at Meiji University in Tokyo. How did he do it???

To answer this question, we should "science the hell out of it", as Matt Damon said in *The Martian*. Figure out how objects change appearance when you look at them in a mirror... and design an object that does this! So David Richeson, a math professor at Dickinson College in Pennsylvania, scienced the hell out of it:

- David Richeson, Sugihara's circle/square optical illusion, *Division by Zero*, July 5, 2016.

The basic idea is this. The top rim of this object is not flat. More precisely, it's not horizontal: it curves up and down! This affects how it looks. If you're looking down on this object, you can make part of the top look farther away by having it be lower.

But a mirror reflects front and back. So in the mirror, part of the top looks closer if it's lower.

By cleverly taking advantage of this, we can make this object look round, but diamond-shaped in the mirror.

And if we turn it around, this effect is reversed!

Here's a bit more of the math. David Richeson gives the details, so I'll try to present just the basic idea.

Suppose you're making a video. Suppose you're looking down at an angle of 45 degrees, just as in this video. Suppose you're videotaping an object that's fairly far away.

Think about one pixel of the object's image on your camera's viewscreen.

Its height on your viewscreen depends on two things. It depends on how far *up* that piece of the object actually is. But it also depends on how far *back* that piece of the object is: how far away it is from your camera. Things farther away give higher pixels on your viewscreen.

There's a simple formula for how this works:

pixel height = actual object height + actual distance back

(It's only this simple when you're looking down at an angle of 45 degrees and the thing you're videotaping is fairly far away.)

But what if we're looking in a mirror? You may think a mirror reverses left and right, but that's wrong: it reverses front and back. So we basically get

mirror image pixel height = actual object height - actual distance back

So, you just need to craft an object for which

actual object height + actual distance back

and

actual object height - actual distance back

give two different curves: one round and one a diamond!

Now for some puzzles:

Puzzle 1. All that sounds fine: by cleverly adjusting the top rim of the object we can make it look different in a mirror. But look at the bottom of the object! What's going on there? How do you explain that?

Puzzle 2. Sometimes I know the answers to the puzzles I'm posing. Sometimes I don't. Do I know the answer to Puzzle 1, or not?

Puzzle 3. Same question for Puzzle 2.

Finally, I should admit that I simplified the formula for the mirror image pixel height. Actually we have

mirror image distance back = constant - actual distance back

and thus

mirror image pixel height = actual object height + constant - actual distance back

In other words, I ignored a constant. This constant is why the whole mirror image looks higher on your viewscreen than the original object! For more, see:

- Stephen L. Macknik, A review of 2016's top ten illusions of the year, *Scientific American*, July 6, 2016.

Kokichi Sugihara's work got second place.

July 8, 2016

I'm back in Singapore, the land of explosive cuisine. This is the menu from our favorite Chinese restaurant. It's on Southbridge Road across from the Sri Mariamman Temple — a popular Hindu temple where they do firewalking on the holiday called Theemithi. Maybe they do it to cool down after eating here.

I hadn't known it was called the Explosion Pot Barbecue. They sell excellent barbecued fish, roast skewers of lamb with cumin, roast chives, dumplings, and other Szechuan delights. The food is a bit spicy, but I haven't seen any exploding pots, so this may be a mistranslation of something that makes more sense in Chinese.

As usual, I'm working at the Centre for Quantum Technologies and Lisa is teaching at the philosophy department at NUS. You can see her in the background ordering our food.

Meanwhile, my student Blake Pollard is in a small town in the hills of Yunnan Province in southern China, helping teach some local students science, English... and American folk songs!

This seems much more adventurous than what I'm doing. But he has a good reason for doing it. His great grandfather, Sam Pollard, was a Methodist missionary in this area — and he invented a script that is still used by the locals:

- Wikipedia, Pollard script.

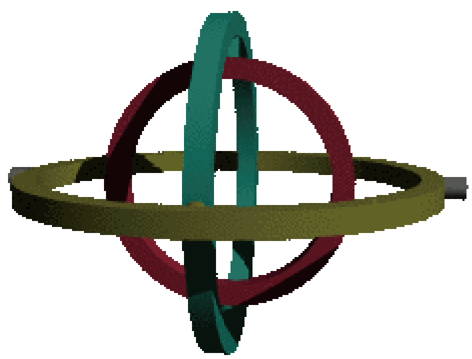
The Miao are an ethnic group that includes the Hmong, Hmub, Xong, and A-Hmao. These folks live in the borderlands of southern China, northern Vietnam, Laos, Myanmar and Thailand. The A-Hmao had a legend about how their ancestors knew a system of writing but lost it. According to this legend, the script would eventually be brought back some day. When Sam Pollard introduced his script for writing A-Hmao, it became extremely popular, and he became a kind of hero. Blake and his family visited this part of China last year. He enjoyed it a lot, so he decided to do some teaching there this summer.

Watch firewalking at the Sri Mariamman Temple:

- Firewalking / Theemithi @ Singapore Sri Mariamman Temple, YouTube.

and if you live around here, check out the Explosion Pot Barbecue!

July 11, 2016



Here you see 3 rotating rings called gimbals. Gimbals are used in gyroscopes and inertial measurement units, which are gadgets that measure the orientation of some object — like a drone, or a spacecraft. Gimbals are also used to orient thrusters on rockets.

With 3 gimbals, you can rotate the inner one to whatever orientation you want. The basic reason is that it takes 3 numbers to describe a rotation in 3 dimensional space. This is a special lucky property of the number 3.

But when two of the gimbal's axes happen to be lined up, you get gimbal lock. In other words: you lose the ability to rotate the inner gimbal a tiny bit in any way you want. The reason is that in this situation, rotating one of the two aligned gimbals has the same effect on the inner gimbal as rotating the other!

I've always found gimbal lock to be a bit mysterious: people refer to it in ominous tones without explaining it, like some incurable deadly disease. So I'm trying to demystify it here.

As the wise heads at Wikipedia point out,

> The word *lock* is misleading: no gimbal is restrained. All three gimbals can still rotate freely about their respective axes of suspension. Nevertheless, because of the parallel orientation of two of the gimbals' axes there is no gimbal available to accommodate rotation along one axis.

Gimbal lock can actually be dangerous! When it happens, or even when it almost happens, you lose some control over what's going on.

It caused a problem when Apollo 11 was landing on the moon. This spacecraft had 3 nested gimbals on its inertial measurement unit. The engineers were aware of the gimbal lock problem but decided not to use a fourth gimbal. They wrote:

> "The advantages of the redundant gimbal seem to be outweighed by the equipment simplicity, size advantages, and corresponding implied reliability of the direct three degree of freedom unit."

They decided instead to trigger a warning when the system came close to gimbal lock. But it didn't work right:

> "Near that point, in a closed stabilization loop, the torque motors could theoretically be commanded to flip the gimbal 180 degrees instantaneously. Instead, in the Lunar Module, the computer flashed a 'gimbal lock' warning at 70 degrees and froze the inertial measurment unit at 85 degrees."

The spacecraft had to be manually moved away from the gimbal lock position, and they had to start over from scratch, using the stars as a reference.

After the Lunar Module landed, Mike Collins aboard the Command Module joked:

> "How about sending me a fourth gimbal for Christmas?"

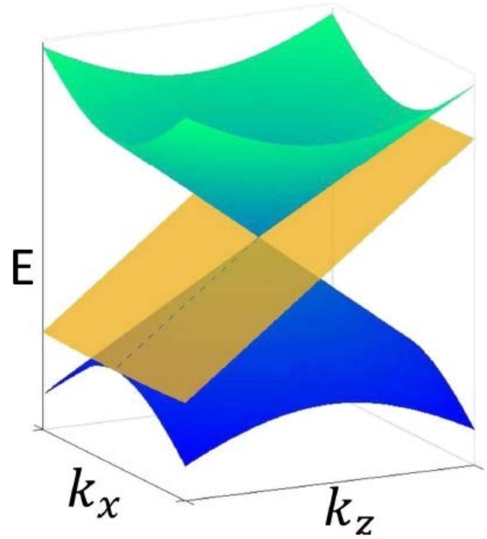
Fun story! But ultimately, it's all about math.

Puzzle. Show that gimbal lock is inevitable with just 3 gimbals by showing that every smooth map from the 3-torus to SO(3) has at least one point where the rank of its differential drops below 3.

See what I mean? Math. This result shows not only that gimbal lock occurs with the setup shown above, but that *any* scheme for describing a rotation by 3 angles — or more precisely, 3 points on the circle — must suffer gimbal lock.

Here's a sketch of an answer to the puzzle: if X and Y are smooth n-manifolds and the rank of the differential of a smooth map $f: X \to Y$ is equal to n everywhere, it's locally a diffeomorphism. If X is compact and Y is connected you can show this makes X into a covering space of Y. So, if there were a smooth map $f: T^3 \to \mathrm{SO}(3)$ whose differential has rank 3 everywhere, the 3-torus would be a covering space of SO(3), but this is not possible, since a covering $f: T^3 \to \mathrm{SO}(3)$ would induce an inclusion of $\pi_1(T^3) \quad \mathrm{Z}^3$ in $\pi_1(\mathrm{SO}(3)) \quad \mathrm{Z}/2$, which is impossible.

July 24, 2016



You can get electrons to behave in many strange ways in different materials. They act like various kinds of particles... but they're not truly fundamental particles, so they're called quasiparticles.

For example, the spin, charge and position of electrons can move in completely independent ways.

Imagine an audience at a football game holding up signs, and then creating a wave by wiggling their signs. This wave can move even even if the people stand still!

Similarly, we can have electrons more or less standing still, with their spins lined up. Then their spins can wiggle a bit, and this wiggle can move through the material, even though the electrons don't move. This wave of altered spin can act like a particle! It's called a spinon.

You can also imagine a hole in a dense crowd of people, moving along as if it were an entity of its own. When this happens with electrons it's called a holon, or more commonly just a hole. A hole acts like a particle with positive charge, since electrons have negative charge.

Since holes have positive charge and electrons have negative charge, they attract. Sometimes they orbit each other for long enough that this combined thing acts like a particle of its own! This kind of quasiparticle is called an exciton.

There are also other quasiparticles. If you're a student who wants to do particle physics, please switch to studying quasiparticles! The math is almost the same, and you don't need huge particle accelerators to make cool new discoveries. Some are even useful.

One of the most fundamental things about a quasiparticle, or for that matter an ordinary particle, is its energy. Its energy depends on its momentum. The relation between them is called the dispersion relation. This says a lot about how the quasiparticle acts.

Right next door to the Centre for Quantum Technologies where I'm working in Singapore there's a lab that studies graphene — a crystal made of carbon that's just one atom thick. When you've got a very thin film like this, a quasiparticle inside it acts like it's living in a 2-dimensional world! Since it can't go up and down, only 2 components of its momentum can be nonzero.

The picture above shows a graph of energy as a function of momentum for a new kind of quasiparticle they're studying. They haven't made it in the lab yet; they've just shown it's possible. It involves a 3-dimensional material, not a thin sheet, so there are really 3 components of momentum, k_x, k_y and k_z. But only two are shown in this picture.

The three colored sheets show that 3 different energies are possible for each momentum — except momentum zero, where all three sheet meet, and also a line of momenta where two sheets meet.

If we only had the green and blue sheets, that would be the dispersion relation for a massless particle. People already know how to make massless quasiparticles with graphene.

The new thing is the yellow sheet! This will make very strange things happen, I'm sure.

I got interested in these new quasiparticles thanks to this article pointed out by Rasha Kamel:

- Unconventional quasiparticles predicted in conventional crystals, *ScienceDaily*, July 21, 2016.

But I got the picture from here:

- Guoqing Chang *et al*, New fermions on the line in topological symmorphic metals.

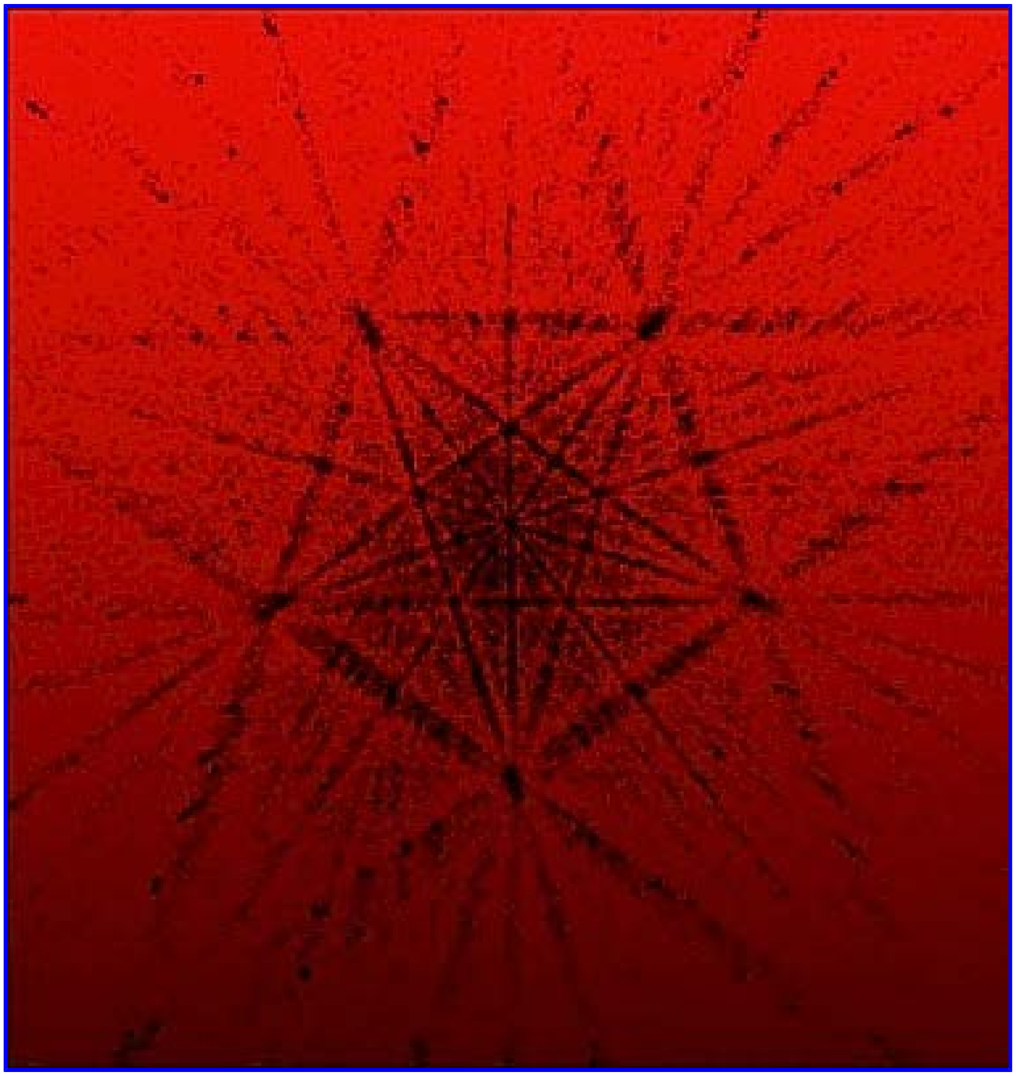
Here's the abstract, for you physics nerds out there:

> **Abstract.** Topological metals and semi-metals (TMs) have recently drawn significant interest. These materials give rise to condensed matter realizations of many important concepts in high-energy physics, leading to wide-ranging protected properties in transport and spectroscopic experiments. The most studied TMs, i.e., Weyl and Dirac semi-metals, feature quasiparticles that are direct analogues of the textbook elementary particles. Moreover, the TMs known so far can be characterized based on the dimensionality of the band crossing. While Weyl and Dirac semimetals feature zero-dimensional points, the band crossing of nodal-line semimetals forms a one-dimensional closed loop. In this paper, we identify a TM which breaks the above paradigms. Firstly, the TM features triply-degenerate band crossing in a symmorphic lattice, hence realizing emergent fermionic quasiparticles not present in quantum field theory. Secondly, the band crossing is neither 0D nor 1D. Instead, it consists of two isolated triply-degenerate nodes interconnected by multi-segments of lines with two-fold degeneracy. We present materials candidates. We further show that triply-degenerate band crossings in symmorphic crystals give rise to a Landau level spectrum distinct from the known TMs, suggesting novel magneto-transport responses. Our results open the door for realizing new topological phenomena and fermions including transport anomalies and spectroscopic responses in metallic crystals with nontrivial topology beyond the Weyl/Dirac paradigm.

Weirdly, I had learned the word 'symmorphic' just yesterday. Greg Egan were working on crystals, and he explained that a crystal is symmorphic if it contains a point where every symmetry of the crystal consists of a symmetry fixing this point followed by a translation. It was important for our work to notice that a diamond is not symmorphic.

July 26, 2016

Satanic crystal found in ancient meteorite



Just kidding! There's nothing devilish about the pentagram here. It's what scientists saw when they shot X-rays through a tiny piece of a meteorite found in the far northeast of Russia.

No ordinary crystal can produce this pattern - it takes a quasicrystal, where the atoms are packed in a way that never quite repeats. Scientists have made lots of quasicrystals in the lab, but only two have been found in nature, both in meteorites!

This is the second one. It contains a mineral called icosahedrite, made of aluminum, copper and iron. It's only stable at high temperatures and pressures, so it must have formed in a collision. It's been slowly decaying ever since, but very slowly. It could be billions of years old.

To see how this mineral could have formed, scientists simulated the collision between two asteroids in their lab. They took thin slices of minerals found in the Khatyrka meteorite and sandwiched them together in a gadget that looks like a a steel hockey puck. They attached it to the muzzle of a four-meter-long gun and blasted it with a projectile moving nearly one kilometer per second! Yup. Icosahedrite. For details and more pictures, see:

- Paul D. Asimow, Chaney Lin, Luca Bindi, Chi Ma, Oliver Tschauner, Lincoln S. Hollister and Paul J. Steinhardt, Shock synthesis of quasicrystals with implications for their origin in asteroid collisions, *Proceedings of the National Academy of Sciences* **113** (2016), 7077—7081. Freely available at

Puzzle: how did pentagrams get associated with Satan in the first place?

July 30, 2016

Death of the diphoton bump

In June 2015, after a two-year upgrade, the Large Hadron Collider turned on again. In its first run it had discovered the Higgs boson, a particle 133 times heavier than the proton — and the main missing piece of the Standard Model. When the collider restarted, with a lot more energy, everyone was hoping to see something new.

In December 2015, two separate detectors saw something: pairs of photons, seemingly emitted by the decay of a brand new particle 6 times heavier than the Higgs boson.

But was it for real? Maybe it was just a random fluctuation: noise, rather than a true signal.

It seemed unlikely to be just chance. Combining the data from both detectors, the chance of coincidentally seeing a bump this big at this location in the photon spectrum was one in 100 thousand.

But in particle physics that's not good enough. Physicists are looking for lots of different things in these big experiments, so rare coincidences do happen. To feel safe, they want to push the chance down to one in 3 million. That's called a '5 sigma event'.

So they looked harder.

Meanwhile, theoretical physicists wrote 500 papers trying to explain this so-called diphoton bump. It turned out to be easy to make up theories that have a particle of the right sort. Not so easy, though, to make a convincingly elegant theory.

New data have come in. The bump is gone.

Theorists are bummed. On his blog, a particle physicist named Adam Falkowski wrote:

> The loss of the 750 GeV diphoton resonance is a big blow to the particle physics community. We are currently going through the 5 stages of grief, everyone at their own pace, as can be seen e.g. in this comments section. Nevertheless, it may already be a good moment to revisit the story one last time, so as to understand what went wrong.
>
> In the recent years, physics beyond the Standard Model has seen 2 other flops of comparable impact: the faster-than-light neutrinos in OPERA, and the cosmic microwave background tensor fluctuations in BICEP. Much as the diphoton signal, both of the above triggered a binge of theoretical explanations, followed by a

> massive hangover. There was one big difference, however: the OPERA and BICEP signals were due to embarrassing errors on the experimentalists' side. This doesn't seem to be the case for the diphoton bump at the Large Hadron Collider. Some may wonder whether the Standard Model background may have been slightly underestimated, or whether one experiment may have been biased by the result of the other... But, most likely, the 750 GeV bump was just due to a random fluctuation of the background at this particular energy. Regrettably, the resulting mess cannot be blamed on experimentalists, who were in fact downplaying the anomaly in their official communications. This time it's the theorists who have some explaining to do.

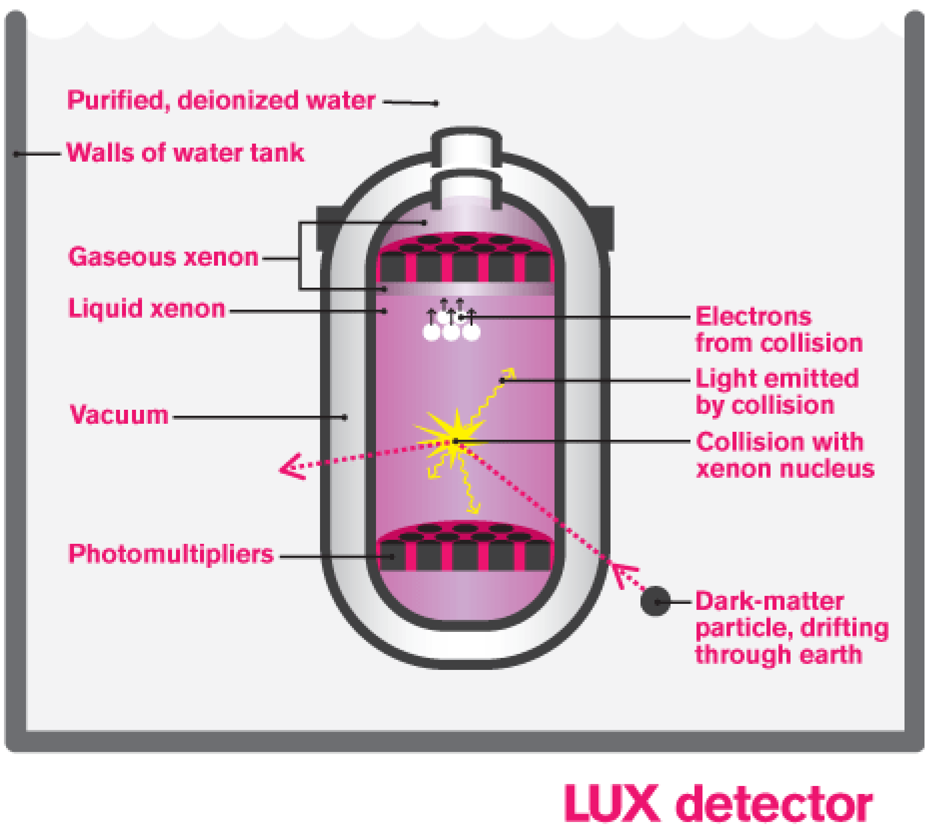
For more, see Adam Falkowski's blog. He goes by the name of 'Jester':

- Adam Falkowkski (Jester), [After the hangover](), *Résonaances*, July 29, 2016.

By now we have to admit it's quite possible that the Large Hadron Collider will not see any new physics not predicted by the Standard Model. Unfortunately, this triumph of the Standard Model would leave a lot of big questions unanswered... for now.

The video above explains the diphoton bump in simple terms. It was made back in the early optimistic days.

July 31, 2016



In South Dakota, in a town named Lead, there was a gold mine. Now it's abandoned. But at the bottom of this mine, more than a mile underground, there sits a one-meter-tall, 12-sided container. It contains 370 kilograms of a noble gas chilled to liquid form. Liquid xenon!

It's called the Large Underground Xenon experiment, or [LUX](). It's been looking for particles that could explain dark matter. If such a particle interacts with a xenon atom, LUX can detect it.

Of course, we also need to distinguish these particles from other things. A mile of rock helps block cosmic rays. But xenon, a gas at room temperatures, chilled to liquid form, is a great choice here. For one thing, it's *self-shielding!* Xenon is so dense that gamma rays and neutrons don't usually get through more than a few centimeters of the stuff. But it's perfectly transparent to ordinary light... so if a dark matter particle hits an atom of xenon in the middle of the tank, LUX will see a flash of light. It can also detect electrons that shoot out from the collision.

Four other experiments had reported hints of dark matter particles about 5 times heavier than a proton. But LUX is much more sensitive!

The LUX team, with over a hundred scientists, has been looking for dark matter since 2014. Ten days ago they announced their results: *no dark matter particles seen*.

This "non-discovery" is actually an important discovery. The most popular theory of dark matter — that it consists of weakly interacting massive particles — has taken a serious hit.

We now know that if these hypothetical particles, affectionately called WIMPs, are responsible for most of the dark matter and have a mass between 1/5 and 1000 times the mass of a proton, they must be very, very unwilling to interact with ordinary matter.

There's no rule saying particles have to interact with ordinary matter. So, we can't rule out such WIMPs, but they're looking less plausible. People are getting more interested in other theories:

1. theories with very light WIMPs, such as axions or neutrinos,
2. theories with very heavy WIMPs, jokingly called WIMPzillas,
3. theories where dark matter consists of large objects such as black holes.

In case you're wondering whether dark matter really exists: there's so much evidence for this that very few scientists question it anymore.

Theory 3) is getting a lot of attention, because the gravitational wave detector called LIGO is now able to detect black hole collisions! It's seen two collisions so far, and the first one involved black holes that seem quite strange, not like the ones we know. They might be primordial black holes, left over from the early Universe. Perhaps dark matter consists of primordial black holes!

More on that later. For now, try these. The new announcement from the LUX team is here:

- Aaron Manalaysay, Dark-matter results from 332 new live days of LUX data, talk at Identification of Dark Matter, Sheffield, July 21, 2016.

For how the LUX detector works, read this nice article:

- Nicole Larsen, Searching for dark matter with the Large Underground Xenon experiment, *Quantum Diaries*, April 17, 2014.

For a nice intro to the new LUX results, on a website that requires you to look at ads, try this:

- Ethan Siegel, Dark matter may be completely invisible, concludes world's most sensitive search, *Forbes*, July 21, 2016.

For primordial black holes as dark matter, try this:

- Jester (Adam Falkowski), Black hole dark matter, *Résonaances*, June 10, 2016.

The picture above comes from here:

- Bill Harlan, Dark matter search goes deep underground in South Dakota, *Symmetry*, April 1, 2012.

For my August 2016 diary, go here.

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home

For my July 2016 diary, go here.

Diary - August 2016

John Baez

August 1, 2016

This surface is beautiful — but it's also the best known solution to a hard math problem. It's called the Endrass octic.

Why 'Endrass'? Because was discovered in 1995 by Stephan Endrass while he was writing his Ph.D. thesis.

Why 'octic'? Because it's described by a polynomial equation of degree 8.

You'll notice it has lots of points where the tips of two cones meet. It has 168 of them, though not all are visible here. And this is, so far, the largest number of such points that people have gotten for an octic.

 e best so far. In 1984, a guy named Miyaoka showed that you can't get get more than 174 of these conical points in an octic. So, there's a gap between what we know is possible and what might be possible. (If you're into algebraic geometry you might like Miyaoka's paper — he used some pretty fancy techniques.)

Endrass actually found two octics with 168 conical points. You can see pictures of both, drawn by Abdelaziz Nait Merzouk, over at my blog:

- Endrass octic, *Visual Insight*, August 1, 2016.

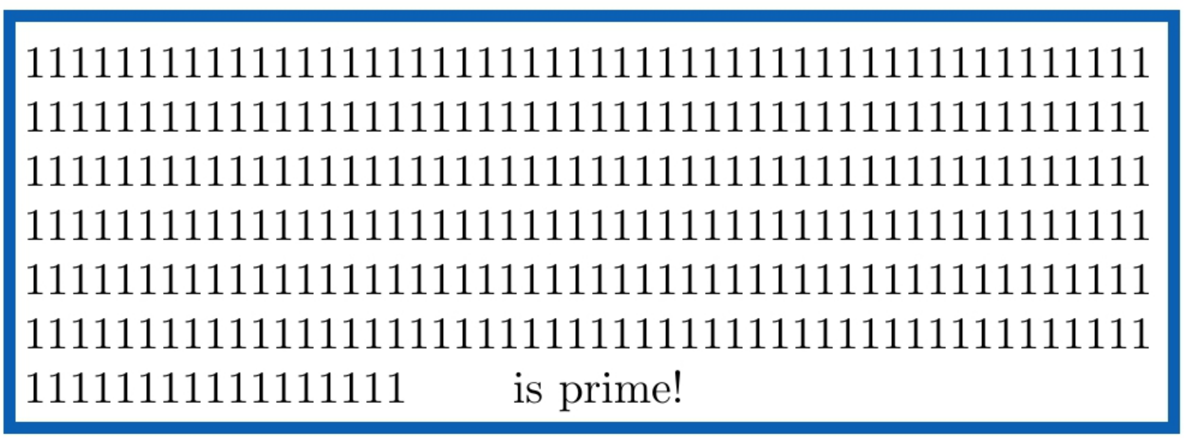
They're very beautiful. You can also see the equations of these surfaces. Those are not very beautiful, at least not to me. Endrass found them with the help of a computer algebra system.

The animated gif above was made by Abdelaziz Nait Merzouk. It's copyrighted under a Creative Commons Attribution-ShareAlike 3.0 Unported license.

Here's another depiction of the Endrass octic, from a German math website:

- IMAGINARY: open mathematics, Endrass octic.

August 3, 2016



This is a prime number whose decimal digits are all ones. It has 317 ones. It's not the world record. The number with 1031 ones is also known to be prime!

Even larger guys like this are suspected to be prime. Are there infinitely many? Mathematicians believe so, but they can't prove it.

Why do they believe it? The main reason is that they have an estimate of the 'probability' that a number with some number of digits is prime. We can use this to guess the answer to this puzzle.

Of course the whole idea of 'probability' is a bit weird here. A number is either prime or not: the math gods do not flip coins to decide which numbers are prime!

Nonetheless, treating primes as if they were random turns out to be useful. Mathematicians have made many guesses using this idea, and then proved these guesses are right, using a lot of extra work.

Of course it's subtle. If I wrote down a number with 317 twos in its decimal expansion, you'd instantly know it's not prime — because it would be even.

In the European Congress of Mathematics, a number theorist named James Maynard just announced something cool. There are infinitely many prime numbers with no sevens in their decimal expansion!.

And his proof works equally well for any other number: there infinitely many primes without 0 as a digit, or 1, or 2, or 3, or 4, or 5, and so on.

This is big news, but not because mathematicians really care about primes with no sevens in them. It's because proving something like this requires a deep and delicate understanding of 'the music of primes' — the way prime numbers are connected to wave patterns. For more on that, here's something easy to read:

- Marianne Freiberger, [Primes without 7s](), *Plus Magazine*, August 1, 2016.

Thanks to Luis Guzman for pointing out this article, and thanks to David Roberts for finding James Maynard's paper on this subject, which is here:

- James Maynard, [Primes with restricted digits]().

He shows that if your base b is sufficiently large, you can find infinitely many primes that are lacking a chosen set of digits, where this set can contain up to $b^{23/80}$ of the digits. Unfortunately I don't see how large b must be — he may not have worked this out. If $b = 10$ counts as sufficiently large, then since $10^{23/80}$ is about 1.94, this result would let you avoid any one digit in base 10, but not two. In any event, he does prove, separately, that you can find infinitely many primes that avoid any one digit in base 10.

It uses cool techniques, like "decorrelating Diophantine conditions which dictate when the Fourier transform of the primes is large from digital conditions which dictate when the Fourier transform of numbers with restricted digits is large". It also uses ideas from Markov process theory — that is, the theory of random processes — as well as hard-core number theory concepts.

By the way, a number whose decimal digits are are 1 is called a [repunit](). Here's a cool fact: the repunit with n digits can only be prime if n is prime. This is easy to see using an example. Consider the repunit with 35 digits:

$$11111111111111111111111111111111111$$

Since $35 = 5 \times 7$, this repunit is not prime, and here's why:

$$11111111111111111111111111111111111 = 11111 \times 1000010000100001000010000100001$$

See? That's five 1's, times a number with 1 every fifth time and a total of seven 1's. If you imagine multiplying these out, you'll see why it works.

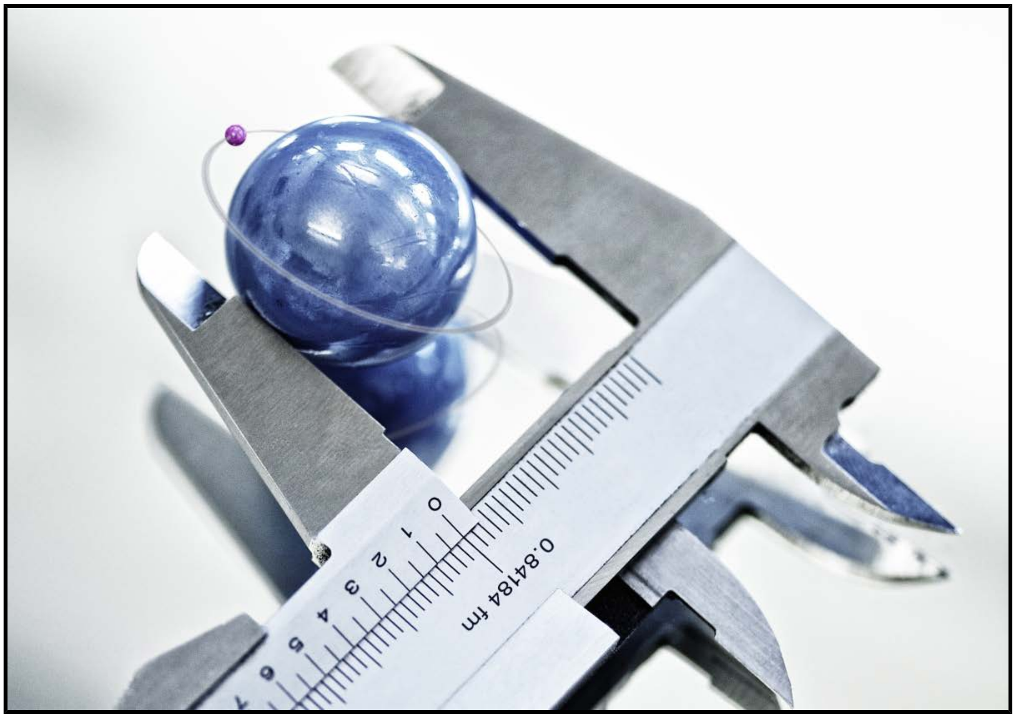
We can also factor the same number another way:

$$11111111111111111111111111111111111 = 1111111 \times 10000001000000100000010000001$$

That's seven 1's, times a number with 1 every seventh time and a total of five 1's.

August 12, 2016

How big is a proton?



We thought we knew. New measurements say we were 4% off. That may not seem like much — but it's enough to be a serious problem!

We can measure the proton radius by bouncing electrons off it, or by carefully studying the energy levels of a hydrogen atom. People have measured it many times, and the different measurements agree pretty well. Here's the answer:

$$0.8775 \pm 0.0051 \text{ femtometers}$$

A **femtometer** is 10^{-15} meters, or a quadrilionth of a meter.

But you can make a version of hydrogen with a muon replacing the electron. The muon is the electron's big brother. It's almost the same, but 207 times heavier. So, muonic hydrogen is about 1/207 times as big across. And that makes the effects of the proton radius easier to detect!

So, in principle, we should be able to measure the radius of a proton more accurately using muonic hydrogen.

So that's what they did — in Switzerland, back in 2010. They repeated the experiment in 2013. Here's what they got:

$$0.84087 \pm 0.00039 \text{ femtometers}$$

In theory, this is about ten times more accurate. However, it's way off from all the earlier measurements! 7 standard deviations off, in fact.

This story is in the news again today. The same team just used muons to measure the radius of deuterium — a proton and a neutron stuck together. And again, they're getting a different answer than what people get using electrons.

Could some new physics be responsible? Some serious mistake in our theory of particles? The guy who led the new experiment, Randolph Pohl, said:

> That would, of course, be fantastic. But the most realistic thing is that it's not new physics.

I like that. A good experimentalist does *not* jump to dramatic conclusions. Pohl guesses that we're wrong about the value of the Rydberg constant, a number that goes into calculating the proton mass from the experimental data. Another possibility, of course, is that he and his team are making some systematic error. It would be nice to have some completely different group of people check their results.

However, it's worth noting that there's another puzzle about muons. Electrons and muons are like tiny magnets. When we calculate how strong the magnetic field of an electron is, we get results that match experiment incredibly well. But when we do the same calculation for the muon, we're off by 3.4 standard deviations.

So maybe, *just maybe*, there's something funny going on with muons, which hints at new physics beyond the Standard Model. I doubt it. But you never know.

Check out this for more:

- Natalie Walchover, [New measurement deepens proton radius puzzle](), *Quanta*, August 11, 2016.

If our estimate of the Rydberg constant were 4 standard deviations off, that would do the job. That sounds like a lot... but if you look at the graphs here, you'll see other cases when we were way off about things!

For even more, check out this:

- Carl E. Carlson, [The proton radius puzzle]().

There are also some interesting comments on my [G+ post]().

August 14, 2016

On Thursday, NASA will announce a planet orbiting Proxima Centauri — the star closest to our Sun, 4.24 light years away. They're trying to make this planet sound like Earth... and that's cool. But I'll tell you some ways it's not.

Mainly, Proxima Centauri is really different from our Sun!

It's a red dwarf. It puts out just only 0.17% as much energy as our Sun. So any planet with liquid water must be very close to this star.

And because it's cooler than the Sun, Proxima Centauri mainly puts out infrared light — in other words, heat radiation. Its visible luminosity is only 0.005% that of our Sun!

So if you were on a planet as warm as our Earth orbiting Proxima Centauri, it would look very dim — about 3% as bright as our Sun.

Of course, if there's life on this planet, it would probably evolve to see infrared.

But there's a more serious problem. Proxima Centauri sometimes puts out big flares, with lots of X-rays! That's not very nice.

Why does a wimpy little red dwarf have bigger flares than the Sun?

The Sun has a core where fusion happens, and helium produced down in the core mainly stays there. A red dwarf doesn't have a core: it's fully convective. In other words, heat moves through the star not by radiation, but by hot gas actually moving up to the surface.

All this ionized gas moving around makes big magnetic fields. The magnetic field lines get twisted up and sometimes explode out in flares! These flares get so hot that they emit X-rays. That's very hot.

Our Sun has flares too, but on a smaller scale. Even on a calm day, Proxima Centauri puts out as much X-ray energy as our Sun. But when a big flare occurs, it can put out 10 times more. This happens pretty often.

So: any 'Earth-like' planet orbiting this star will be a lot closer than the Earth is to our Sun, and get a lot more X-rays.

Puzzle 1. Use what I've told you to estimate how much closer a planet must be, to be at the same temperature as the Earth.

Puzzle 2. Estimate how much more X-rays it will get.

On top of this, Proxima Centauri could be part of a triple star system!

The closest neighboring stars, Alpha Centauri A and B, orbit each other every 80 years. One is a bit bigger than the Sun, the other a bit smaller. They orbit in a fairly eccentric ellipse. At their closest, their distance is like the distance from Saturn to the Sun. At their farthest, it's more like the distance from Pluto to the Sun.

Proxima Centauri is fairly far from both: a quarter of a light year away. That's about 350 times the distance from Pluto to the Sun! We're not even sure Proxima Centauri is gravitationally bound to the other stars. If it is, its orbital period could easily exceed 500,000 years.

On the bright side, Proxima Centauri will last a lot longer than our Sun. As it ages, it will get smaller and hotter, gradually changing from red to blue. After about *four trillion years* it will grow to 2.5% of the Sun's luminosity. When its hydrogen is exhausted, it will then become a white dwarf, without ever puffing up into a red giant like our Sun.

So, any planet orbiting this star will be a weirdly different world. But if we ever get there, we could stay for trillions of years, long after our Sun has become a red giant, roasting life on Earth.

For rumors of NASA's announcement, see this:

- Scientists to unveil new Earth-like planet: report, *Phys.org*, August 12, 2016.

For more, try these:

- Wikipedia, Proxima Centauri.

- Wikipedia, Habitiability of red dwarf systems.
- M. Guedel, M. Audard, F. Reale, S.L. Skinner and J.L. Linsky, Flares from small to large: X-ray spectroscopy of Proxima Centauri with XMM-Newton, *Astron. Astrophys.* **416** (2004), 713–732.

The picture of a cold desert on a planet orbiting Proxima Centauri was apparently created by Space Engine, a program by Vladimir Romanyuk.

For answers to the puzzles, see my G+ post.

August 18, 2016

We live in a world of shadowy struggles. A team of hackers called the Equation Group has remarkable powers:

- They can reprogram your hard drive firmware. This lets them put software on your machine that will survive even if you reformat your hard drive and reinstall your operating system. They can create an invisible, persistent area in your hard drive, store data there, and collect it later.

- They can retrieve data from networks not connected to the internet. They can use an infected USB stick with a hidden storage area to collect information from a computer. When this USB stick is later plugged into a computer they've subverted that does have an internet connection, they can retrieve this information.

- Since 2001, the Equation Group has infected thousands of computers in over 30 countries, focusing on

> government and diplomatic institutions, telecommunications, aerospace, energy, nuclear research, oil and gas, military, nanotechnology, Islamic activists and scholars, mass media, transportation, financial institutions and companies developing encryption technologies.

They also seem to be connected to StuxNet, the computer worm that destroyed centrifuges used by the Iranians for uranium enrichment.

Given all this, it's a good guess that the Equation Group is closey connected to the NSA, the National Security Agency of the US. I sort of hope so — because while that's scary, the alternatives scare me more.

Now another mysterious group called Shadow Brokers has released 256 megabytes of hacking tools that may be used by the Equation Group — and has offered to sell the rest for $500 million! They wrote:

> We follow Equation Group traffic. We find Equation Group source range. We hack Equation Group. We find many many Equation Group cyber weapons. You see pictures. We give you some Equation Group files free, you see. This is good proof no? You enjoy!!! You break many things. You find many intrusions. You write many words. But not all, we are auction the best files.

At first researchers doubted that these guys had been able to steal software from the Equation Group. But new research at the cybersecurity firm Kaspersky Labs seems to confirm it.

Read this for more:

- Dan Goodin, Confirmed: hacking tool leak came from "omnipotent" NSA-tied group, *Ars Technica*, August 17, 2016.

Also try these:

On how the Equation Group was found by researchers at Kapersky Labs last year:

- Dan Goodin, How "omnipotent" hackers tied to NSA hid for 14 years.and were found at last, *Ars Technica*, February 17, 2015.

- Kaspersky Labs, Equation group: the crown creator of cyber-espionage, February 16, 2015.

On how Shadow Brokers released 256 megabytes of hacking software on their blog:

- Dan Goodin, Group claims to hack NSA-tied hackers, posts exploits as proof, *Ars Technica*, August 16, 2016.

Wikipedia is collecting information on the Equation Group here, so this should eventually be the best place to access information about them:

- Wikipedia, Equation Group.

August 20, 2016

Today we're half-way through Hungry Ghost Month here in Singapore! This is the time when ghosts and spirits of deceased ancestors come out from the lower realm... and pester us if we haven't been feeding them with enough offerings.

Today seems to have been the main festival day, the Ghost Festival. There was a lot of fun stuff happening today in the local market! People are giving offerings to the kings of the underworld:

People are praying and giving offerings to their ancestors:

There were monks in white robes walking around. And there was live music — drums, clackers, cymbals and a horn!

But as usual, the market was also selling lots of food to the hungry people out there, like Lisa and me:

So we had Assam laksa for breakfast and enjoyed the music and ceremonies, then did our grocery shopping.

Here's some Assam laksa made by Bee Yinn Low, who has a nice blog on Malay food:

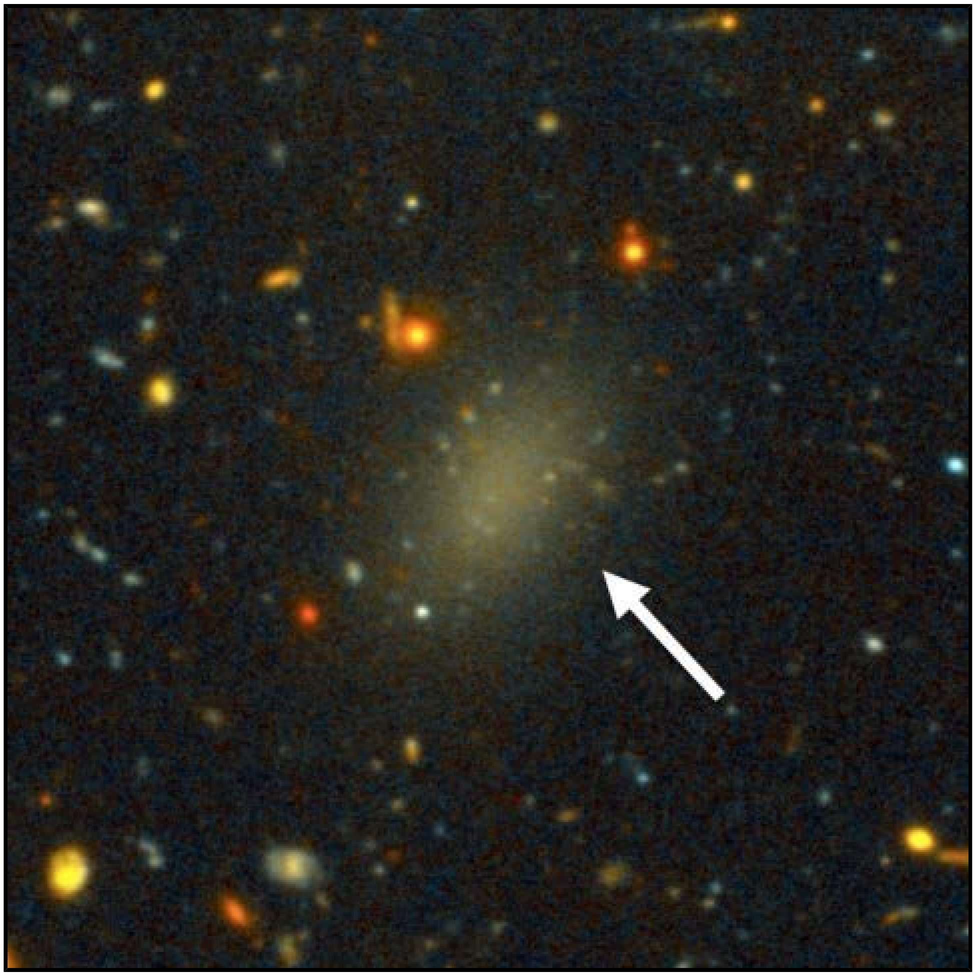
She writes:

> A staple — and arguably the most famous — hawker food in Penang, Penang Assam laksa is very addictive due to the spicy and sour taste of the fish broth. Tamarind is used generously in the soup base and hence the word Assam (means tamarind in Malay). In addition to tamarind, assam keping or peeled tamarind is also

> commonly added to give it extra tartness. Another secret ingredient is Polygonum leaf (marketed as Vietnamese mint leaf in the United States) or daun kesom/daun laksa. While the best Assam laksa broth is infused with the aromatic ginger flower (bunga kantan), I made without it because I couldn't find this special ingredient in the market. Of course, no Assam Laksa is complete without belacan and dollops of heh ko/prawn paste (the dark paste on the spoon).

The stuff we had was quite different — no Vietnamese mint, more sardines, really fat white noodles — but it still had that sweet sour flavor of tamarind. Very good! And very different than the usual Singaporean laksa, which is made with coconut milk.

August 27, 2016



This week some astronomers published an exciting paper: they found a galaxy that's 98% dark matter.

It's called Dragonfly 44. It's extremely faint, so it doesn't have many stars. But we can use redshifts to see how fast those stars are moving — over 40 kilometers per second on average. If you do some calculations, you can see this galaxy would fly apart unless there's a lot of invisible matter providing enough gravity to hold it together. (Or unless something *even weirder* is happening.)

Something similar is true for most galaxies, including ours. What makes Dragonfly 44 special is that *98 percent* of the matter must be invisible. And this is just in the part where we see stars. If we count the outer edges of the galaxy, the halo, the percentage could rise to 99% or more!

For comparison, the Milky Way is roughly 90% dark matter if you count the halo. We know this pretty well, because we can see a few stars out in there and measure how fast they're moving.

There are also galaxies like Messier 105 that may have less than the average amount of dark matter in their halo, though this is debatable.

And most excitingly, sometimes clusters of galaxies collide and stop moving, but their dark matter keeps on going!

We can see this because light from more distant galaxies is bent, not toward the colliding clusters, but toward something else. The most famous example is the Bullet Cluster, but there are others.

All these discoveries — and more — make dark matter seem more and more like a real thing. So it's more and more frustrating that we don't know what it is. As I explained a while ago, recent experiments to detect particles of dark matter have failed. So it could be something else, like black holes about 30 solar masses in size. And intriguingly, the first black hole collision seen by LIGO involved a 35-solar-mass and a 30-solar-mass black hole. These are too big to have formed from the collapse of a single star. They might be primordial black holes, left over from the early Universe.

But more on that later.

For more on Dragonfly 44, see:

- Pieter van Dokkum, Roberto Abraham, Jean Brodie, Charlie Conroy, Shany Danieli, Allison Merritt, Lamiya Mowla, Aaron Romanowsky and Jielai Zhang, A high stellar velocity dispersion and ~100 globular clusters for the ultra diffuse galaxy Dragonfly 44.

For our failure to find dark matter particles, see my July 31 diary entry. For more on dark matter on the outer edges of galaxies, see:

- Wikipedia, Dark matter halo.

For the Milky Way's dark matter halo, see:

- G. Battaglia *et al*, The radial velocity dispersion profile of the Galactic halo: constraining the density profile of the dark halo of the Milky Way.

August 23, 2016

Dolphins do this. Why? Maybe just for fun. But people actually debate this question. Here's what they say at Dolphins-World:

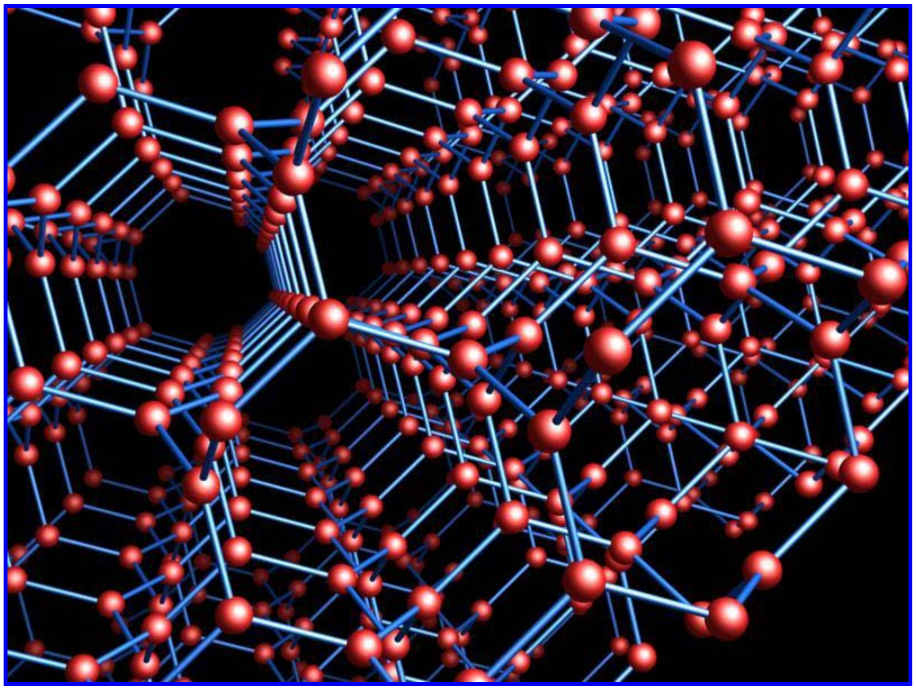
> Why do dolphins jump out of the water?
>
> There is an ongoing debate about why dolphins jump out of the water. Scientists think about different reasons for this behavior.
>
> Among them, some think that dolphins jump while traveling to save energy as going through the air

> consume less energy than going through the water.
>
> Some other think that jumping is to get a better view of distant things, mainly food. So, in this way, dolphins jump to locate food or food related activity like seagulls eating or pelicans hunting.
>
> Other explanation suggest that dolphins use jumping to communicate either with a mate or with another pod.
>
> Some people even think that dolphins jump for cleaning, trying to get rid of parasites while jumping.
>
> Finally some scientists think that they are only having good fun, as playing helps to keep senses at their best.

The idea that this double flip "saves energy" would be idiotic. The idea that they can find prey better by jumping out of the water than by using their sonar underwater also seems implausible. The other ideas are possible. But I think it's likely that all sufficiently intelligent life forms do stuff "just for fun". There are plenty of good biological reasons for this, I think.

And if you've ever seen the amazing games dolphins play with air bubbles, you'll know what I mean. If you haven't, check this out:

August 28, 2016



Greg Egan and I spent a lot of time working on 'topological crystals', but at some point he decided not to be a coauthor of the paper we were writing. So I finished it up myself, and now you can read about the idea here:

- Part 1 - the basic idea.
- Part 2 - the maximal abelian cover of a graph.
- Part 3 - embedding topological crystals.
- Part 4 - examples of topological crystals.

For my September 2016 diary, go here.

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home

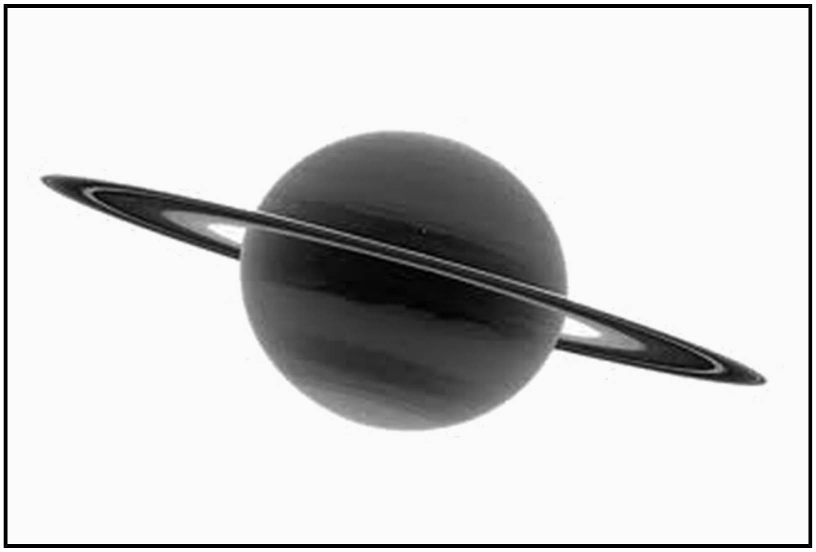
[For my August 2016 diary, go here.]

Diary — September 2016

John Baez

September 2, 2016

Black Saturns



Imagine a black hole with a black ring. Physicists call such a thing a 'black Saturn'.

Nobody has ever seen one. But we can still study them.

You see, we know the equation that describes black holes. It's called Einstein's equation, the basic formula in Einstein's theory of gravity.

We know this equation has solutions with a round event horizon — a surface that you can't escape if you fall through it. These are black holes. And we've seen plenty of black holes — or at least the hot gas falling into black holes.

Could there be a black ring — an event horizon shaped like a ring? It would need to spin so it wouldn't collapse.

Nobody has ever seen a black ring... and there's a reason why! They're mathematically impossible. There's no solution of Einstein's equation that describes a black ring just sitting there, or just spinning but staying the same shape. Physicists have known this since the 1970's. The options for stationary black-hole-like solutions are very limited. You can have a black hole that just sits there, or you can have one that spins... and it can also have electric charge if you want. That's it.

But suppose we had an extra dimension.

Suppose space were 4-dimensional, instead of 3-dimensional. We can still write down Einstein's equation and try to solve it. You can still get round black holes. But in 2001, two physicists proved that black rings are also possible!

Once you have round black holes and black rings, it's irresistible. You've got to see if you can create a black Saturn! Can you get a black ring to orbit a black hole?

Yes you can! In 2007, Henriette Elvanga and Pau Figueras found black Saturn solutions of Einstein's equations in 4d

 n questions. Can you get the ring to rotate the opposite way than the black hole is spinning? Can you get a black hole with more than one ring orbiting it? Are black Saturns stable, or unstable? And so on.

You might say this is just a game. Or you might say it's important to understand what's so special about 3-dimensional space. Either way, it's pretty cool.

Puzzle 1: Could a ring of dust be stable if there weren't a planet in the middle? Does having a planet inside help stabilize the ring — and if so, how?

I think The Black Saturns would be a good name for a band... and here's one reason why:

Puzzle 2: Why does the phrase 'black Saturn' make sense in terms of astrology? A hint: Jupiter, or Jove, was supposedly responsible for making people 'jovial', or happy.

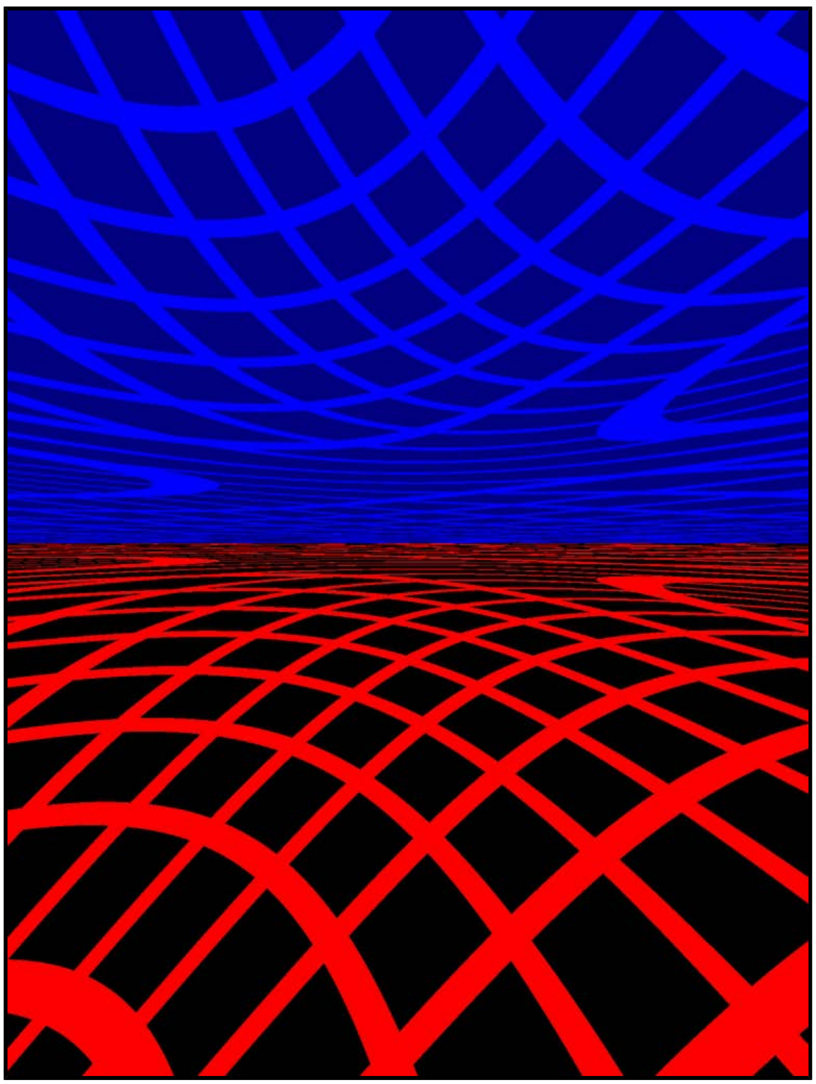
Here is the first paper on black rings:

- R. Emparan and H. S. Reall, A rotating black ring in five dimensions, *Phys. Rev. Lett.* **88** (2002) 101101.

and here's the first paper on black Saturns:

- Henriette Elvang and Pau Figueras, Black Saturns, *JHEP* (2007), 0705:050.

The photon sphere



A nonrotating black hole is surrounded by an imaginary sphere called the event horizon. If you cross this sphere, you are doomed to fall in.

If you carry a flashlight and try to shine light straight out, light emitted at the instant you cross the event horizon will basically stay there! Why? Because to stay on the horizon you must move outwards at the speed of light. As the Red Queen said in Alice in Wonderland:

"Now, here, you see, it takes all the running you can do, to keep in the same place."

But there's another imaginary sphere outside the event horizon, called the 'photon sphere'. This is where light can go in circles around the black hole!

This picture by David Madore shows the view from the photon sphere. The black hole occupies exactly half the sky! As he says:

> This is the distance at which, for an observer standing still, the black hole occupies precisely one half of the visual field. This is because it is the distance at which photons themselves will orbit the black hole circularly (this orbit is unstable, however).

In other words, the horizon is the distance at which photons emitted outward from the black hole are standing still, whereas the photon sphere is the distance at which photons emitted orthogonally from the black hole remain at this constant distance and circle around the black hole in an orbit: but since light rays always appear to be straight, to an observer standing still on the photon sphere, the photon sphere seems like an infinite plane, with the black hole occupying half of space beyond it, and the outside world occupying the other half of space.

Now I should admit, as David does, that it's unstable for light to stay exactly on the horizon, or to orbit the photon sphere in a circle. It's like balancing a pencil on its tip! In reality you can't make things so perfect.

And this is especially true because light is a wave, not a particle - so it doesn't have a precise location, it's always a bit smeared out. So, if you have a beam of light orbiting the photon sphere, it will spread out. Some will fall in, and some will escape outwards. I highly recommend David Madore's page on black holes:

- David Madore, Kerr black holes images and videos.

and if I have the energy I will try to explain more about them here. They're on my mind right now, because I'm writing a paper where I discuss them.

The photon sphere of a nonrotating black hole is one and a half times as big across as the event horizon. The radius of the event horizon is called the 'Schwarzschild radius' and it's

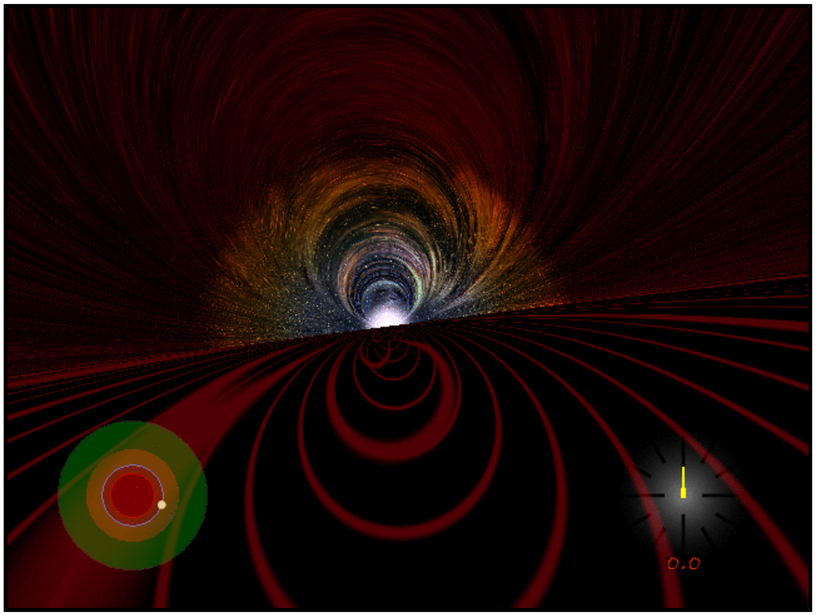
$$\frac{2Gm}{c^2}$$

where m is the black hole's mass, G is Newton's gravitational constant and c is the speed of light. The radius of the photon sphere is

$$\frac{3Gm}{c^2}.$$

September 11, 2016

Just above the photon sphere



This gif shows what it's like to orbit a non-rotating black hole just above its photon sphere.

That's the imaginary sphere where you'd need to move at the speed of light to maintain a circular orbit. At the photon sphere, the horizon of the black hole looks like a perfectly straight line!

But since you can't move at the speed of light, this gif shows you orbiting slightly above the photon sphere, a bit slower than light.

We cannot go to such a place — not yet, anyway. The gravity would rip us to shreds if we tried. But thanks to physics, we can figure out what it would be like to be there! And that is a wonderful thing.

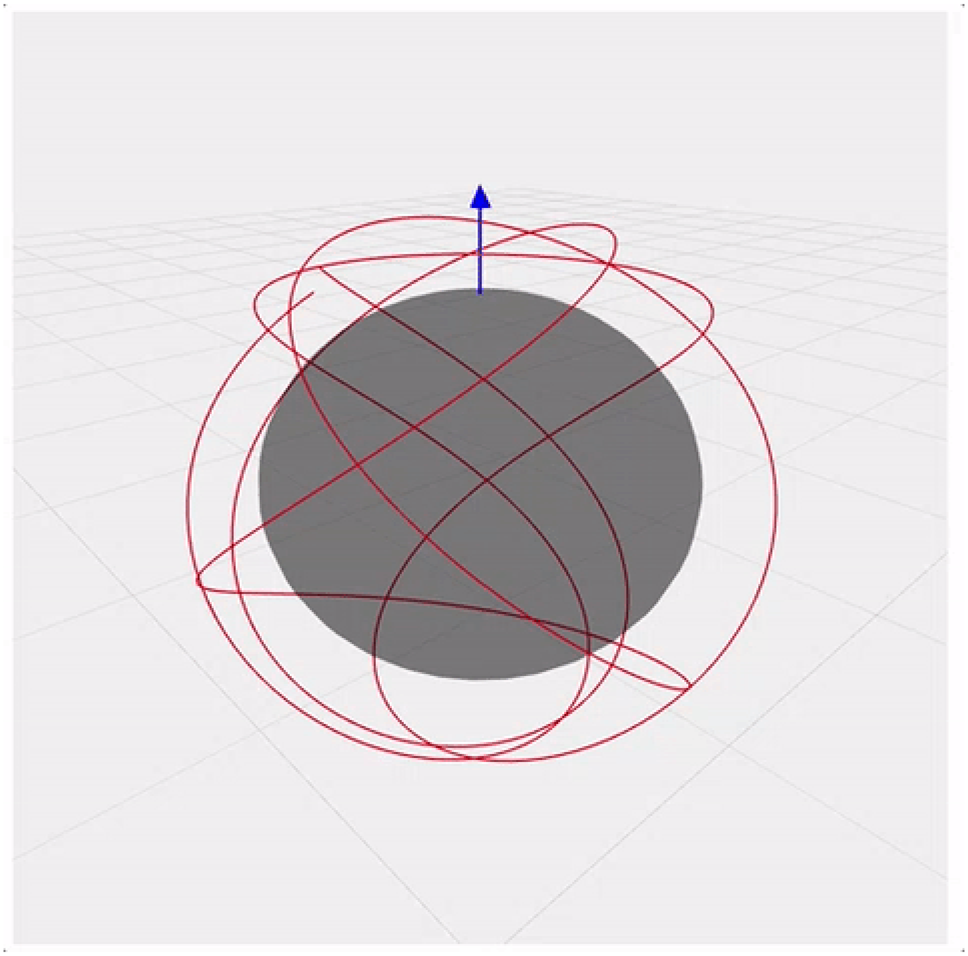
The red stuff drawn on the black hole is just to help you imagine your motion. You would not really see that stuff.

The light above the black hole is starlight — bent and discolored by your rapid motion and the gravitational field of the black hole.

This gif was made by Andrew Hamilton, an expert on black holes at the University of Colorado. You can see a lot more explanations and movies on his webpage:

- Andrew Hamilton, Journey into a Schwarzschild black hole.

September 16, 2016



This gif by Leo Stein shows a photon orbiting a black hole. Since the black hole is rotating, the photon traces out a complicated path. You can play around with the options here:

- Leo C. Stein, Kerr spherical photon orbits.

If a black hole isn't rotating, light can only orbit it on circles that lie on a special sphere: the photon sphere.

But if the black hole is rotating, photon orbits are more complicated! They always lie on some sphere or other — but now there's a range of spheres of *different radii* on which photons can move!

The cool part is how a rotating massive object — a black hole, the Sun or even the Earth — warps spacetime in a way that tends to drag objects along with its rotation. This is called 'frame-dragging'.

Frame-dragging was one of the last experimental predictions of general relativity to be verified, using a satellite called Gravity Probe B. Frame-dragging was supposed to make a gyroscope precess a bit more. This experiment was really hard. It suffered massive delays and cost overruns. When it was finally done, the results were not as conclusive as we'd like. I believe in frame-dragging mainly because everything else about general relativity works great, and it's hard to make up a theory that differs in just this one prediction.

It's pretty bizarre that instead of following orbits that move in and out from the black hole — like ellipses, or something similar — photons can move only in orbits of constant radius, with a range of different possible radii being allowed. Leo Stein explains:

> After you study the radial equation, you learn that the only bound photon trajectories — that is, orbits! — are those for which r = constant in Boyer-Lindquist coordinates. This is why these photon orbits are sometimes called 'circular' or 'spherical'.

> In the end, you see that for each angular momentum parameter a for the black hole, there is a one-parameter family of trajectories given by the radius r, which must be between the two limits
>
> $$r_1(a) \leq r \leq r_2(a)$$
>
> The innermost photon orbit is a prograde circle lying in the equatorial plane, and the outermost orbit is a retrograde circle lying in the equatorial plane.

'Prograde' means that the orbit goes around the same way that the black hole is rotating; retrograde means it moves in the opposite direction.

These orbits are all unstable. Push the photon slightly inward and it will fall into the black hole. Push it outward just a bit and it will fly away. So, this stuff is mainly interesting for the math. You won't actually find a lot of light orbiting a black hole.

For more of the math, see Leo Stein's website. It's great! But the most fun part is using some sliders to play with photon orbits. For more on frame-dragging, see:

- [Frame-dragging](), Wikipedia.

At first I didn't understand how the photon orbiting a rotating black hole has orbits of different allowed radii, with the radius of each orbit being constant as a function of time. But after a conversation [on G+](), I think I get it now.

The 'radial equation' Stein mentions expresses conservation of energy. Usually for an orbiting object in a Newtonian potential this equation takes a form roughly like this:

$$\dot{r}^2 + U(r) = \text{constant}$$

where the effective potential U is concave up with a single minimum, so the radial distance r oscillates. But in this case U is concave down with a single maximum, so r either sits still on top of that maximum or rolls downhill to 0 or infinity.

That's not surprising, since that's what happens already with a photon orbiting a nonrotating black hole. The photon either stays on the photon sphere, or it spirals into the black hole, or it spirals out to infinity.

What's new must be this: the precise form of $U(r)$ depends on some angle that says the 'slant' at which the photon crosses the equator of the rotating black hole. The location of the maximum of $U(r)$ depends on this slant angle. So, depending on this slant angle, we get orbits of different radii.

September 17, 2016

Exploring black holes — with cats!

There should be a series of videos exploring black holes with cats.

So far all we have is this gif made by Dragana Biocanin. A cat can orbit just above the photon sphere of a non-rotating black hole, moving at almost the speed of light. It's impossible for a cat to orbit below the photon sphere. As long as it's outside the event horizon it can accelerate upwards and escape the black hole's gravitational pull. But if it crosses the event horizon, it's doomed!

The event horizon is an imaginary surface in spacetime that's defined by this property: once a cat crosses this surface, it can't come back without going faster than light! This property involves events in the future, so there's no guaranteed way for the cat to tell when it's crossing an event horizon.

For example, if two supermassive black holes were shooting toward our Solar System right now and collided in an hour, forming a black hole that swallowed the Earth, at some moment your cat would cross the event horizon. That's the moment when, no matter how hard it tried, it could no longer escape. But this moment could be happening right now, and your cat might not notice! No alarm bells ring at this moment.

What happens inside the event horizon?

For a non-rotating black hole formed by the collapse of matter, the answer is pretty well understood — except at the 'singularity', where the laws of physics we know break down.

Your cat will fall in, getting stretched ever thinner. For a hypothetical non-rotating black hole with the mass of our Sun, once it crosses the event horizon it will hit the singularity in about 10 microseconds. That's not much time!

In fact, all known black holes are heavier than our Sun. If you double the mass of the black hole, you double the amount of time it takes to hit the singularity, and so on. So, for a non-rotating black hole 100,000 times the mass of our Sun, it takes 1 second to hit the singularity after crossing the horizon.

The biggest known black holes are about 30 billion times the mass of our Sun. For a non-rotating black hole this big, it would take three and a half days for your cat to hit the singularity after it crosses the horizon! You might want to send it in with some cat food.

But there's a catch. Real-world black holes are always rotating! This makes them much more complicated. For starters, frame-dragging tends to pull you along with the black hole's rotation.

We began to see that in yesterday's diary entry about photons orbit a rotating black hole. There's not just one photon sphere — there's a bunch!

There's also a region called the 'ergosphere' where frame-dragging becomes so strong that your cat can't stand still. And Penrose discovered something interesting about this.

You can send a cat into the ergosphere with rockets strapped to its back. When it shoots back out, it can carry angular momentum and energy out of the black hole! It's a bit like how we use Jupiter to fling satellites to Pluto — except we're using the rotation rather than the motion of the black hole!

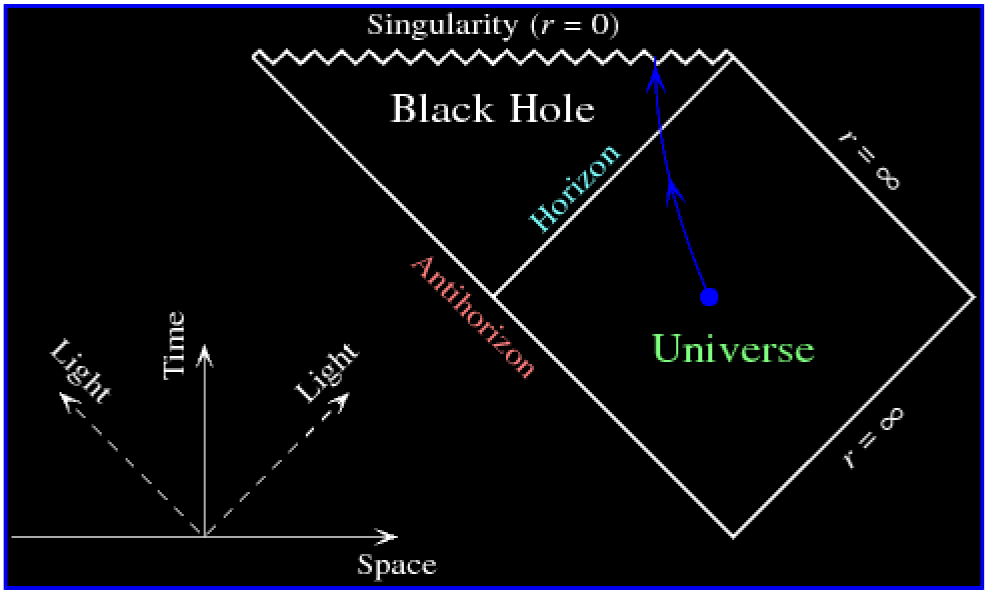
So, we can in theory 'mine' a rotating black hole, removing energy from it until it's not rotating.

Beneath the ergosphere lies the horizon. Inside the horizon of a rotating black hole, things get even weirder. More on that later, I hope. But probably not with cats.

For now, try this:

- Ergosphere, Wikipedia.

September 20, 2016



This is a diagram of a Schwarzschild black hole: a non-rotating, uncharged black hole that has been around forever.

Real-world black holes are different. They aren't eternal — they were formed by collapsing matter. They're also rotating. But the Schwarzschild black hole is simple: you can write down a formula for it. So this is the one to start with, when you're studying black holes.

This is a Penrose diagram. It shows time as going up, and just one dimension of space going across. The key to Penrose diagrams is that light moves along diagonal lines. In these diagrams the speed of light is 1. So it moves one inch across for each inch it moves up — that is, forwards in time.

The whole universe outside the black hole is squashed to a diamond. The singularity is the wiggly line at top. The blue curve is the trajectory of a cat falling into the black hole. Since it's moving slower than light, this curve must move more up than across. So, once it crosses the diagonal line called the horizon, it is doomed to hit the singularity.

Indeed, anyone in the region called "Black Hole" will hit the singularity. Notice: when you're in this region, the singularity is not in front of you! It's in your future. Trying to avoid it is like trying to avoid tomorrow.

But what is the diagonal line called the antihorizon? If you start in our universe, there's no way to reach the antihorizon without going faster than light. But we can imagine things crossing it from the other direction: entering from the left and coming in to our universe!

The point is that while this picture of the Schwarzschild black hole is perfectly fine, we can imagine extending it and putting it inside a larger picture. We say it's not maximally extended.

The larger picture, the maximally extended one, describes a very strange world, where things can enter our universe through the antihorizon. But that's another story, which deserves another picture.

If we stick with the diagram here, nothing can come out of the antihorizon, so it will look black. In fact, to anyone in the "Universe" region, it will look like a black sphere. And that's why a Schwarzschild black hole looks like a black sphere from outside!

The weird part is that this black sphere you see, the antihorizon, is different than the sphere you can fall into, namely the horizon.

If this seem confusing, join the club. I think I finally understand it, but nobody ever told me this — at least, not in plain English — so it took me a long time.

What could be behind the antihorizon? If you want to peek, try Andrew Hamilton's page on Penrose diagrams, where I got this picture:

- Andrew Hamilton, [Penrose diagrams].

I wish that Wikipedia had a really nice Penrose diagram like this! It's very important. They have some more complicated ones, but the most basic important ones are not drawn very nicely. You need to think about Penrose diagrams to understand black holes and the Big Bang!

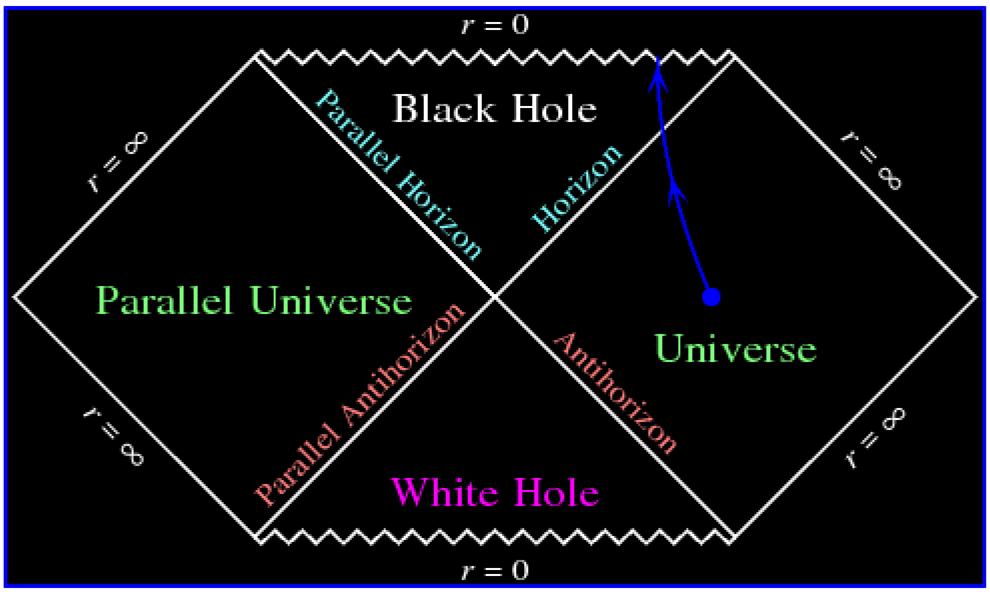
Still, their article is worth reading:

- [Penrose diagram], Wikipedia.

For more on the Schwarzschild black hole, read this:

- [Schwarzschild metric], Wikipedia.

September 21, 2016



Last time I showed you a Schwarzschild black hole... but not the whole hole.

Besides the horizon, which is the imaginary surface that light can only go in, that picture had a mysterious 'antihorizon', where light can only come out. When you look at this black hole, what you actually see is the antihorizon. The simplest thing is to assume no light is coming out of the antihorizon. Then the black hole will look black.

But I didn't say what was behind the antihorizon!

In a real-world black hole there's no antihorizon, so all this is just for fun. And even in the Schwarzschild black hole, you can never actually cross the antihorizon — unless you can go faster than light. So there's no real need to say what's behind the antihorizon. And we can just decree that no light comes out of it.

But inquiring minds want to know... *what could be behind the antihorizon?*

This picture shows the answer. This is the maximally extended Schwarzschild black hole — the biggest universe we can imagine, that contains this sort of black hole.

It's really weird.

It contains not only a black hole but also a white hole. The wiggly lines are singularities. Matter and light can only fall into the black hole from our universe... passing through the horizon and hitting the singularity at the top of the picture. And they can only fall out of the white hole into our universe... shooting out of the singularity at the bottom of the picture and passing through the antihorizon.

If that weren't weird enough, there's also a parallel universe, just like ours.

Someone from our universe and someone from the parallel universe can jump into the black hole, meet, say hi, then hit the singularity and die. Fun!

But we can never go from our universe to the parallel universe. 

Why not? Remember, the only allowed paths for people going slower than light are paths that go more up the page than across the page - like the blue path in the picture. To get from our universe to the parallel universe, a path would need to go more across than up.

If you could go faster than light for just a very short time, you could get from our universe to the parallel universe by zipping through the point in the very middle of the picture, where the horizon and antihorizon meet.

Puzzle 1: Suppose the parallel universe has stars in it more or less like ours. You can't see it from our universe — but you could see it if you jumped into the black hole! What would it look like?

Puzzle 2: How would my story change if the "arrow of time" in the parallel universe pointed the other way from ours? In other words, what if the future for them was at the bottom of the picture, rather than the top?

I should emphasize that we're playing games here, but they're games with rules. We're not talking about the real world, but the math of this stuff is well-understood, so you can't just make stuff up. Or you can, but it might be wrong. These puzzles have right and wrong answers!

Unfortunately I haven't really explained things very well, so you may need to guess the answers instead of just figure them out. For more info, try Andrew Hamilton's page, from which I took this picture:

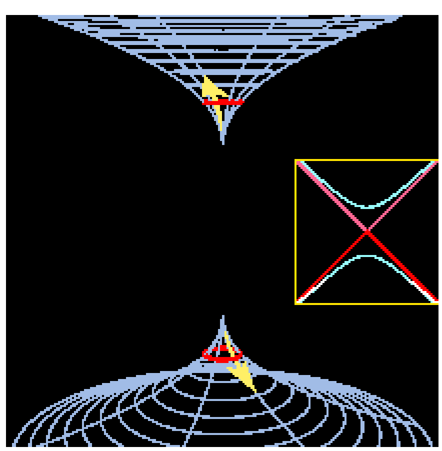
- Andrew Hamilton, Penrose diagrams.

For more on the Schwarzschild black hole, read this:

- Schwarzschild metric, Wikipedia.

September 25, 2016

Black hole + white hole = wormhole



When you learn general relativity — and when they invent immortality, you'll have time — one of the tricky parts is understanding how a black hole and a white hole combine to give a wormhole.

It's hard to get an intuitive feel for it. But this little movie by Andrew Hamilton helps. A bit.

We're quite sure black holes are real. White holes are purely theoretical. The point is this: if you have a solution of the equation of general relativity, and you 'play the movie backwards', switching the future and the past, you get another solution. And if you do this for a black hole, you get a 'white hole'.

Let's see what a white hole would be like.

In a real-world black hole, matter collapses and forms a singularity, where according to the theory spacetime becomes infinitely curved — but in fact, we don't know what really happens. It would be fun to look at a singularity and find out what it's really like. But unfortunately, the singularity is surrounded by an event horizon. That's an imaginary sphere, where if you enter this sphere you can never get back out. You're doomed to fall into the singularity.

You see, when you cross the event horizon, spacetime is so curved that the singularity is not in front of you. It's in your *future* — so trying to avoid it is just like trying to avoid next Tuesday!

In summary, viewed from outside: a bunch of ordinary matter collapses into a small region called a black hole. From then on, nothing ever comes out of the black hole: stuff only falls in. (This is ignoring 'Hawking radiation'.)

Now let's play this movie backwards. We start with a small region called a 'white hole'. Nothing ever goes into this white hole: stuff only comes out. Then, eventually, the white hole explodes into a bunch of ordinary matter!

Astronomers have looked for white holes. They've never seen a thing like this. It's not so surprising: the laws of physics say that theoretically, a scrambled egg could be uncooked and stuck back into the shell — and we don't see that either. Some things seem to be more probable than their time-reversed versions.

But all this was just the warmup.

When you take a class in general relativity, they make you find a solution of general relativity that describes a black hole. And the simplest solution doesn't describe a star collapsing and forming a black hole — that's complicated! The simplest solution describes a black hole that has always been there and always will be. That's a lot simpler, because it doesn't change with time: it's perfectly 'static'. You can solve the equations with pencil and paper, not a supercomputer.

But now look! On the one hand, the time-reversed version of this perfectly unchanging thing is again perfectly unchanging. On the other hand, the time-reversed version of a black hole should be a white hole.

So somehow this solution describes both a black hole and a white hole! You can actually chop this solution into two parts, a black hole part and a white hole part. But they fit together.

If we take only the black hole part, we get a picture like this: a black hole that lasts forever. Stuff can fall though the event horizon, and then it's doomed to hit the singularity. Nothing can come out.

If we take only the white hole part, we get a picture like this: a white hole that lasts forever. Stuff can come out of the singularity and come out through the 'reverse event horizon'. But nothing can go into the reverse event horizon.

It's when we we take both parts that things get funny. Now there are two singularities, one in the past and one in the future. But event horizon and the 'reverse event horizon' are the same thing! This horizon is a sphere. Stuff can fall from our universe into this sphere, hit the future singularity and disappear. But stuff can also appear at the past singularity, shoot out of this sphere and enter our universe!

I hope you sort of understood that. It's weird but it's actually logical and symmetrical. You could have guessed it, if you just kept cool and tried to dream up the most symmetrical possibility.

But here's the part you probably couldn't have guessed: this solution also describes two separate universes, connected by a wormhole!

That's the part that freaks me out. Needless to say, this is not something anyone has ever seen. Right now it's just a solution of the equations that describe gravity. But still, I'd like to understand it.

The movie shows how it works. In the little picture at right:

- the up-down direction is 'time': the future is up, the past is down
- the left-right direction is one dimension of 'space'
- light can only move along diagonal lines
- matter can only move slower than light, meaning more vertically than horizontally
- the blue hyperbola at top is the future singularity
- the orange diagonal lines are the event horizon: if you cross this moving more vertically than horizontally you're doomed to hit the future singularity
- the blue hyperbola at bottom is the past singularity
- the red diagonal lines are the 'reverse event horizon': you can only cross this moving more vertically than horizontally if you came out of the past singularity
- the region to the right of the diagonal lines is 'our universe'
- the region to the left of the diagonal lines is the 'other universe'.

The slice moving up through this little picture shows one way to slice spacetime. That is, it shows the passage of time. The big movie shows that as this happens, the two universes meet and become connected by a wormhole — but then this wormhole snaps and the universes separate!

Unfortunately, you can't actually go from one universe to the other universe. Because you can only go slower than light, once you cross the event horizon you are doomed to hit the future singularity. But before you do, you can meet other doomed people who came from the other universe!

Unfortunately you can never report back and tell people outside the black hole that you met people from another universe... because signals can't get out across the horizon! Bummer.

I should explain this even more, but I'm getting tired, so why don't you just read Andrew Hamilton's description:

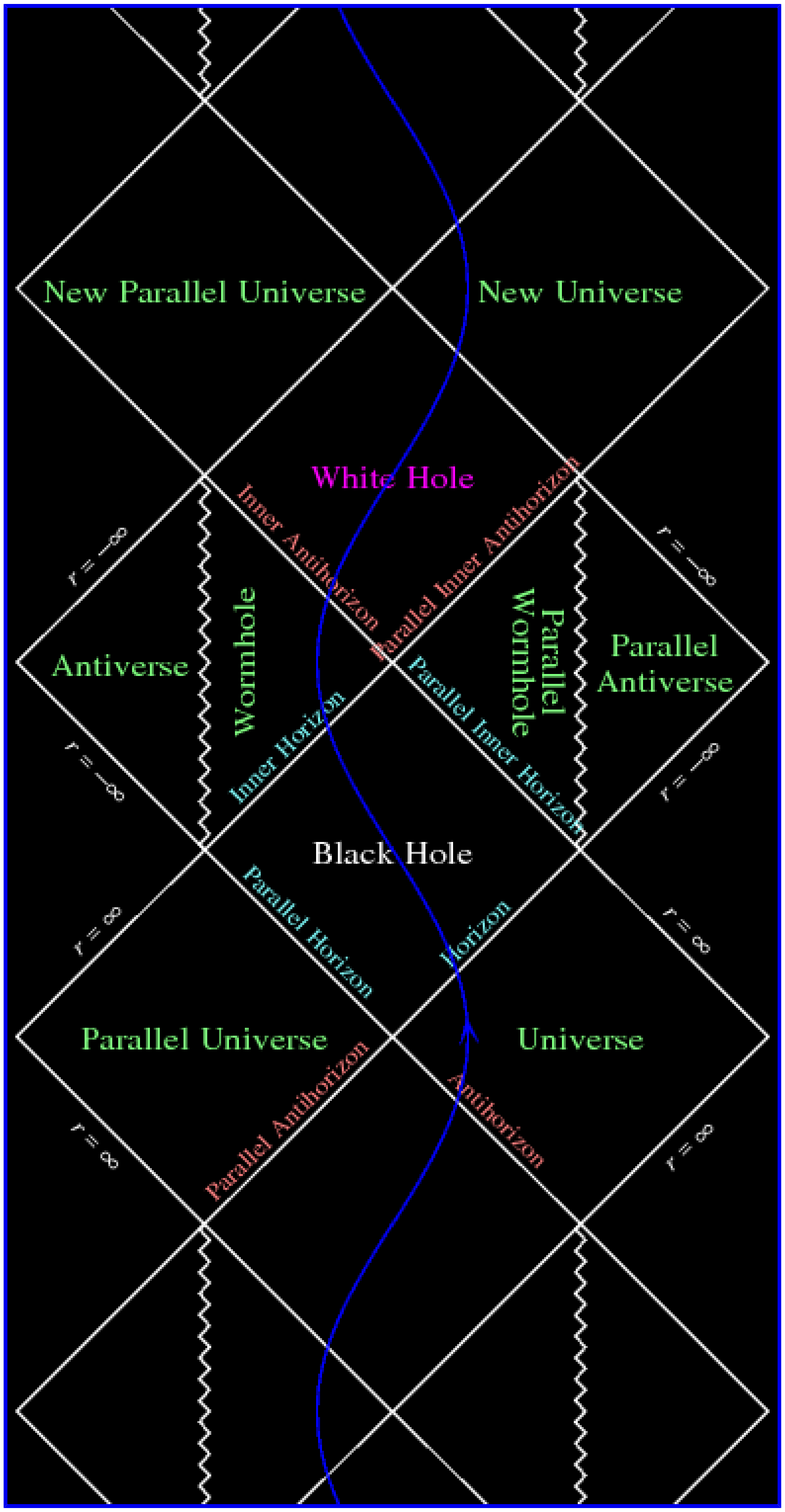
- Andrew Hamilton, Instability of the Schwarzschild wormhole.

There's also a fun discussion in the comments on my G+ post.

By the way, while this gif is a great idea, it's pretty small and a bit scraggly. I think someone should create a better one and put it on Wikicommons. This stuff is so cool everyone should have a chance to learn about it! Even before they invent immortality.

September 26, 2016

An infinite corridor of universes



Einstein's equations for gravity have some amazing solutions. Some describe things we see: the Big Bang and black holes. Others don't — like white holes, wormholes, and the infinite corridor of universes shown here.

As far as we know, all real-world black holes were formed at some moment in time by collapsing matter. But it's easier to find solutions of Einstein's equations that describe an eternal black hole whose shape doesn't change with time.

A rotating eternal black hole is called a Kerr black hole, because this solution of Einstein's equation was first found by Roy Kerr in 1963. However, he just found part of the solution — not the whole picture here!

You see, when you solve Einstein's equations, you get a world obeying the rules of general relativity. But sometimes, if you're not careful, somebody else can find a bigger world that contains yours! It's like you drew a map of the world but you forgot there was anything south of the equator. A solution is called 'maximally extended' if you can't make it any bigger.

This picture shows the maximally extended Kerr solution. It's a Penrose diagram, so moving up the page takes you forward in time, while moving to the right or left edge of the page takes you away from the black hole. Light moves along diagonal lines.

It's a single world, but it has portions called 'Universe', 'Parallel Universe', 'Antiverse', and 'Parallel Antiverse'. Each of these is roughly like our universe, but with no Big Bang. Each lasts forever: time is not drawn to scale.

Each universe, and each parallel universe, has a black hole in it — and also a white hole! Each antiverse, and each parallel antiverse, has a black hole with *negative mass*, and also a white hole with negative mass.

Only a few of these universes and antiverses are shown here. But there are infinitely many. The pattern repeats forever as you continue to go up or down the picture — that is, forwards or backwards in time.

There's also an infinite repeating sequence of black holes and white holes. And there's more — you can see singularities drawn as wiggly lines. But let's not worry about those yet. There's too much to take in at once.

Let's just follow the blue curve as it goes up the page. This describes a path you could take through space and time.

You could shoot out of a white hole at the very bottom of the picture and wind up in our universe.

Then you could jump into the black hole.

If you dodge the singularities, you could wind up in a new white hole!

And at this point, you have a choice. Swerve right and you go into a new universe. Swerve left and you go into a new parallel universe. They're different — but there's no big difference. In this picture, you choose to enter the new universe.

And so on!

It would be great fun if *our* universe were part of a grand infinite corridor of universes like this. As far as we know, it's not. I suspect the real universe will be even more amazing. However, we will need much better science and technology to discover what's out there. Right now most of us are stuck here on Earth, and we need to learn to live here. That's a tough challenge too.

My picture is from Andrew Hamilton's wonderful website:

- Andrew Hamilton, Penrose diagrams.

I would like to tell you more about the Kerr black hole — but if I don't get around to it, also check out David Madore's page:

- David Madore, Kerr black holes: images and videos.

and the discussion in the comments to my G+ post.

September 28, 2016

David Madore has a lot of great stuff on his website — videos of black holes, a discussion of infinities, and more. He has an interesting story that claims to tell you the Ultimate Question, and its Answer. (No, it's not 42.) I like it — but how much sense does it make?

Here's the key part:

> What is the Ultimate Question, and what is its Answer? The answer to that is, of course: "The Ultimate Question is 'What is the Ultimate Question, and what is its Answer?' and its answer is what has just been given.". This is completely obvious: there is no difference between the question "What color was Alexander's white horse?" and the question "What is the answer to the question 'What color was Alexander's white horse?'?". Consequently, the Ultimate Question is "What is the Answer to the Ultimate Question?" — but so that we can understand the Answer, I restate this as "What is the Ultimate Question, and what is its Answer?", at which point it becomes obvious what the Answer is.

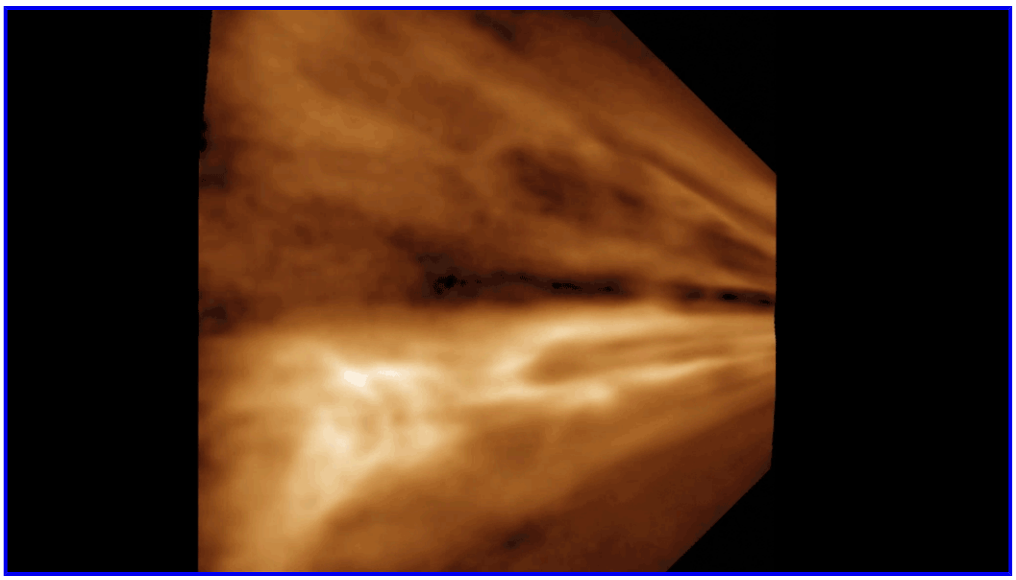
Of course it's meant to be funny. I like it. But I wasn't sure how logical it is. The logic is quite twisty — but how much sense does it make? It's more funny if the logic is sound.

Joel David Hamkins and Mike Shulman helped me figure out what was going on, in part by revealing previous work on this puzzle. To learn all about it, read this:

- John Baez, The ultimate question, and its answer, *The n-Category Café*, September 9, 2016.

and especially the comments. Also try the comments on my G+ post, though they're much less profound.

September 24, 2016



This is the solar wind, the stream of particles coming from the Sun. It was photographed by STEREO. That's the 'Solar Terrestrial Relations Observatory', a pair of satellites we put into orbit around the Sun at the same distance as the Earth, back in 2006. One stays ahead of the Earth, one is behind. Together, they can make stereo movies of the Sun!

One interesting thing is that there's no sharp boundary between the 'outer atmosphere' of the Sun, called the corona, and the solar wind. It's all just hot gas, after all! STEREO has been studying how this gas leaves the corona and forms the solar wind. This picture is a computer-enhanced movie of that process, taken near the Sun's edge.

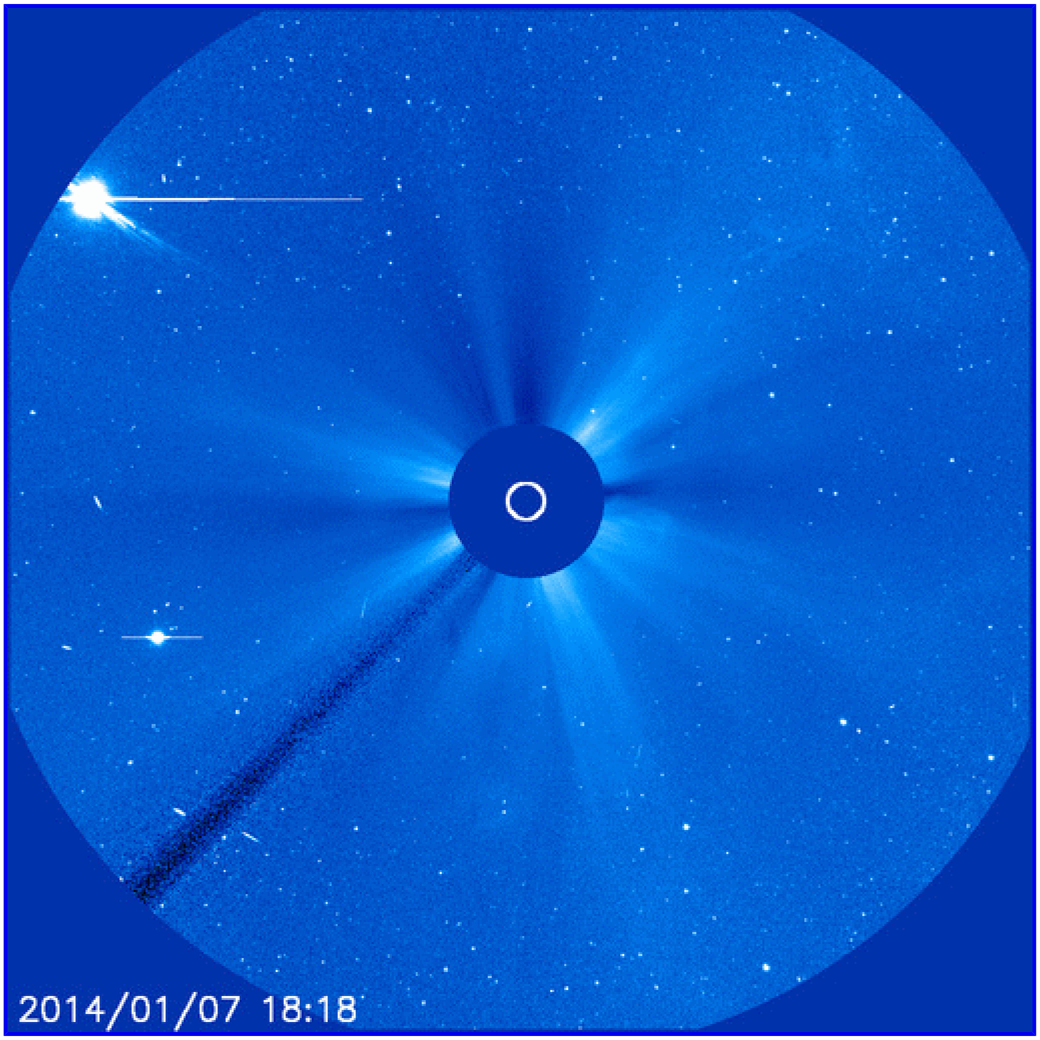
What's the solar wind made of? When you take hydrogen and helium and heat them up so much that the electrons get knocked off, you get a mix of electrons, hydrogen nuclei (protons), and helium nuclei (made of two protons and two neutrons). So that's all it is.

The Sun's corona is very hot: about a million kelvin. That's hotter than the visible surface of the Sun, called the photosphere! Why does it get so hot? When I last checked, this was still a bit mysterious. But it has something to do with the Sun's powerful magnetic fields.

When they're this hot, some electrons are moving fast enough to break free of the Sun's gravity. Its escape velocity is 600 kilometers per second. The protons and helium nuclei, being heavier but having the same average energy, move slower. So, few of these reach escape velocity.

But with the negatively charged electrons leaving while the positively charged protons and helium nuclei stay behind, this means the corona builds up a positive charge! So the electric field starts to push the protons and helium nuclei away, and some of them — the faster-moving ones — get thrown out too.

Indeed, enough of these positively charged particles have to leave the Sun to balance out the electrons, or the Sun's electric charge would keep getting bigger. It would eventually shoot out huge lightning bolts! The solar wind deals with this problem in a less dramatic way — but sometimes it gets pretty dramatic. Check out this proton storm:



When such storms happen, the US government sends out warnings like this:

> Space Weather Message Code: WATA50
> Serial Number: 48
> Issue Time: 2014 Jan 08 1214 UTC
> WATCH: Geomagnetic Storm Category G3 Predicted
> Highest Storm Level Predicted by Day:
> Jan 08: None (Below G1) Jan 09: G3 (Strong) Jan 10: G3 (Strong)
> THIS SUPERSEDES ANY/ALL PRIOR WATCHES IN EFFECT
> Potential Impacts: Area of impact primarily poleward of 50 degrees geomagnetic latitude.
> Induced Currents — Power system voltage irregularities possible, false alarms may be triggered on some protection devices.
> Spacecraft — Systems may experience surface charging; increased drag on low Earth-orbit satellites and orientation problems may occur.
> Navigation — Intermittent satellite navigation (GPS) problems, including loss-of-lock and increased range error may occur.

> Radio — HF (high frequency) radio may be intermittent.
> Aurora — Aurora may be seen as low as Pennsylvania to Iowa to Oregon.

The solar wind is really complicated, and I've just scratched the surface. I love learning about stuff like this, surfing the web as I lie in bed sipping coffee in the morning. Posting about it just helps organize my thoughts — when you try to explain something, you come up with more questions about it.

For more on space weather, visit this fun site:

- Spaceweather.com — news and information about the Sun-Earth environment.

You can see space weather reports here:

- Space Weather Prediction Center, National Oceanic and Atmospheric Administration, Alerts, watches and warnings.

Space weather is probably just as complicated as the Earth's weather! For example, there are really at least two kinds of solar wind. According to Wikipedia:

> The solar wind is divided into two components, respectively termed the slow solar wind and the fast solar wind. The slow solar wind has a velocity of about 400 km/s, a temperature of $1.4\text{–}1.6 \times 10^6$ K and a composition that is a close match to the corona. By contrast, the fast solar wind has a typical velocity of 750 km/s, a temperature of 8×10^5 K and it nearly matches the composition of the Sun's photosphere. The slow solar wind is twice as dense and more variable in intensity than the fast solar wind. The slow wind also has a more complex structure, with turbulent regions and large-scale structures.
>
> The slow solar wind appears to originate from a region around the Sun's equatorial belt that is known as the 'streamer belt'. Coronal streamers extend outward from this region, carrying plasma from the interior along closed magnetic loops. Observations of the Sun between 1996 and 2001 showed that emission of the slow solar wind occurred between latitudes of 30-35° around the equator during the solar minimum (the period of lowest solar activity), then expanded toward the poles as the minimum waned. By the time of the solar maximum, the poles were also emitting a slow solar wind.
>
> The fast solar wind is thought to originate from coronal holes, which are funnel-like regions of open field lines in the Sun's magnetic field. Such open lines are particularly prevalent around the Sun's magnetic poles. The plasma source is small magnetic fields created by convection cells in the solar atmosphere. These fields confine the plasma and transport it into the narrow necks of the coronal funnels, which are located only 20,000 kilometers above the photosphere. The plasma is released into the funnel when these magnetic field lines reconnect.

Even when it reaches Earth, the slow solar wind is too hot for hydrogen atoms to form. Around this distance from the Sun, the temperature of protons in the slow solar wind about 40,000 kelvin, while the temperature of the electrons is about 150,000 kelvin. The temperature it takes to for hydrogen atoms to ionize depends on the density, going to zero at zero density, but these temperatures are high enough to keep it ionized it even at densities much higher than that of the solar wind. So, very few atoms will have formed.

It's interesting that the protons and electrons are so far from equilibrium. That alone proves they haven't bumped into each other enough to equilibriate — much less combine to form atoms.

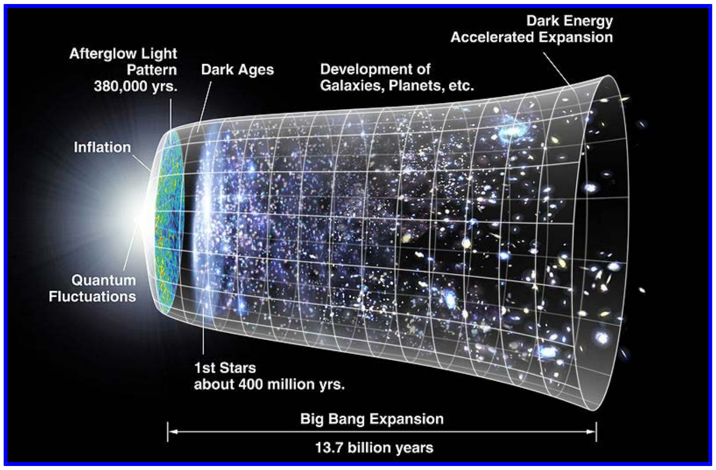
The story for helium is rather similar, but helium nuclei make up only 4% of the slow solar wind. The fast solar wind is a bit cooler, but not much.

The numbers here are from this article:

- Solar wind: global properties

For more, read the comments on my G+ post.

September 25, 2016



For many years I've been wanting to write a paper on 'struggles with the continuum' — that is, the problems in making physical theories mathematically rigorous, due to our assumption that spacetime is a continuum. I offered to contibute such a paper to a book *New Spaces in Mathematics and Physics*, edited by Mathieu Anel and Gabriel Catren. When the time came to write it, I found myself resisting the duty and procrastinating — in part because it made me feel sad that I'm no longer working on 'fundamental physics' of this sort. But once I got into it, I enjoyed it a lot — except at the end, when I needed to learn more general relativity. This made me ashamed I didn't already know this material better! But when I finally bit the bullet and started work on that part, even that was fun. The paper is more or less done now, except for some small improvements I'd like to make. And I broke it up into a series of short articles which I posted both on my own blog and also *Physics Forums*:

- Part 1 — Problems with infinity. Point particles interacting gravitationally.
- Part 2 — The quantum mechanics of nonrelativistic charged point particles.
- Part 3 — The relativistic electrodynamics of point particles.
- Part 4 — The ultraviolet catastrophe, and quantum field theory.
- Part 5 — Quantum field theory: renormalization.
- Part 6 — Quantum field theory: summing over Feynman diagrams.
- Part 7 — General relativity: the singularity theorems.
- Part 8 — General relativity: the cosmic censorship hypothesis. Conclusion.

For my October 2016 diary, go here.

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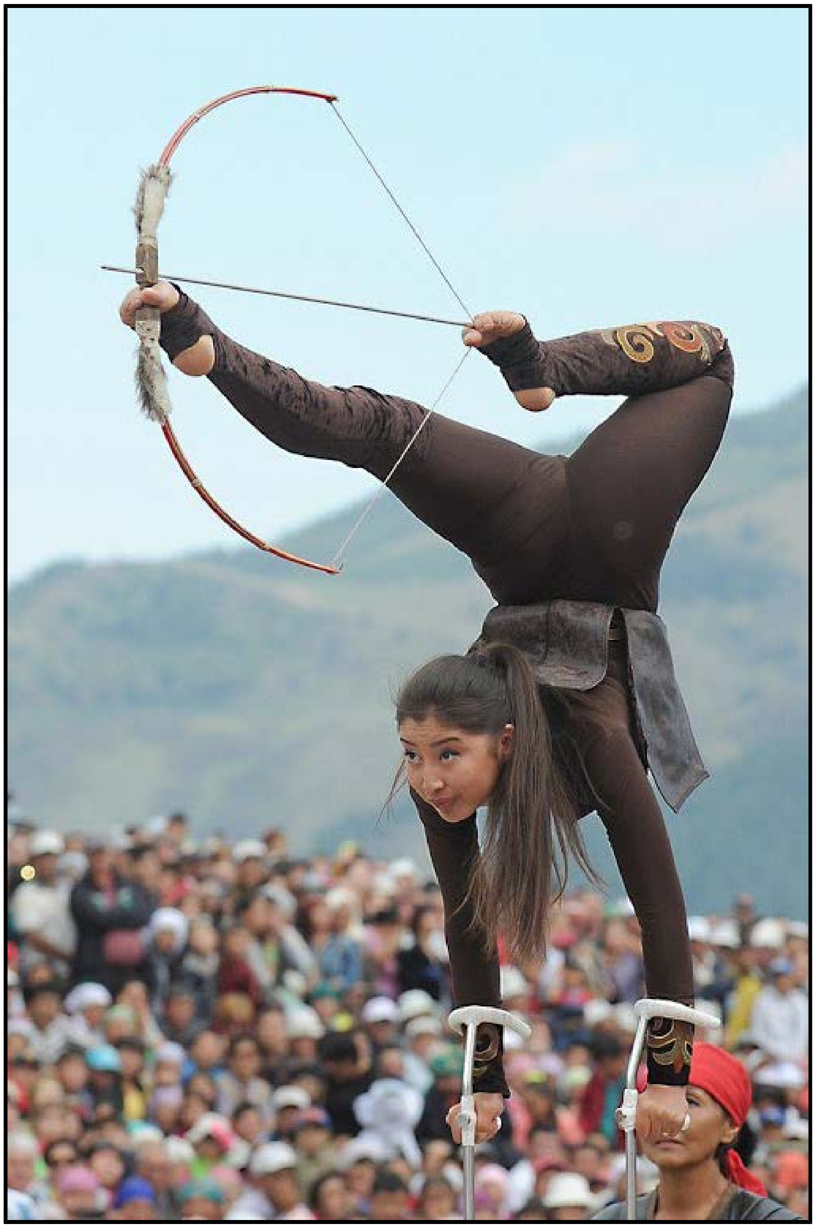
home

For my September 2016 diary, go here.

Diary — October 2016

John Baez

October 1, 2016



Nomads kick ass! James Dator explains:

> The World Nomad Games concluded on Friday in what can only be described as the greatest week-long sporting event on the planet. The games, intended to showcase ethnic sports of Central Asia, featured things you have never heard of, athletes you'll never learn about and sports that sound absolutely terrifying.
>
> There were 16 sports with medals up for grabs. These are the ones that are the absolute wildest.
>
> **Cirit**

This Turkish equestrian sport involves teams of riders chasing each other and throwing javelins at each other while on horseback. Yes, seriously.

Er Enish

It's wrestling, except you're on a horse. You win by pulling your opponent off their horse.

Kok-boru

There's no delicate way to explain Kok-boru. It's horseback basketball using a goat carcass. You win by tossing the dead goat into your opponent's well. It comes from a tradition of beating up wolves that attacked your herd of sheep and throwing a dead wolf to your friends who went wolf hunting with you.

Mas-wrestling

In this form a wrestling, athletes fight over a stick. Each wrestler is given part of the stick to hold and are seated facing each other with their feet on a plank. Whoever gets the stick wins.

Salbuurun

A three-step hunting sport involving animals. Competitions are held in the following disciplines:

- **Burkut saluu** - hunting with golden eagles. Composition of the team - 6 people: 1 leader and 5 berkutchi (hunter with eagles).

- **Dalba oynotuu** - falcon flying to the lure. Composition of the team - 6 people: 1 leader and 5 Kushchu (falconer).

- **Taigan jarysh** - dog racing among breeds of greyhound. Composition of the team - 6 people: 1

leader and 5 owners of dogs.

James Dator explains more games here:

- James Dator, The World Nomad Games are like the Olympics, except with more fire and flying goat carcasses, *SB Nation*, September 12, 2016.

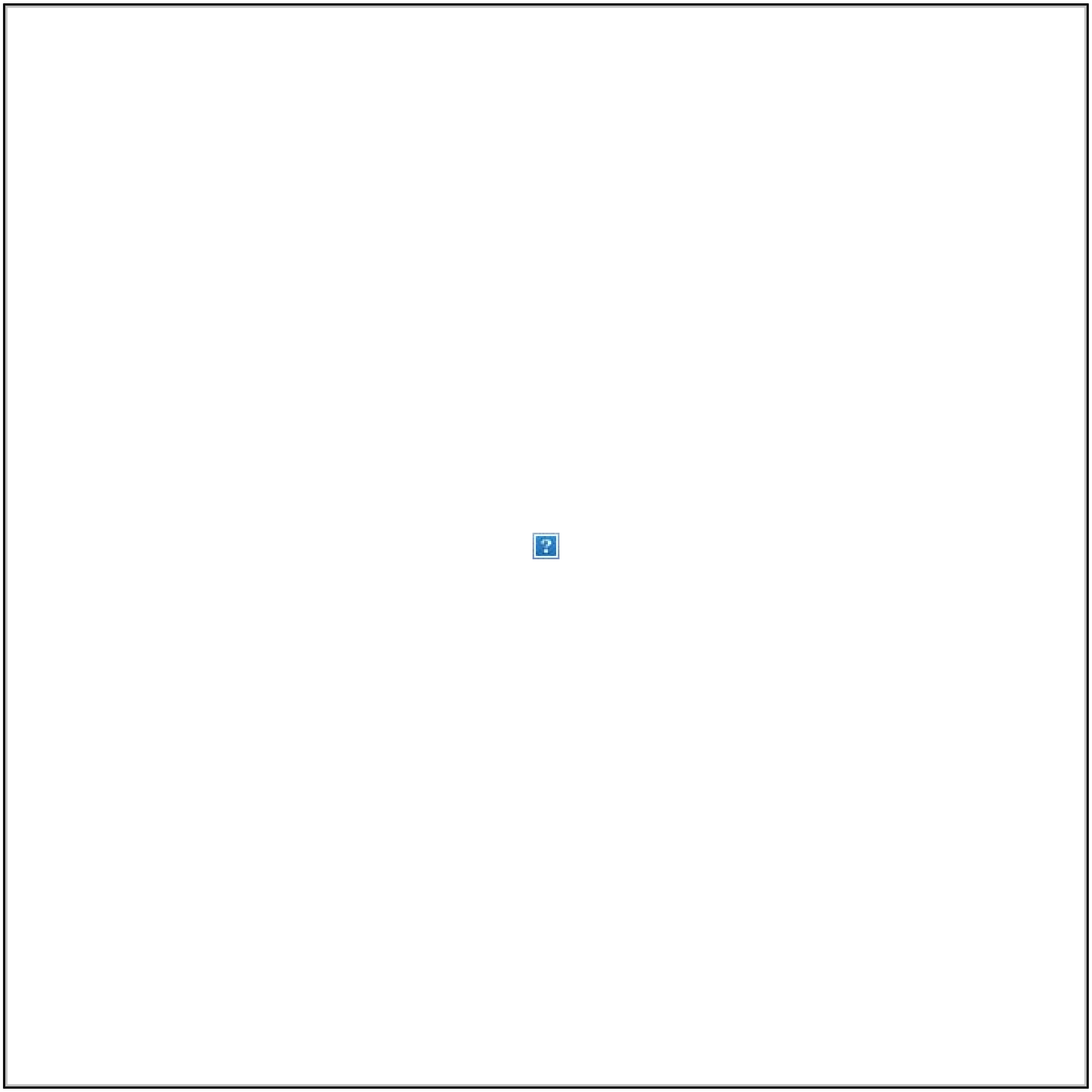
For example, setting yourself on fire and riding a horse!

There's more here:

- Marissa Payne, Like 'rugby on horses' with a decapitated goat: Inside first U.S. team at World Nomad Games, *Washington Post*, September 14, 2016.

Not for the squeamish! However, more excellent pictures of hunters with their eagles, horse riders, etc.

October 12, 2016

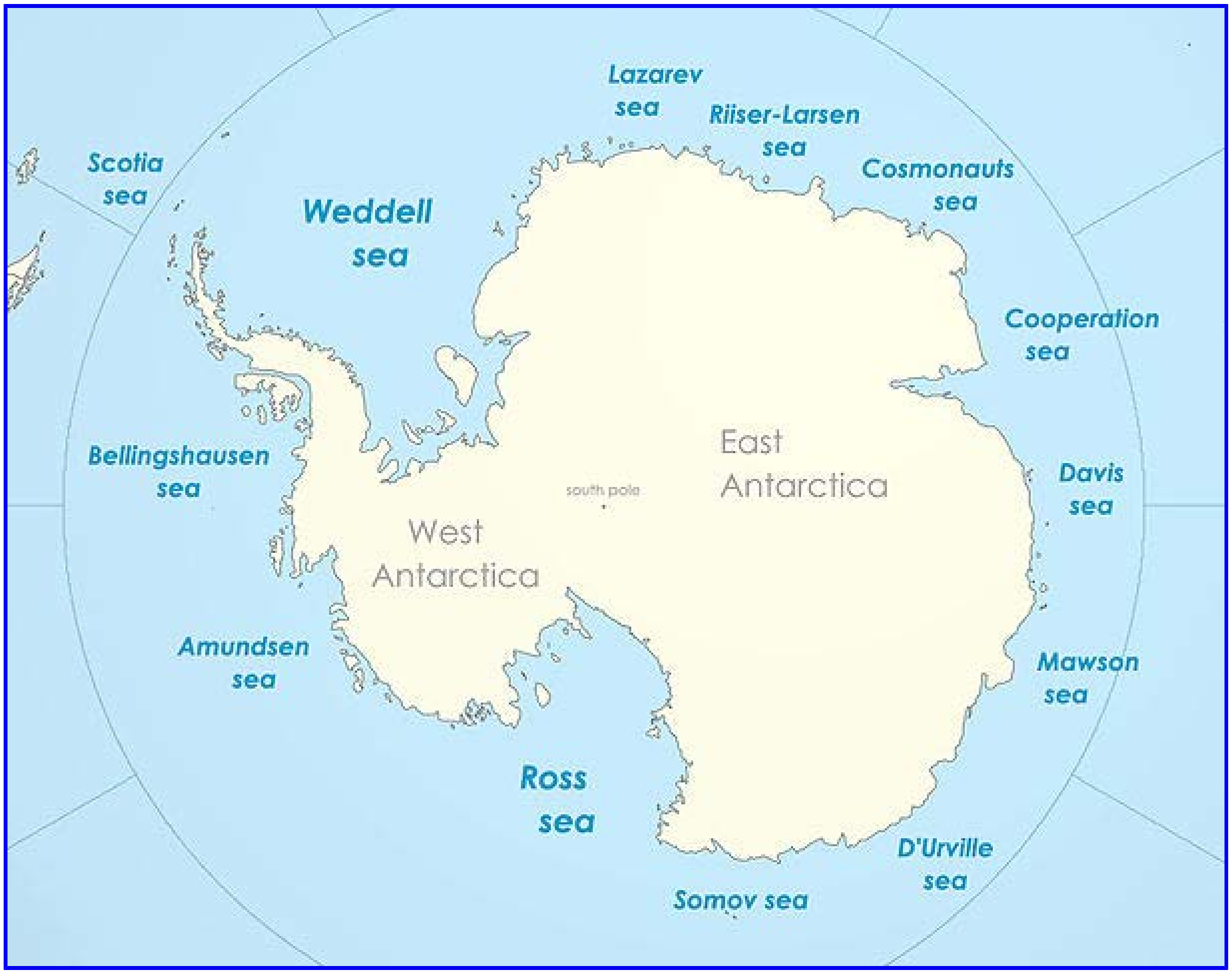


Here Canadian photographer David Burdeny captured an iceberg rising straight out of the ocean. It seems to divide the world into four parts.

He took this photo in 2007 in the Weddell Sea, one of the two big dents in Antarctica separated by the huge peninsula called West Antarctica. Scientists have found that the Weddell Sea has the clearest water of any sea. But it's a dangerous place, according to historian Thomas R. Henry's book *White Continent*:

> The Weddell Sea is, according to the testimony of all who have sailed through its berg-filled waters, the most treacherous and dismal region on earth. The Ross Sea is relatively peaceful, predictable, and safe.

The Ross Sea is the other big dent in Antarctica:



David Burdeny took this photo in 2007 and called it 'Mercator's Projection'. It appeared as part of a series of photos from Antarctica and Greenland. As a fan of the Earth's desolate regions, I like these a lot:

- David Burdeny, North/South.

You can see more of his work here:

- David Burdeny, *Instagram*.

Most is not as thrilling to me, but there are some stunning images of tulip fields, which look don't look like tulip fields.

October 13, 2016

Super Saturn

About 400 light years away, there's something with rings like Saturn — but much, much bigger!

It's called J1407b. It could be a huge planet. Or it could be a star so small that it never lit up: a brown dwarf.

One of Saturn's largest visible rings, the F ring, is about 140 thousand kilometers in radius. But J1407b's rings are almost a thousand times bigger. It has rings 90 million kilometers in radius!

That's 2/3 as big as the Earth's orbit around the Sun. That's insane! It's so huge that scientists don't know why the ring doesn't get ripped apart by the gravity of the star it orbits.

One theory is that the rings are spinning in a retrograde way — in other words, backwards. If you have a planet moving clockwise around a star, and its rings are turning counterclockwise, this helps keep them from getting pulled apart. You can see a simulation here:

- Nicholas St. Fleur, Distant ringed object could be 'Saturn on steroids', *New York Times*, October 13, 2016.

However, it's not obvious why the rings would turn backwards.

There's no sharp boundary between a very large planet and a very small star. If it produces heat using nuclear fusion, it's considered a star... but there are some funny borderline cases.

Stars about 13 times heavier than Jupiter get hot enough to fuse deuterium — but they quickly fizzle out, since that isotope of hydrogen is rare. Stars about 65 times heavier than Jupiter can also fuse lithium... but then fizzle out. So, these things are called brown dwarfs. Stars over 80 times heavier than Jupiter can actually fuse hydrogen, so they light up and form very small red dwarfs.

The atmosphere of a hot brown dwarf is similar to a sunspot — a cold spot on our Sun. It contains molecular hydrogen, carbon monoxide and water vapor. This is called a class M brown dwarf.

But after they run out of fuel, they cool down. The cooler class L brown dwarfs have clouds!

But the even more chilly class T brown dwarfs do not. Why not?

Here's a popular theory: the clouds may rain down, with material moving deeper into the star! People seem to be seeing this in Luhman 16B, a brown dwarf 7 light years from us. It's half covered by huge clouds. These clouds are hot — 1200

°C — so they're probably made of sand, iron or salts. But some of them have been seen to disappear!

Finally, as brown dwarfs cool below 300 °C, astronomers expect that ice clouds start to form: first water ice, and eventually ammonia ice. These are called class Y brown dwarfs.

Wouldn't that be neat to see? A star with icy clouds! And maybe it could have huge rings, too!

For more on J1407b, try Wikipedia:

- 1SWASP J140747.93-394542.6, Wikipedia.

Also try the fun comments on my G+ post!

The picture above is an artist's impression by Ron Miller.

October 14, 2016

Mini Saturn

Chariklo orbits the Sun between Saturn and Uranus. Just 250 kilometers across, it has two tiny rings!

Is it an asteroid? Not quite: it's a 'centaur'. In Greek mythology, a centaur was half-human, half-horse. In astronomy, a centaur is halfway between an asteroid and a comet. Centaurs live in the outer solar system between Jupiter and Neptune. They don't stay there long - at most a million years. They come from further out, pulled in by the gravity of Neptune, but their orbits are chaotic and they eventually move in toward Jupiter.

Over 300 centaurs have been seen, and scientists believe there are over 40,000 that are bigger than a kilometer across. But Chariklo is the biggest. And it has two rings!

In my last entry I discussed a 'super Saturn' — an object in another solar system with rings almost a thousand times bigger than Saturn. Chariklo, on the other hand, is a 'mini Saturn'. Its rings are just 800 kilometers across — just 0.3%

the size of Saturn's F ring.

These rings are narrow and dense. One is about 6 kilometers wide and the other — which you can barely see in this artist's picture — is just 3 kilometers wide. They're separated by a 9-kilometer gap.

How did they get there? Some smaller objects—probably made of ice—may have collided and broken apart. But they must have collided not too fast, or they would have shot all over instead of forming neat rings.

The rings are probably not very stable, unless Chariklo has one or more moons to stabilize them. Saturn has such moons, called shepherd moons.

The second largest centaur, called Chiron, may also have rings.

Puzzle 1. Who was Chariklo in Greek mythology?

Puzzle 2. Who was Chiron?

For answers, see the comments on my G+ post.

Chariklo's full name is 10199 Chariklo:

- Wikipedia, 10199 Chariklo.

Its rings are tentatively named Oiapoque and Chuí, after two rivers in Brazil:

- Wikipedia, Rings of Chariklo.

They were discovered in 2013. How come nobody told me?

Centaurs are lots of fun if you like celestial mechanics:

- Wikipedia, Centaur (minor planet).

I can't resist quoting a bit:

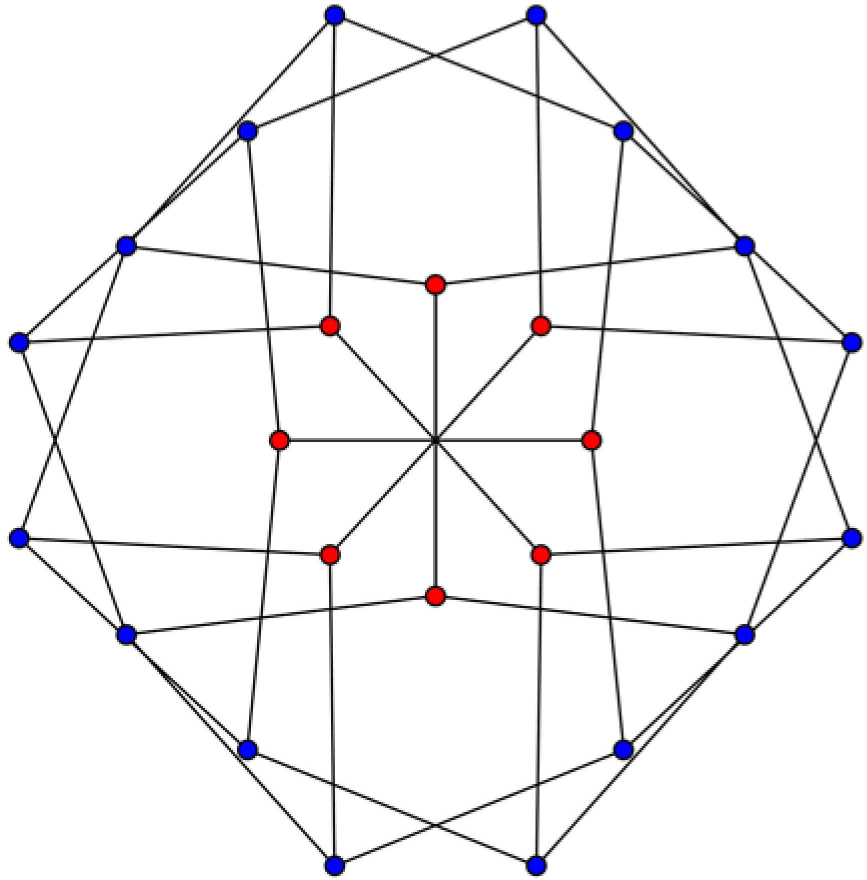
> Because the centaurs are not protected by orbital resonances, their orbits are unstable within a timescale of 10^6 to 10^7 years. For example, 55576 Amycus is in an unstable orbit near the 3:4 resonance of Uranus. Dynamical studies of their orbits indicate that being a centaur is probably an intermediate orbital state of objects transitioning from the Kuiper belt to the Jupiter family of short-period comets. Objects may be perturbed from the Kuiper belt, whereupon they become Neptune-crossing and interact gravitationally with that planet. They then become classed as centaurs, but their orbits are chaotic, evolving relatively rapidly as the centaur makes repeated close approaches to one or more of the outer planets. Some centaurs will evolve into Jupiter-crossing orbits whereupon their perihelia may become reduced into the inner Solar System and they may be reclassified as active comets in the Jupiter family if they display cometary activity. Centaurs will thus ultimately collide with the Sun or a planet or else they may be ejected into interstellar space after a close approach to one of the planets, particularly Jupiter.

The picture here was created by Nick Risinger for ESO, the European Southern Observatory in Chile. They reported their discovery here:

- ESO, First ring system around asteroid, March 26, 2014.

October 15, 2016

Alien machinery



That's what it looks like to me. But it's an image created by Greg Egan, the science fiction author. And there's a story behind it.

Egan and I figured out a bunch of stuff about the '[McGee graph]', a highly symmetrical graph with 24 vertices and 36 edges. I wrote an article about it on *Visual Insight*, my blog for beautiful math pictures. Later I got an email from Ed Pegg, Jr. saying he'd worked out a 'unit-distance embedding' of the McGee graph: a way of drawing it in the plane so that any two vertices connected by an edge are distance 1 apart. He wanted to know if this was 'rigid' or 'flexible'. In other words, he wanted to know whether you can change its shape slightly while it remains a unit-distance embedding.

Egan thought about it a lot and did a lot of computations and discovered that this unit-distance embedding is flexible. And here it is, flexing!

For Pegg and Egan's work, go here:

- Ed Pegg, Jr., Is unit McGee rigid?, *Mathematics Stack Exchange*, October 17, 2015.

What's the practical use of all this? Mainly, it's a practice problem in 'structural rigidity': the study of whether a structure is flexible or rigid. This is important in engineering:

- Wikipedia, Structural rigidity.

A structure is 'infinitesimally flexible' if, roughly, we can bend it a teeny weeny bit. As the name suggests, infinitesimal rigidity can be determined by using calculus to take the derivative of all the edge lengths as a function of all the vertex positions and then using linear algebra to see in which directions this derivative is zero. This is easy in principle, though complicated when you have 24 vertices and 36 edges.

Puzzle 1. With a minimum of explicit computation, prove that any unit-distance embedding of the McGee graph is infinitesimally flexible.

Infinitesimal flexibility is a necessary but not sufficient condition for true flexibility.

Puzzle 2. Find a unit-distance embedding of a graph that is infinitesimally flexible but not flexible.

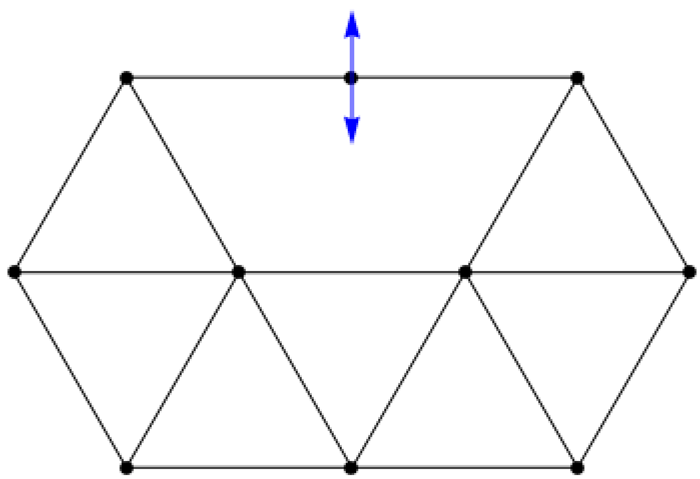
So, Egan had to do more work to show Pegg's unit-distance embedding of the McGee graph was actually flexible. There is probably a high-powered theoretical way to do this, and it's probably not even very complicated, but I don't know it. Do you?

For discussions of the puzzles, look at the comments on my G+ post. For my *Visual Insight* post on the McGee graph, go here:

- John Baez, McGee graph, *Visual Insight*, September 15, 2015.

By the way, I don't like the phrase 'unit-distance embedding' — we're not really *embedding* the McGee graph in the plane, because we're letting the edges cross. The word 'immersion' would be better.

Here, by the way, is Greg Egan's answer to Puzzle 1:



For my November 2016 diary, go here.

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home

For my October 2016 diary, go here.

Diary — November 2016

John Baez

November 1, 2016

So cute! This small lizard, called the 'thorny devil' or *Moloch horridus,* lives in the deserts and scrub lands of Australia.

It may look fierce, but it's not dangerous. It only eats ants. It's spiny so it doesn't get eaten. It can change color, for camouflage! And it has a "false head" on the back of its neck, which it shows to potential predators by dipping its real head. I'm not sure why.

It's also called a 'thorny dragon':

- Thorny dragon, Wikipedia.

I thank Rasha Kamel for introducing me to this beast. She pointed out this article:

- Bob Yirka, Thorny devil found to drink through its skin with assist from gravity, *Phys.org*, November 4, 2016.

Scientists have recently figured out more about how this lizard gets water:

> Researchers discovered long ago that because its mouth has evolved to eat ants, it cannot sip or even lick water from a source — instead it has to rely on other means. Prior research had found that the lizard has tiny folds on its skin that overlap, creating tube-like structures capable of carrying water — the tubes all lead to the back of the mouth. It was noted that setting the lizard in a small bucket of water caused the tubes to fill and the lizard to start swallowing. But what has remained a mystery is how such a technique could work in the desert, where there are rarely puddles to stand in. To solve the mystery, the researchers captured some specimens and took them back to their lab for study.
>
> In the lab, the researchers first tried allowing the lizards to stand on sand that had been wetted — this resulted in some water being drawn into the tubes, but not enough to get the lizard to start swallowing, which meant it wasn't enough. The answer, it turned out, was the lizard's habit of pushing sand onto its back — this caused any water from recent rains or even from dew to move slowly downward, due to gravity. Eventually, it would reach the skin, where it would be sucked into the tubes like a child with a straw. At some point, the tubes would fill and the lizard would swallow it.
>
> The researchers note that such a drinking technique is likely merely supplementary — most of the water they get comes from the ants they eat; thus, using skin for drinking would likely only occur during extreme draught conditions.

I got this picture from a website with a lot of great photos of thorny devils:

- Thorny devil lizard — prickly desert ant-eater, *FactZoo.com*.

November 2, 2016

These are Pristerognathus, very ancient mammal-like reptiles. They lived in the middle Permian, around 260 million years ago. That's long before the dinosaurs!

These animals were roughly dog-sized, and had long, narrow skulls and large canine teeth. They probably lived in woodlands, and preyed on smaller animals.

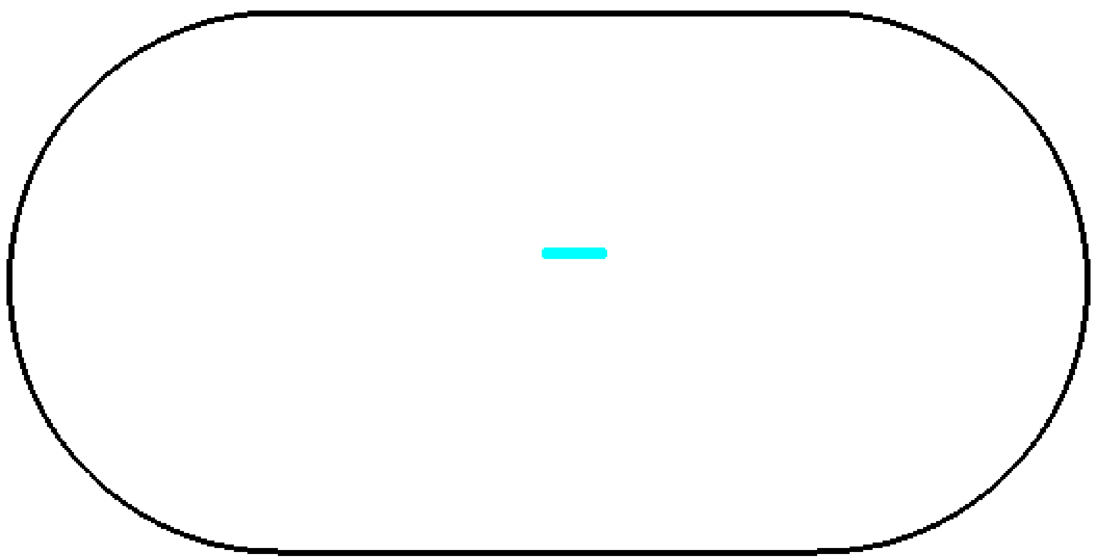
There were many kinds of mammal-liked reptiles back then. In general they are called therapsids. Some of them evolved to become mammals — like you and me! Fur has been found in the fossilized poop of some of these animals. So, at least some of them had hair.

These particular guys are called *Pristerognathus vanderbyli*. This picture is from Wikicommons.

The first dinosaurs showed up around 240 million years ago — and they only became common after the great Triassic-Jurassic extinction, 200 million years ago. Therapsids started around 275 million years ago. Some of them evolved into mammals 225 million years ago, and all the non-mammalian ones went extinct by the early Cretaceous, 100 million years ago. Most dinosaurs went extinct at the end of the Cretaceous, 65 million years ago. Some, however, are still sold as food at grocery stores.

November 15, 2016

Chaotic billiards



Nice animation by Phillipe Roux! Take some balls moving in the same direction and let them bounce around in this shape: a rectangle with ends rounded into semicircles. They will soon start moving in dramatically different ways. To keep things simple we don't let the balls collide — they pass right through each other. In a while they will be close to evenly spread over the whole billiard table.

This is an example of chaos: slightly different initial conditions lead to dramatically different trajectories.

It's also an example of ergodicity: for almost every choice of initial conditions, the trajectory of a ball will have an equal chance of visiting each tiny little region. That is, if we take a random choice of initial conditions, there is a 100% probability that the trajectory will have this property.

The Bunimovich stadium is a rectangle capped by semicircles in which a point particle moves at constant speed along straight lines, reflecting off the boundary in a way that the angle of incidence equals the angle of reflection. The animation shows a collection of such particles initially moving in the same direction. With each bounce their trajectories diverge, and after a while they are distributed almost evenly through the whole stadium, though for a while one can still see a density wave moving back and forth.

The Bunimovich stadium appears in the 1979 work of Leonid Bunimovich:

• Leonid A. Bunimovich, On the ergodic properties of nowhere dispersing billiards, *Commun. Math. Phys.* **65** (1979), 295–312.

He showed that the motion of a billiard in this stadium is ergodic. This is a way of making precise the intuition that given a billiard with randomly chosen initial position and velocity, over time its position almost surely becomes uniformly distributed over the whole stadium.

More precisely, we can define the phase space Ω for the Bunimovich stadium to be the space of position-velocity pairs where the velocity is a unit vector. (Since the speed of the billiard does not change, we may assume it is normalized to 1.) There is a probability measure on Ω for which time evolution defines measure-preserving dynamical system:

$$T_t : \Omega \to \Omega, \qquad\qquad t \quad \mathrm{R}.$$

Given a measure-preserving dynamical system, we say a measurable subset $A \quad \Omega$ is 'invariant' if for all $t \quad \mathrm{R}$ the sets $T_t(A)$ and A differ only by a null set, meaning that the symmetric difference $T_t(A) \quad A$ has measure zero. A measure-preserving dynamical system is ergodic if the only invariant measurable subsets $A \quad \Omega$ are null sets and the complements of null sets.

The meaning of this is clarified by 'ergodic theorem'. Suppose $T_t : \Omega \to \Omega$ is a measure-preserving dynamical system on a probability measure space Ω, μ, and suppose $f : \Omega \to \mathrm{R}$ is an integrable function. Then we can define two averages of f, the 'time average' and 'phase space average'.

Time average: This is the following average (if it exists):

$$\hat{f}(x) = \lim_{t \to \infty} \frac{1}{t} \int_0^t f(T_s x)\, ds.$$

Phase space average: This is the integral of f over the phase space:

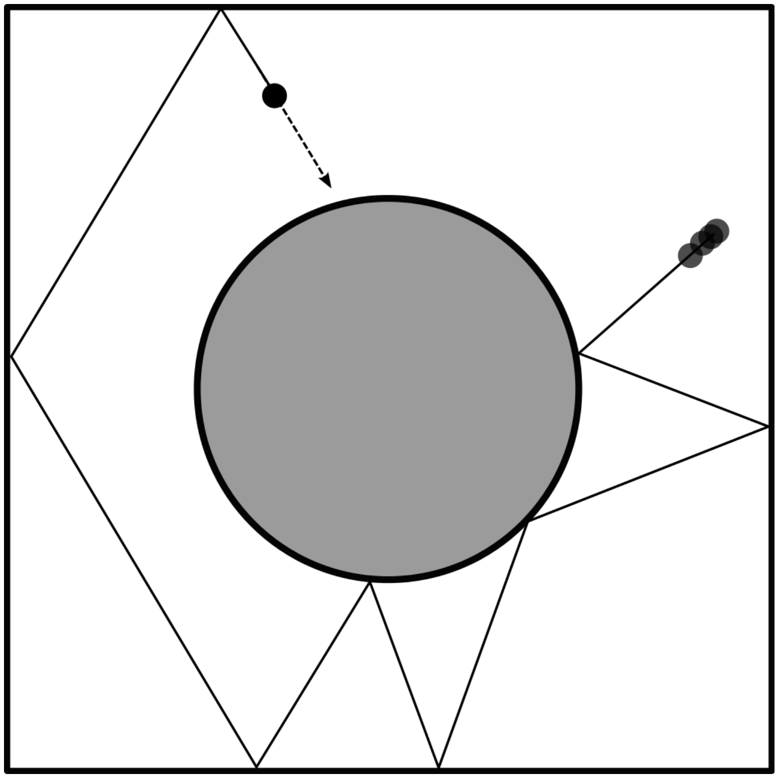
$$\bar{f} = \int_\Omega f \, d\mu(x).$$

In general the time average and phase space average may be difference, and the time average may not exist. But if T_t is ergodic, Birkhoff's **ergodic theorem** says that

$$\hat{f}(x) = \bar{f}$$

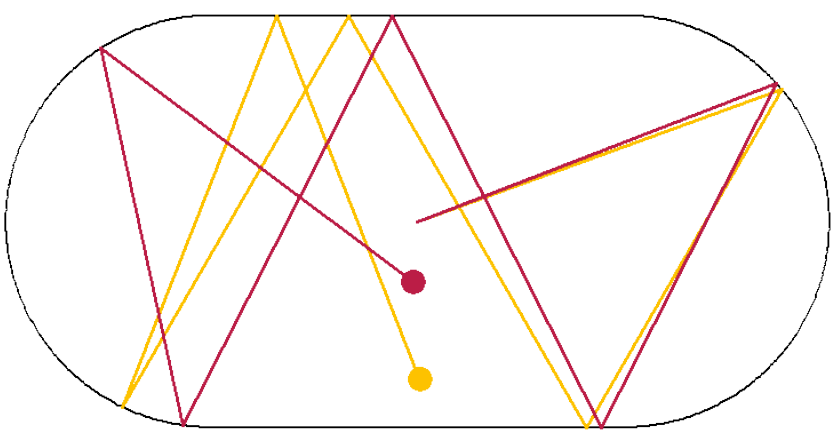
for almost every $x \quad \Omega$.

Proving that a measure-preserving dynamical system is ergodic can be difficult. Bunimovich's thesis advisor, Yakov G. Sinai, showed that a billiard moving on a square table with a reflecting disk inside is ergodic.



Sinai Billiard - George Stamatiou

The curvature of the disk tends to amplify the angle between slightly different trajectories. The Bunimovich stadium is subtler because it lacks this feature: since its rounded ends are convex, they tend to focus billiards that bounce off them. The rectangular portion of the table counteracts this focusing effect, and over long enough times there tend to be an exponentially growing distance between initially nearby trajectories.



Bunimovich Stadium Trajectories - Jakob Scholbach

As Buminovich writes:

> Moreover, a closer analysis of these billiards revealed a new mechanism of chaotic behavior of conservative dynamical systems, which is called a **mechanism of defocusing**. The key observation is that a narrow parallel beam of rays, after focusing because of reflection from a focusing boundary, may pass a focusing (in linear approximation) point and become divergent provided that a free path between two consecutive reflections from the boundary is long enough. The mechanism of defocusing works under condition that divergence prevails over convergence.

This is from:

* Leonid Buminovich, Dynamical billiards, *Scholarpedia*.

However, this analysis is not sufficient to understand the ergodicity of the Bunimovich stadium, because in 1973 Lazutkin showed that a convex billiard table with infinitely differentiable boundary cannot be ergodic. In fact he showed this for a convex table whose boundary has 553 continuous derivatives! In 1982 Douady showed 6 continuous derivatives is enough — and he conjectured that 4 is enough. For references, see:

* Nikolai Chernov and Roberto Makarian, *Introduction to the Ergodic Theory of Chaotic Billiards*, AMS, Providence, Rhode Island, 2006. Shorter version free online.

For quantum aspects of the Bunimovich stadium see:

* Terence Tao, Open question: scarring for the Bunimovich stadium, *What's New*, March 28, 2007.

This explained an interesting question which was addressed by later work:

* Terence Tao, Hassell's proof of scarring for the Bunimovich stadium, *What's New*, July 7, 2008.

Also try Carlos Scheidegger's great webpage that lets you play around with billiards on the Bunimovich stadium as well as elliptical table, where their motion is completely integrable:

* Carlos Scheidegger, Bunimovich stadium.

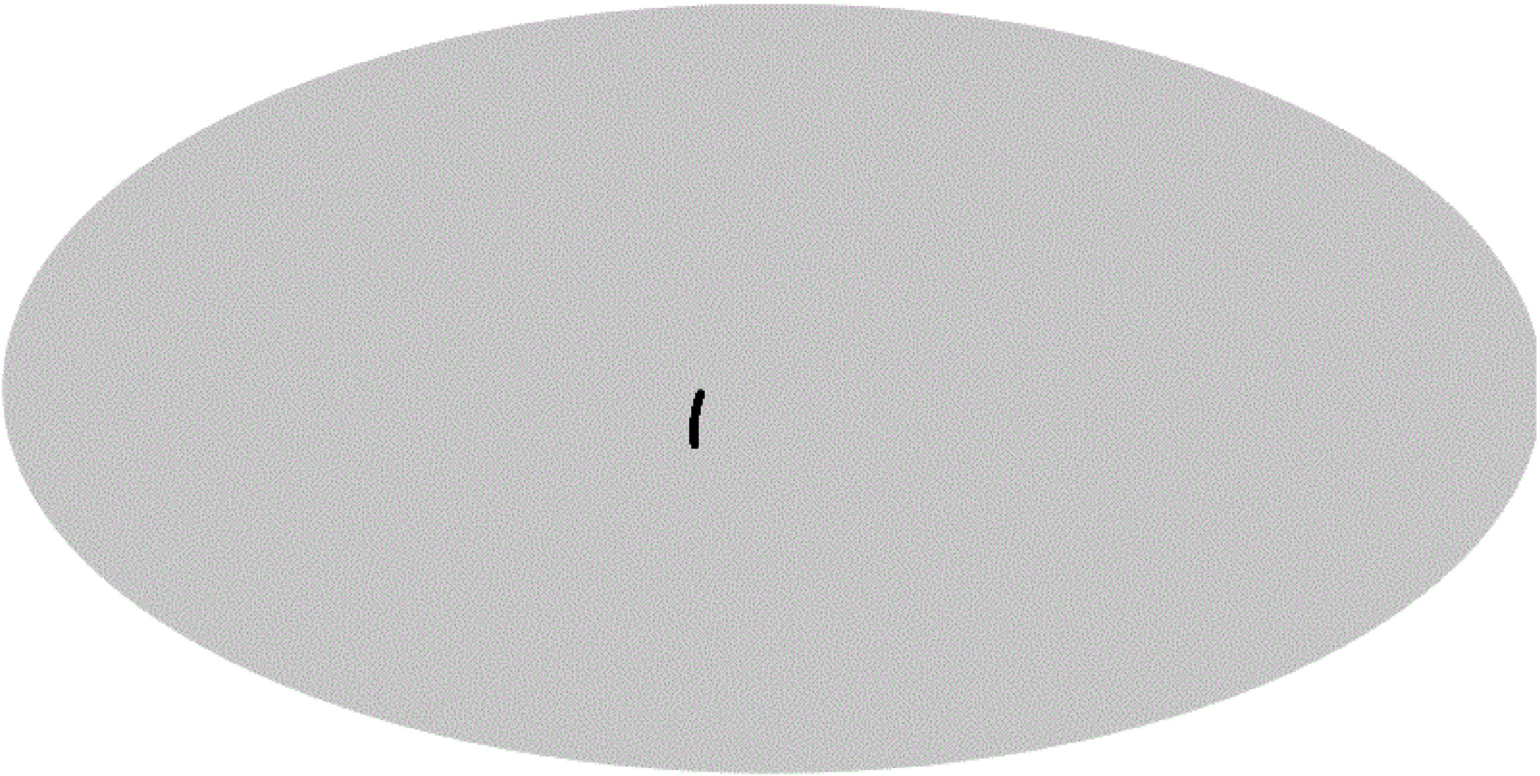
Check out more of Phillipe Roux's animations here:

* Phillipe Roux, Billiards, November 12, 2016.

George Stamatiou put his picture of the Sinai billiard on Wikicommons under a Creative Commons Attribution 2.5 Generic license. Jakob Scholbach put his picture of billiard trajectories in the Bunimovich stadium on Wikicommons under a Attribution-ShareAlike 3.0 Unported license.

November 16, 2016

Completely integrable billiards



Check out Carlos Scheidegger's great webpage that lets you play around with billiards on two tables:

- Carlos Scheidegger, Bunimovich stadium

The table here is elliptical, and you'll see that the billiards trace out nice patterns — not at all random. Often there's a region of the table that they never enter! Not in this particular example, but try others and you'll see what I mean.

Puzzle 1. What shape is this 'forbidden region', and why?

It will be easier to answer if you experiment a bit.

The other table is a rectangle with rounded ends, called the 'Bunimovitch stadium'. For that one the billiards move chaotically. After a while they seem randomized.

This illustrates two very different kinds of dynamical systems. The 'completely integrable' systems, like the elliptical billiards, do very predictable things. The 'ergodic' ones seem random.

With some math, we can make these ideas precise. I'll be quick: a system whose motion is described by Hamiltonian mechanics is completely integrable if it has the maximum number of conserved quantities. It's ergodic if it has the minimum number. All sorts of in-between cases are also possible!

For a particle moving around in n dimensions the maximum number of conserved quantities is n. More precisely, we can write every conservated quantity as a function of n such quantities. The minimum number is 1, since energy is always conserved.

So, for a billiard ball, the maximum number is 2, and that's what we have for the elliptical billiard ball table. One of them is the energy, or if you prefer, the speed of the billiard ball.

Puzzle 2. What's the other?

This is related to Puzzle 1, since it's this extra conserved quantity that sometimes forbids the billiard ball from entering certain regions in the ellipse.

For more on complete integrability versus ergodicity, try these:

- Integrable system, Wikipedia.
- Ergodic theory

For some very nice answers to the puzzles, see the comments on my G+ post.

November 17, 2016

This photo by Kei Nomiyama shows fireflies just above the ground in a bamboo forest.

Photographing fireflies is popular in Japan, and this article shows some other nice examples:

- Courtney Constable, Skilled photographers capture Japan's gorgeous summer firefly phenomenon, *Thecoolist*.

She writes:

> Japan is a beautiful country full of breathtaking buildings, landscapes, and scenery any time of year. In the height of summer, however, something particularly magical happens. Throughout the countryside, twinkling fireflies take to the evening skies in search of a mate. This natural phenomenon creates a beautifully ethereal glow through trees and leaves that is nothing short of breathtaking.
>
> Of course, in this phenomenon, Japanese and visiting photographers have found a gorgeous source of inspiration. Capturing the lights of the fireflies, however, can be extremely difficult. Fireflies are very sensitive to other sources of light besides themselves, meaning that camera flashes, cell phones, flashlights, and other things that photographers often need to get their equipment set up can drive the little creatures away.

> The difficulty of capturing photos of the fireflies, however, hasn't deterred the most dedicated photographers. They've simply adapted their strategy to account for the habits of the fireflies. Photographers often scout an area out days in advance to see where the fireflies congregate and then return very early on the day they want to shoot, setting up in daylight before the twinkling lights begin and lying in still, silent wait for hours.

You can see more of Kei Nomiyama's firefly photos here:

- Kei Nomiyama, [Photography: firefly](#).

What puzzles me is this: the glowing fireflies in these photos seem more *orange* than what I see in the eastern US. I'm used to firefly light being yellow-green. So:

Puzzle 1. Are fireflies in Japan from a different species than US fireflies?

and more importantly:

Puzzle 2. Do they use a different chemical mechanism to make light?

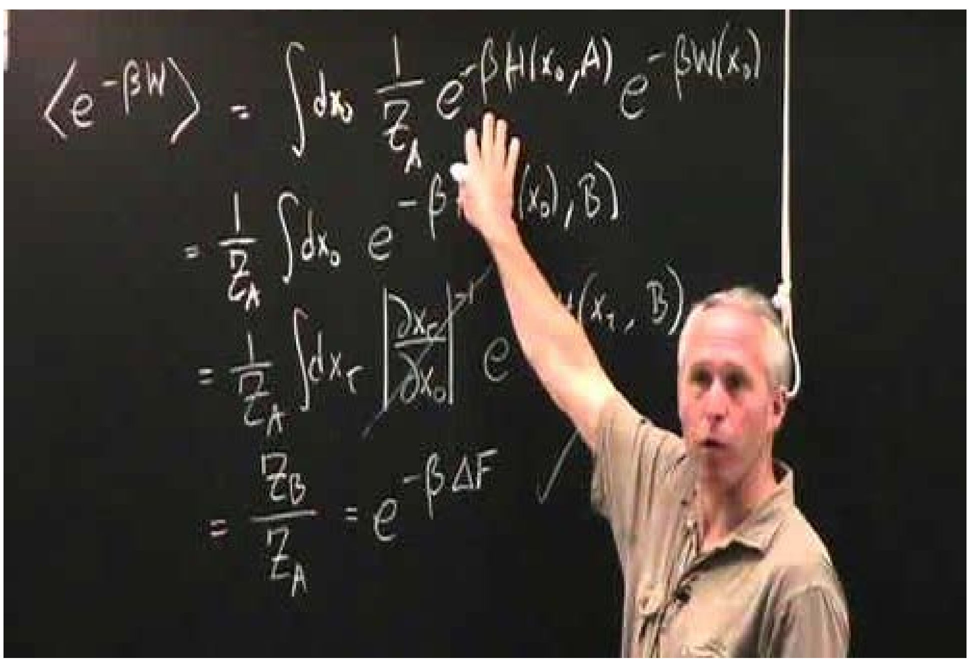
or more generally:

Puzzle 3. How do fireflies make light, and how do they turn the chemical reaction on and off?

For some attempts at answers, see the comments on my [G+ post](#).

November 27, 2016

Jarzynski on thermodynamics



In the old days, despite its name, thermodynamics was mainly about 'thermodynamic equilibrium'. Thermodynamic equilibrium is a situation where nothing interesting happens. For example, if you were in thermodynamic equilibrium right now, you'd be dead. Not very dynamic!

Sure, there were a few absolutely fundamental results like the second law, which says that entropy cannot decrease as we carry a system from one equilibrium state to another. But the complications you see when you boil a pot of water... those were largely out of bounds.

This has changed in the last 50 years. One example is the Jarzynski equality, discovered by Christopher Jarzynski in 1997.

The second law implies that the change in 'free energy' of a system is less than or equal to the amount of work done on it. But the Jarzynski equality gives a precise equation relating these two concepts, which implies that inequality. I won't explain it here, but it's terse and beautiful.

Last week at the Santa Fe Institute, Jarzynski gave an incredibly clear hour-long tutorial on thermodynamics, starting with the basics and zipping forward to modern work. With his permission, you can see his slides here:

- John Baez, Jarzynski on Non-Equilibrium Statistical Mechanics, *Azimuth*, November 18, 2016.

along with links to an explanation of the Jarzynski equality, and a proof.

I had a great time in Santa Fe, and this was one of the high points.

For my December 2016 diary, go here.

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[For my November 2016 diary, go here.]

Diary — December 2016

John Baez

December 1, 2016

Otto Bell had never made a feature film. But when he saw this photo, he flew to Mongolia — and made a movie about a girl.s struggle to capture and tame a golden eagle. It's called *The Eagle Huntress*, and now it may get an Oscar.

Otto Bell was surfing the web at work when he saw this photo. It amazed him: a rosy-cheeked Mongolian girl, perched on a mountain ridge, smiling with delight at a ferocious golden eagle flapping on her arm. Look at her face!

The scene was a world away from the office cubicle in New York where Bell was sitting. The shots were taken in the Altai mountains, "the most remote part of the least-populated country in the world". He had no financing and had only ever made short, commercially funded documentaries. But he was so moved that soon he gathered up a small team and took a flight to Mongolia to track down the girl: a 13-year-old named Aisholpan.

When they finally found her nomadic family, Bell was nervous they might not want to be filmed. Instead her father Nurgaiv made an extraordinary offer. "This afternoon we are going down the mountain to steal an eagle for Aisholpan. Do you want to film that?"

Aisholpan had her eye on a fledgling female. Female eagles are larger, so preferred for hunting. For days, Aisholpan had been watching this one through her father's old broken binoculars. It was the perfect age: able to survive without her mother, but young enough to be trained.

Capturing Aisholpan's climb down a sheer cliff to an eagle's nest, with only a rope tied round her waist, posed problems for them all. For a start, the cameraman was afraid of heights so could only film from solid ground below! The photographer wasn't well-placed to step in, since he'd never shot moving images. So Bell had to get creative. He strapped a GoPro inside Aisholpan's cardigan and climbed with the photographer to a ledge opposite the nest to capture another angle.

It's a heart-stopping scene: a young girl with plaits jauntily tied with pink ribbons makes a terrifying descent while an angry mother eagle circles menacingly overhead. Some movie reviewers assume the scene is a re-creation. But it's the real thing.

I have got to see this movie! I haven't yet. This picture makes me happy. It makes me, too, want to rush off to the Altai mountains near the borders of Mongolia, China, Russia and Kazakhstan. Lake Kucherla looks amazing. But I have to grade finals.

My writeup above is paraphrased from this review:

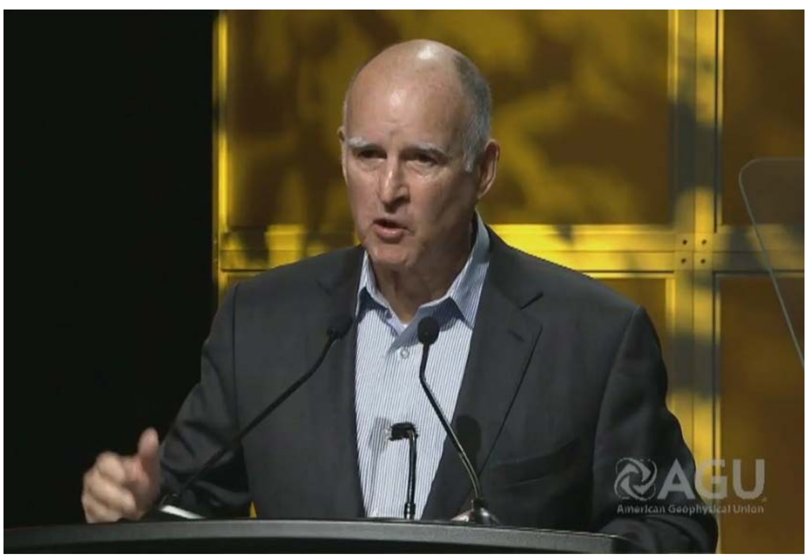
- Homa Khaleeli, The Eagle Huntress: the teenage Mongolian nomad who's preparing to swoop on the Oscars , *The Guardian*, 11 December 2016.

(I wound up seeing this movie and my aunt on January 3rd, and it met my high expectations! Great scenery, a simple and unadorned but thrilling story, and lots of interesting views of life near the Altai mountains. One interesting tidbit at the start: golden eagles raised for hunting are let free after seven years. I'm not sure how well that works out.)

December 2, 2016

December 14, 2016

"If Trump turns off the satellites,
California will launch its own damn satellite!"



That's what Governor Jerry Brown just said to 24,000 climate scientists in San Francisco, to thunderous applause. And:

"We've got the scientists, we've got the lawyers and we're ready to fight."

And to Rick Perry of Texas, newly appointed to lead a department whose name he forgot when listing 3 departments he'd abolish if he were president — oh yeah, the Department of Energy:

"We've got more sun than you've got oil."

Brown is at the annual meeting of the American Geophysical Union. I went there once — it's huge. They usually meet in San Francisco because it has one of the few conference centers big enough to hold 24,000 people.

The move to save climate data continues. Right now the main thing we could use is 3 terabytes of storage space; to get that from Google seems to cost $100/month, since they'll give you 1 terabyte for $10/month and then 10 terabytes for $100/month.

Jan Galkowski, a professional statistician and member of the Azimuth Project, is spending Christmas break downloading data using WebDrive. He could use 3 terabytes of space.

First we're downloading stuff. In the longer term we will try to make this stuff publicly available. And we will try to coordinate with the Climate Mirror project, here:

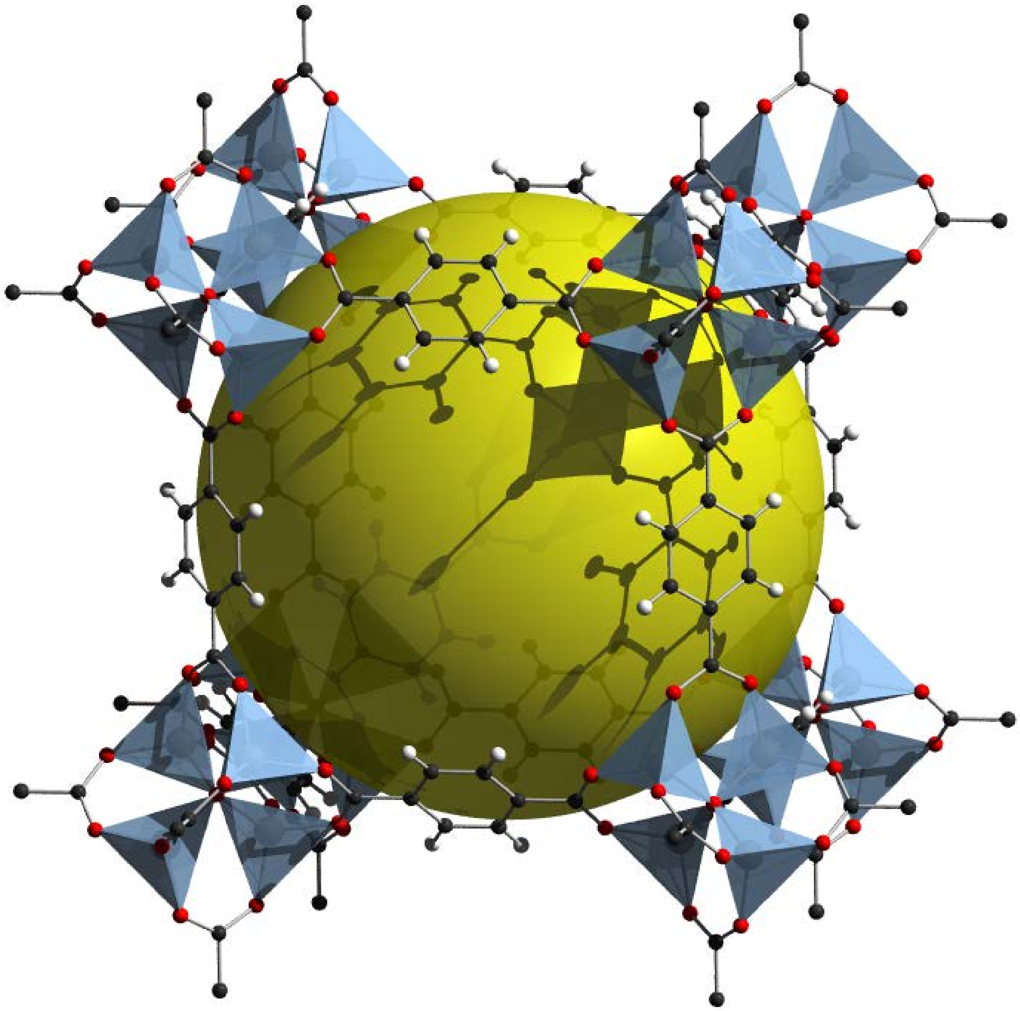
- Climate Mirror

Tomorrow I will talk to someone involved in this project and the head of the Society of American Archivists, since they know a lot about archiving data. I would like to find more ways for ordinary folks to help, but right now it's a confusing scramble to organize things.

You can see Jerry Brown's speech here:

December 27, 2016

Metal-Organic Framework 5



I like the look of this thing! It's a metal-organic framework — a compound made of metal ions connected by organic stuff. The picture here is just part of a structure that keeps repeating in all directions.

The blue tetrahedra are made of an oxygen atom surrounded by 4 atoms of zinc. They're connected by a kind of latticework made of an organic molecule called 1,4-benzodicarboxylic acid.

The whole thing is called 'Metal-Organic Framework 5' or 'MOF5' for short. There are lots of other kinds.

But what about the huge yellow ball?

That's not a real thing. It's empty space where you can put something — like a molecule of hydrogen!

And indeed, metal-organic frameworks are used for storing hydrogen - you can actually pack more hydrogen into a MOF than you can easily squeeze into an empty tank! They can also be used as catalysts. So they're not only beautiful, they're practical.

For a bigger view of MOF5, go here:

- University of Liverpool, MOF-5 (or IRMOF-1) Metal Organic Framework, *ChemTube3d*.

For more about metal-organic frameworks, go here:

- Metal-organic framework, Wikipedia.

Also check out my new [collection of chemistry posts on G+](#).

December 29, 2016

It's better to do something imperfect that helps than not help at all. We so easily forget that. Here's a great story to help us remember: the Hair Dryer Incident, as told by psychatrist Scott Alexander:

> The Hair Dryer Incident was probably the biggest dispute I've seen in the mental hospital where I work. Most of the time all the psychiatrists get along and have pretty much the same opinion about important things, but people were at each other's throats about the Hair Dryer Incident.
>
> Basically, this one obsessive compulsive woman would drive to work every morning and worry she had left the hair dryer on and it was going to burn down her house. So she'd drive back home to check that the hair dryer was off, then drive back to work, then worry that maybe she hadn't really checked well enough, then drive back, and so on ten or twenty times a day.
>
> It's a pretty typical case of obsessive-compulsive disorder, but it was really interfering with her life. She worked some high-powered job — I think a lawyer — and she was constantly late to everything because of this driving back and forth, to the point where her career was in a downspin and she thought she would have to quit and go on disability. She wasn't able to go out with friends, she wasn.t even able to go to restaurants because she would keep fretting she left the hair dryer on at home and have to rush back. She'd seen countless psychiatrists, psychologists, and counselors, she'd done all sorts of therapy, she'd taken every medication in the book, and none of them had helped.
>
> So she came to my hospital and was seen by a colleague of mine, who told her "Hey, have you thought about just bringing the hair dryer with you?"
>
> And it worked.
>
> She would be driving to work in the morning, and she'd start worrying she'd left the hair dryer on and it was going to burn down her house, and so she'd look at the seat next to her, and there would be the hair dryer, right there. And she only had the one hair dryer, which was now accounted for. So she would let out a sigh of relief and keep driving to work.
>
> And approximately half the psychiatrists at my hospital thought this was absolutely scandalous, and This Is Not How One Treats Obsessive Compulsive Disorder, and what if it got out to the broader psychiatric community that instead of giving all of these high-tech medications and sophisticated therapies we were just telling people to *put their hair dryers on the front seat of their car?*
>
> I, on the other hand, thought it was the best fricking story I had ever heard and the guy deserved a medal. Here's someone who was totally untreatable by the normal methods, with a debilitating condition, and a drop-dead simple intervention that nobody else had thought of gave her her life back. If one day I open up my own psychiatric practice, I am half-seriously considering using a picture of a hair dryer as the logo, just to let everyone know where I stand on this issue.
>
> Miyamoto Musashi is quoted as saying:
>
>> The primary thing when you take a sword in your hands is your intention to cut the enemy, whatever the means. Whenever you parry, hit, spring, strike or touch the enemy.s cutting sword, you must cut the enemy in the same movement. It is essential to attain this. If you think only of hitting, springing, striking or touching the enemy, you will not be able actually to cut him.
>
> Likewise, the primary thing in psychiatry is to help the patient, whatever the means. Someone can concern-

troll that the hair dryer technique leaves something to be desired in that it might have prevented the patient from seeking a more thorough cure that would prevent her from having to bring the hair dryer with her. But compared to the alternative of "nothing else works" it seems clearly superior.

This is from:

* Scott Alexander, [The categories were made for man, not man for the categories](), *Slate Star Codex*, November 21, 2014.

Thanks to Richard Mlynarik for leading me to this, indirectly. He actually pointed me to an interesting article about psychology and network theory:

* Scott Alexander, [SSC journal club: mental disorders as networks](), *Slate Star Codex*, December 14, 2016.

The idea is that some mental disorders, instead of having a single "root cause", are a network of symptoms that reinforce each. Some, not all!

That article led me to this tale here

December 30, 2016

Creature of nightmares

This is the scariest insect I've ever seen: the giant toothed longhorn beetle from the Amazon basin in Ecuador. It's not as big as it looks here, but it's big: one of the biggest beetles in the world, up to 17 centimeters long. (That's half a foot, for you Americans.) Its larvae are even longer!

Gil Wizen, who photographed this monster, writes:

> Encountering this species was one of my highlights for the year. I know *Macrodontia cervicornis* very well from museum insect collections. It is one of the most impressive beetle species in the world, both in size and structure. But I never imagined I would be seeing a live one in the wild! Well let me tell you, it is hard to get over the initial impression. The male beetle that I found was not the biggest specimen, but the way it moved around still made it appear like nothing short of a monster. This species is very defensive, and getting close for the wide angle macro shot was a bit risky. The beetle responds to any approaching object with a swift biting action, and those jaws are powerful enough to cut through thick wooden branches, not to mention fingers!

Check out his favorite photos of the year:

- Gil Wizen, 2016 in review.

and for more on this beetle, see:

- Wikipedia, Macrodontia cervicornis

For my January 2017 diary, go here.

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