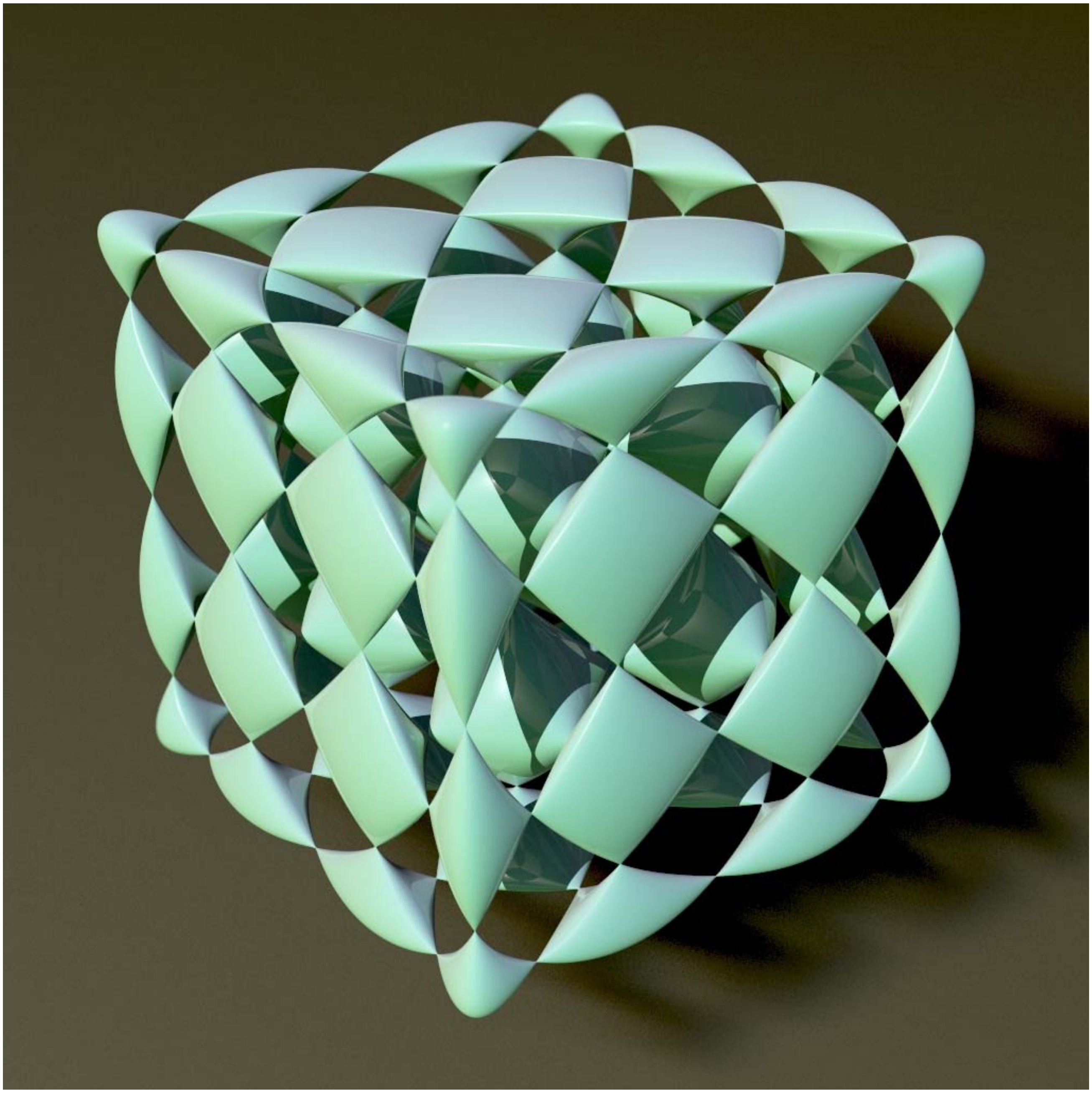
For my December 2016 diary, go here.

Diary — January 2017

John Baez

January 1, 2017

Chmutov octic



You can get some very fancy surfaces using just polynomial equations. Here Abdelaziz Nait Merzouk drew one using polynomials of degree 8. That's why it's called an octic.

Why is it called the Chmutov octic? Well, that's because it was constructed by V. S. Chmutov as part of an effort to build surfaces with lots of ordinary double points, meaning points that look the place where the tips of two cones meet. This one has 144 ordinary double points!

That's not the best you can do: the octic with the highest known number of ordinary double points is the Endrass octic, shown here:

- John Baez, Endrass octic, *Visual Insight*, August 1, 2016.

The Endrass octic has 168 ordinary double points. Nobody knows if that's the best possible.

The Chmutov octic is just one of a series of surfaces invented by Chmutov. There's a Chmutov quadratic, a Chmutov cubic, a Chmutov quartic, a Chmutov quintic, a Chmutov sextic, a Chmutov septic, a Chmutov octic, a Chmutov nonic, a Chmutov decic, a Chmutov hendecic, a Chmutov duodecic, a Chmutov triskaidecic, a Chmutov tetrakaidecic, a Chmutov pendecic, a Chmutov hexadecic, a Chmutov heptadecic, a Chmutov octadecic, a Chmutov enneadecic, a Chmutov icosic, and so on. In fact you can see a quick animated gif of all of these — from the quadratic to the icosic — here:

- John Baez, Chmutov octic, *Visual Insight*, January 1, 2017.

Again, it was made by Abdelaziz Nait Merzouk. You'll notice that most of the Chmutov surfaces of even degree look a lot like the octic here, while those of odd degree extend out to infinity.

Chmutov made these surfaces to get a lower bound on how many ordinary double points we could cram into a surface of a given degree. In most cases other people have beaten him by now. But still, these surfaces are cute! They're defined using some polynomials invented by the Russian mathematician Chebyshev — also known as Chebychev, Chebysheff, Chebychov, Chebyshov, Tchebychev, Tchebycheff, Tschebyschev, Tschebyschef, or Tschebyscheff. Apparently he suffered from a rare psychological disorder that made him forget how to spell his name — so each time he wrote another paper, he signed it a different way!

Happy New Year! (You may not have heard, but this year April Fool's Day has been scheduled on January 1st instead of April 1st.)

This may be my last *Visual Insight* post for a while — I'm getting burnt out on these, and I have a lot of projects on my plate: my work with Metron:

- John Baez, Complex adaptive system design (part 1), *Azimuth*, October 2, 2016.

- John Baez, Complex adaptive system design (part 2), *Azimuth*, October 18, 2016.

the Azimuth Backup Project:

- John Baez, Azimuth Backup Project (part 1), December 16, 2016.

- John Baez, Azimuth Backup Project (part 2), December 20, 2016.

three papers to finish, and five grad students to manage. Like last quarter I'm teaching two courses and also my seminar on network theory; this heavier work load will let me take the spring as a non-teaching quarter, but right now it's making me a bit frenetic. It will pay off in having more time to write during the spring, and also, I hope, visit Hong Kong!

January 11, 2017

A 'luminous red nova' is something brighter than a nova but less bright than a supernova, which can happen when two stars merge. An example is shown above.

In this new paper, astronomers predict a new luminous red nova will occur sometime between September 2021 and September 2022, which could become the brightest object in the night sky here on Earth:

- Lawrence Molnar *et al*, Prediction of a red nova outburst in KIC 9832227.

The stars in KIC 9832227 have been orbiting each other faster and faster over the last three years, and they seem to be surrounded by a shared envelope of gas. If they indeed merge, this will be the first such event predicted ahead of time.

As Greg Egan noted:

> Given that nobody knows exactly when this will happen, the main thing that determines how many people are likely to be able to see it is the declination, 46° N. So anyone in the northern hemisphere will have a good chance... while for someone like me, at 31° S, the odds aren't great: it will never rise higher than 13° above the northern horizon, for me.
>
> Right ascension is the celestial equivalent of longitude, but without knowing the season in advance (and the error bars on the current prediction are much too large for that) we can't tell if the sun will be too close to the object, drowning it in daylight to the naked eye.
>
> If that happens, I guess the only comfort is that there are still sure to be telescopes able to make

observations, maybe including both Hubble and James Webb.

For my February 2017 diary, go here.

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For my January 2017 diary, go here.

Diary — February 2017

John Baez

February 18, 2017

The Azimuth Climate Data Backup Project is backing up 40 terabytes of US government climate data and copying it to a number of locations, to protect it from all possible threats.

It's going well! Our Kickstarter campaign ended on January 31st and the money has recently reached us. Our original goal was $5000. We got $20,427 of donations, and after Kickstarter took its cut we received $18,590.96.

Soon I'll tell you what our project has actually been doing — lots of good news. Right here I just want to give a huge "thank you!" to all 627 people who contributed money on Kickstarter.

I recently sent out thank you notes to everyone, updating them on our progress and asking if they wanted their names listed. The blanks in the following list represent people who either didn't reply, didn't want their names listed, or backed out and decided not to give money. I'll list people in chronological order: first contributors first.

Only 12 people backed out; the vast majority of blanks on this list are people who haven't replied to my email. I noticed some interesting but obvious patterns. For example, people who contributed later are less likely to have answered my email. People who contributed more money were more likely to answer my email.

The magnitude of contributions ranged from $2000 to $1. A few people offered to help in other ways. The response was international — this was really heartwarming! People from the US were more likely than others to ask not to be listed.

But instead of continuing to list statistical patterns, let me just thank everyone who contributed. Here's the list! (I'll keep updating this list on the Azimuth blog, but not here.)

```
Daniel Estrada
Ahmed Amer
Saeed Masroor
Jodi Kaplan
John Wehrle
Bob Calder
Andrea Borgia
L Gardner

Uche Eke
Keith Warner
Dean Kalahan
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James Benson
Dianne Hackborn

Walter Hahn
Thomas Savarino
Noah Friedman
Eric Willisson
Jeffrey Gilmore
John Bennett
Glenn McDavid

Brian Turner

Peter Bagaric

Martin Dahl Nielsen
Broc Stenman

Gabriel Scherer
Roice Nelson
Felipe Pait
Kenneth Hertz

Luis Bruno

Andrew Lottmann
Alex Morse

Mads Bach Villadsen
Noam Zeilberger

Buffy Lyon

Josh Wilcox

Danny Borg

Krishna Bhogaonker
Harald Tveit Alvestrand

Tarek A. Hijaz, MD
Jouni Pohjola
Chavdar Petkov
Markus Jöbstl
Bjørn Borud

Sarah G

William Straub

Frank Harper
Carsten Führmann
Rick Angel
Drew Armstrong

Jesimpson

Valeria de Paiva
Ron Prater
David Tanzer

Rafael Laguna
Miguel Esteves dos Santos
Sophie Dennison-Gibby

Randy Drexler
Peter Haggstrom

Jerzy Micha? Pawlak
Santini Basra
Jenny Meyer

John Iskra

Bruce Jones
M?ris Ozols
Everett Rubel

Mike D
Manik Uppal
Todd Trimble

Federer Fanatic

Forrest Samuel, Harmos Consulting

Annie Wynn
Norman and Marcia Dresner

Daniel Mattingly
James W. Crosby

Jennifer Booth
Greg Randolph

Dave and Karen Deeter

Sarah Truebe

Jeffrey Salfen
Birian Abelson

Logan McDonald

Brian Truebe
Jon Leland

Sarah Lim

James Turnbull

John Huerta
Katie Mandel Bruce
Bethany Summer

Anna Gladstone

Naom Hart
Aaron Riley

Giampiero Campa

Julie A. Sylvia

Pace Willisson

Bangskij

Peter Herschberg

Alaistair Farrugia

Conor Hennessy

Stephanie Mohr

Torinthiel

Lincoln Muri
Anet Ferwerda

Hanna

Michelle Lee Guiney

Ben Doherty
Trace Hagemann

Ryan Mannion

Penni and Terry O'Hearn

Brian Bassham
Caitlin Murphy
John Verran

Susan

Alexander Hawson
Fabrizio Mafessoni
Anita Phagan
Nicolas Acuña
Niklas Brunberg

Adam Luptak
V. Lazaro Zamora

Branford Werner
Niklas Starck Westerberg
Luca Zenti and Marta Veneziano

Ilja Preuß
Christopher Flint

George Read
Courtney Leigh

Katharina Spoerri

Daniel Risse

Hanna
Charles-Etienne Jamme
rhackman41

Jeff Leggett

RKBookman

Aaron Paul
Mike Metzler

Patrick Leiser

Melinda

Ryan Vaughn
Kent Crispin

Michael Teague

Ben

Fabian Bach
Steven Canning

Betsy McCall

John Rees

Mary Peters

Shane Claridge
Thomas Negovan
Tom Grace
Justin Jones

Jason Mitchell

Josh Weber
Rebecca Lynne Hanginger
Kirby

Dawn Conniff

Michael T. Astolfi

Kristeva

Erik
Keith Uber

Elaine Mazerolle
Matthieu Walraet

Linda Penfold

Lujia Liu

Keith

Samar Tareem

Henrik Almén
Michael Deakin

Erin Bassett
James Crook

Junior Eluhu
Dan Laufer
Carl
Robert Solovay

Silica Magazine

Leonard Saers
Alfredo Arroyo García

Larry Yu

John Behemonth

Eric Humphrey

Øystein Risan Borgersen
David Anderson Bell III

Ole-Morten Duesend

Adam North and Gabrielle Falquero

Robert Biegler

Qu Wenhao

Steffen Dittmar

Shanna Germain

Adam Blinkinsop

John WS Marvin (Dread Unicorn Games)

Bill Carter
Darth Chronis

Lawrence Stewart

Gareth Hodges

Colin Backhurst
Christopher Metzger

Rachel Gumper

Mariah Thompson

Falk Alexander Glade
Johnathan Salter

Maggie Unkefer
Shawna Maryanovich

Wilhelm Fitzpatrick
Dylan "ExoByte" Mayo
Lynda Lee

Scott Carpenter

Charles D, Payet
Vince Rostkowski

Tim Brown
Raven Daegmorgan
Zak Brueckner

Christian Page

Adi Shavit

Steven Greenberg
Chuck Lunney

Adriel Bustamente

Natasha Anicich

Bram De Bie
Edward L

Gray Detrick
Robert

Sarah Russell

Sam Leavin

Abilash Pulicken

Isabel Olondriz
James Pierce
James Morrison

April Daniels

José Tremblay Champagne

Chris Edmonds

Hans & Maria Cummings
Bart Gasiewiski

Andy Chamard

Andrew Jackson

Christopher Wright

ichimonji10

Alan Stern
Alison W

Dag Henrik Bråtane

Martin Nilsson

William Schrade

For my March 2017 diary, go here.

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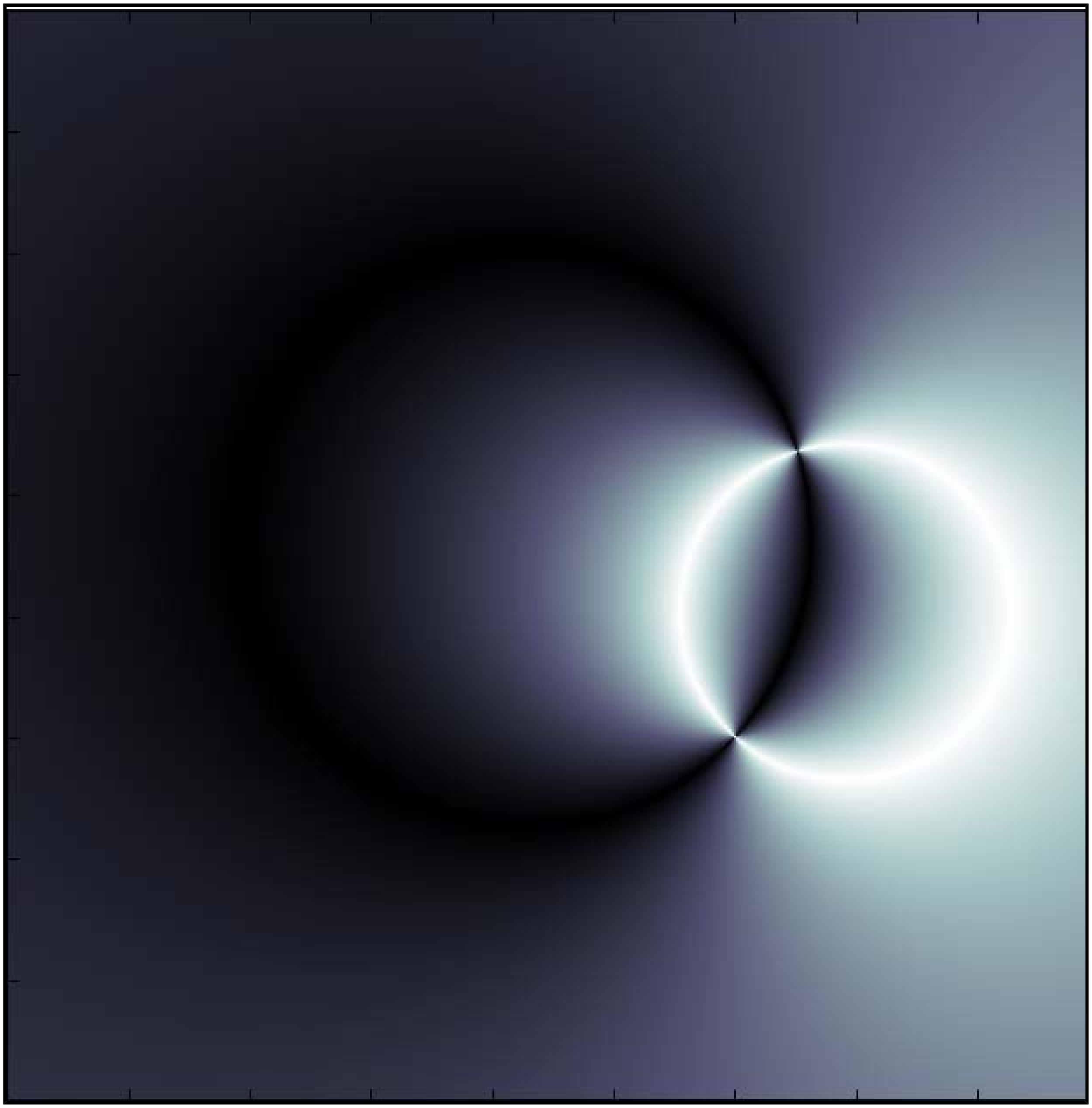
For my February 2017 diary, go here.

Diary — March 2017

John Baez

March 11, 2017

When light kisses darkness



This is one of many beautiful images on Thomas Baruchel's blog. They depict functions on the complex plane. Some are exquisitely baroque. This one is delightfully simple: a circle of light intersecting a larger circle of darkness. Its intense contrast reminds me of a solar eclipse.

The function here, like most on the blog, is supposedly defined by a continued fraction:

Loading [MathJax]/jax/output/HTML-CSS/jax.js

$$\cfrac{z\exp(2\pi i/3)}{z+\cfrac{z\exp(4\pi i/3)}{z/2+\cfrac{z\exp(6\pi i/3)}{z/3+}}}$$

He says that "white parts on the picture are real values; black parts are imaginary ones." That doesn't fully explain how the numbers get turned into shades of gray. It would be nice to know the exact recipe. A more obvious choice would be to use the color wheel to describe the phase of a complex number and brightness or intensity to describe its absolute value. But the simplicity of a grayscale image pays off in a kind of classic beauty.

Here's the image on Baruchel's blog:

- Thomas Baruchel, #146, March 6, 2017.

It's number 146 of a long series. He has threatened to produce three a day — and so far he seems to be keeping up!

For my April 2017 diary, go here.

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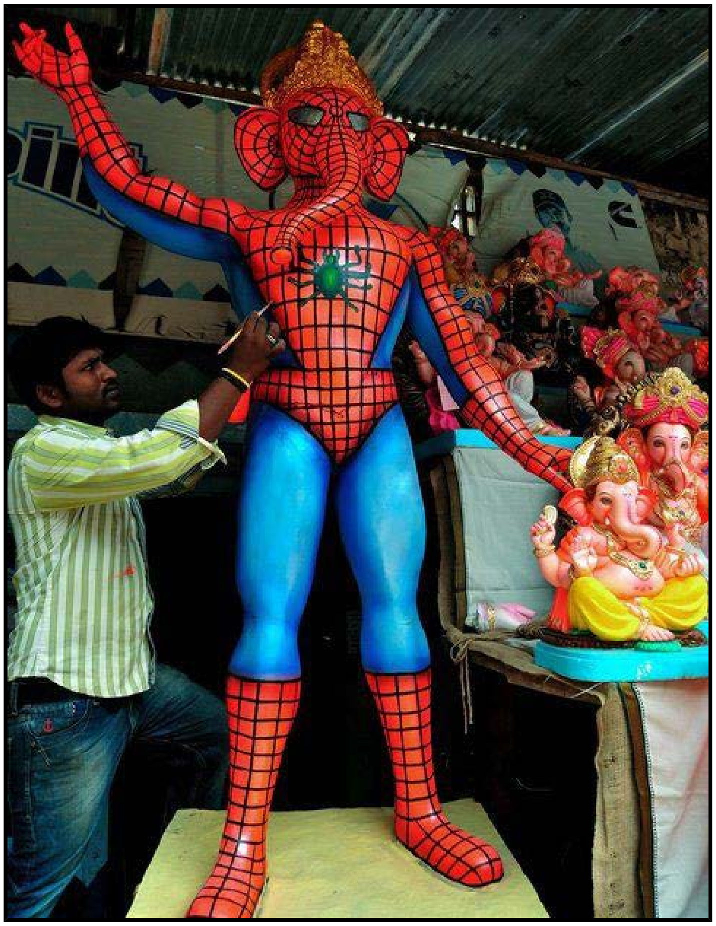
[For my March 2017 diary, go here.]

Diary — April 2017

John Baez

April 2, 2017

Spider-Ganesha



It's a natural combination. Ganesha is one of the most beloved of the Hindu gods. Kids love him, because after all, just how cool is an elephant-headed god? But he's also revered as the remover of obstacles, the patron of arts and sciences — and the god of beginnings, honored at the start of ceremonies. He first appeared around the 5th century AD, and he's been spreading ever since. Even many Buddhists and Jains like him.

Spider-Man is one of the most beloved of the Marvel Comics superheroes. He has super strength, extreme agility, a 'spider-sense' for detecting foes, and the ability to cling to most surfaces and shoot spiderwebs using wrist-mounted devices of his own invention. And yet he's approachable, since he's also Peter Parker, a photographer at the Daily Bugle with problems like our own.

I don't know who invented Spider-Ganesha, and I don't know if this blend will catch on. But it could — the line between gods and superheroes is smaller than monotheists might think. They say that after his death Hercules ascended to Olympus. There was an Egyptian cult honoring Alexander the Great from the 3rd to the 1st centuries BC. And the historical Chinese general Guan Yu was deified about 300 years after his death. I've seen plenty of statues of him in Taoist and Buddhist temples in Shanghai and Singapore — and in Hong Kong, you can find him in every police station!

If we're going to have gods and heroes, I say we should have lots, and do it with a playful, relaxed attitude, enjoying them without 'believing' in them.

So, three cheers for Spider-Ganesha!

Read a great discussion of this in the comments on my G+ post.

April 6, 2017

Trillions of warriors, in a battle visible from space

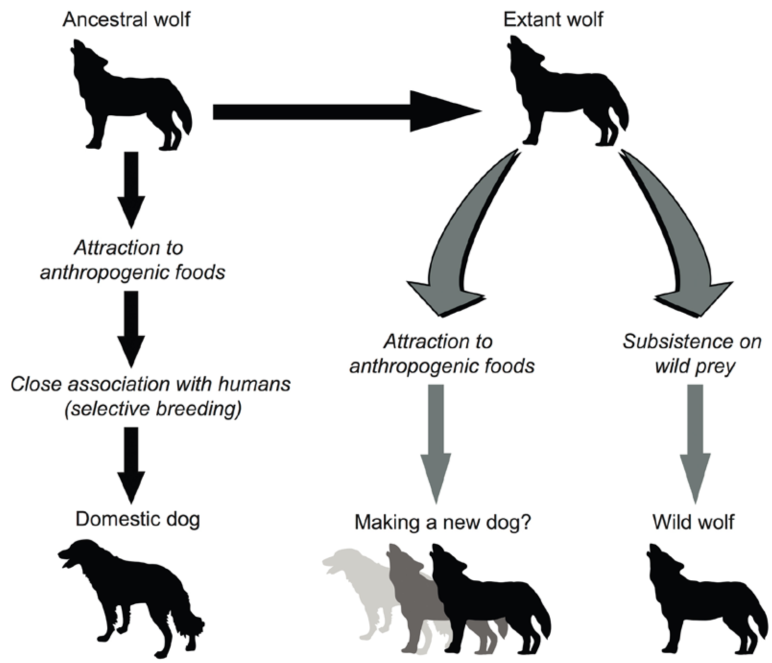
See the murky cloud in the water? It's made of dying warriors — tiny sea creatures called coccolithophores who are fighting viruses, losing, dying and falling to the sea floor.

It's not an unusual event. It happens around the globe all the time. This war has been going on for millions of years. The combatants have evolved intricate strategies to outwit each other. And most interestingly, the way this battle plays out is crucial for all oxygen-breathing life on this planet.

Listen to the story here. You won't regret it! It's well-told, it's thrilling, and it will make you think of the world in a new way:

- A war we need, *Radiolab*, March 5, 2012.

April 8, 2017



Dogs are now considered to be the same species as wolves. They can interbreed with wolves just fine. They've just evolved to look and act different through interaction with us. They eat things wolves wouldn't touch. Dingoes, in Australia, are semi-wild dogs that went through a similar evolution.

Now that humans have taken over the world, there is very little true wilderness. In most places where wolves roam, they encounter people. They have the option of trying to get food from human sources. It's often easier than hunting.

This means that all wolves are evolving into something new. They're roaming less, getting less scared of people. We're "making a new dog".

That's what this paper is about:

- Thomas M. Newsome *et al*, Making a new dog?, *BioScience* **67** (2017), 374–381.

And as humans encroach on their range, wolves are having more trouble finding mates. Sometimes they mate with domestic dogs. But mainly they're starting to interbreed with coyotes! This especially true in the northeast US. There are now zones where coyote populations are more wolf-like. They've got wolf genes affecting their body size and proportions.

So: nature is doing its thing. There is no sharp separation between nature and culture, civilization and wilderness. The rapid changes in human culture are rippling through the whole biosphere in a myriad of ways.

April 30, 2017

Back in old Hong Kong!

Why "old" Hong Kong? One reason is that Hoagie Carmichael song, the Hong Kong Blues. It starts like this:

> *It's the story of a very unfortunate colored man*
> *Who got arrested down in old Hong Kong*
> *He got twenty years privilege taken away from him*
> *When he kicked old Buddha's gong.*

It's featured in the great Bogart–Bacall movie *To Have and Have Not*. You can see the scene here:

It's a cheesy bit of orientalism made tolerable by Hoagie's charm. But Hong Kong is indeed full of history and mystery, so "old Hong Kong" sounds right.

Anyway, we're back! Today Lisa and I went to the jade market in Yau Ma Tei. Our favorite jade seller was not there: she's visiting relatives in China. Her husband was, and he showed us some nice white jade from Xinjiang — the wild west of China. This is getting rare these days, but we decided not to buy any until the woman comes back in 10 days. It gives us an excuse to postpone difficult decisions — and an excuse to return.

We also took a look in the Tin Hau temple near the jade market. Tin Hau is the god of the sea, a favorite of sailors. But the little figures shown here are some of the sixty Tai Sui deities one for each year in a 60-year cycle formed by multiplying the 12 signs of the zodiac by the 5 phases: wood, fire, earth, metal and water. Like many Chinese temples I've seen, this one has statues of all sixty. But how they look varies immensely from temple to temple!

Then we walked north to Mong Kok, a very busy area full of shops. It was densely packed with people — maybe because it's a long weekend with May Day coming on Monday? There was a long line for this food stall:

Lisa was happy to see that the Mong Kok computer center, a building packed with useful small stores, has been reopened. We bought some crucial VGA/micro-D converters and went back to our hotel, exhausted but glad to be back in old Hong Kong.

For my May 2017 diary, go here.

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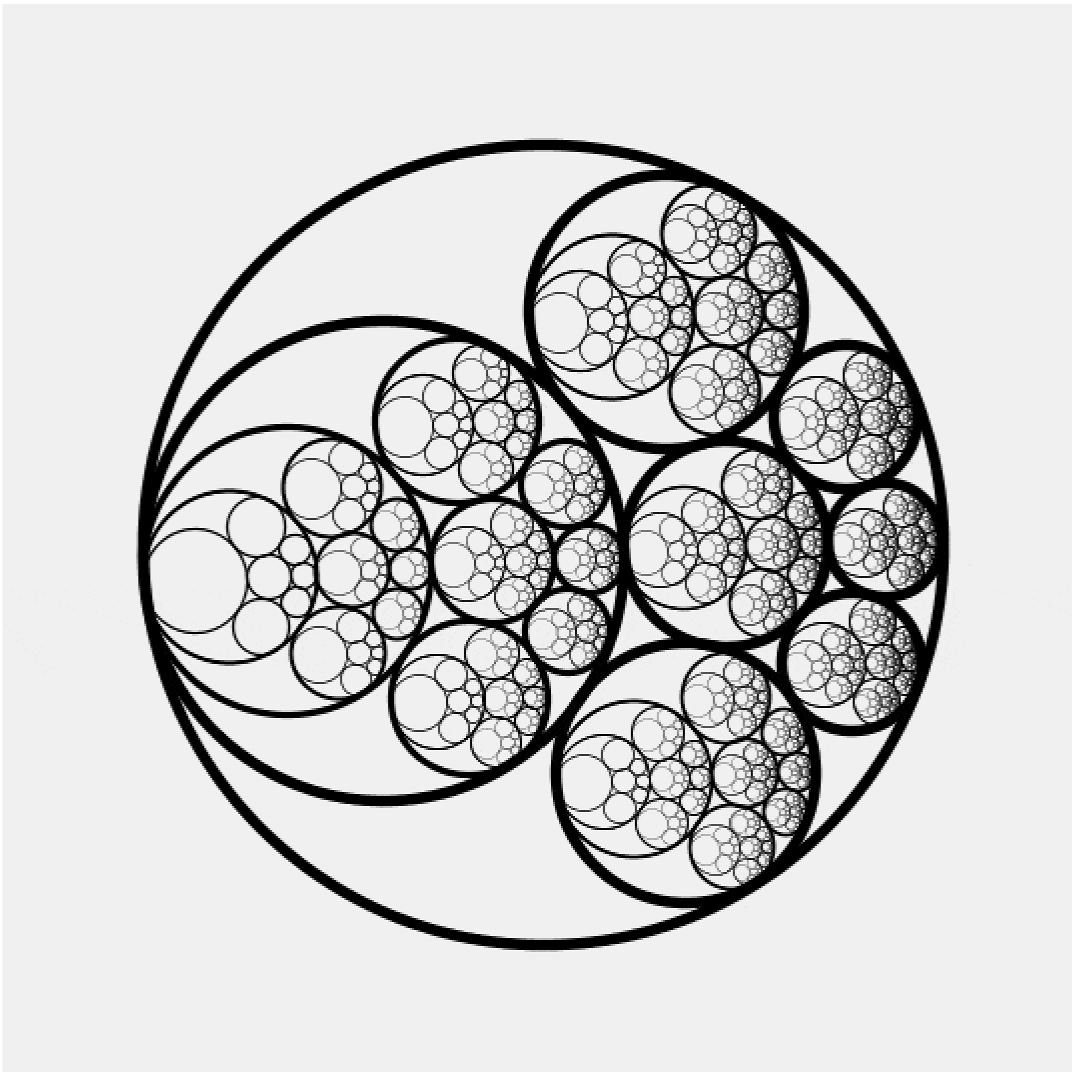
Diary — May 2017

John Baez

May 17, 2017



This beautiful animation by Gábor Damásdi illustrates an amazing result of Jakob Steiner. Namely: if you can snugly fit some circles inside one circle and outside another, you can move these circles around while they stay touching! They may need to change size, though.

Just for fun, this animation goes ahead and recursively uses the same pattern inside each of the smaller circles, ad infinitum.

This result by Steiner, proved in the 1800s, is usually called 'Steiner's porism'.

What the heck is a 'porism'?

This is one of those scary Greek math words like 'syzygy' and 'plethysm' — words that nobody ever seems to explain in a clear, intuitive way. It's not promising that the Wikipedia entry for 'porism' begins:

> The subject of porisms is perplexed by the multitude of different views which have been held by geometers as to what a porism really was and is.

In brief, a porism is something in between a problem and a theorem. Here's what Wikipedia says:

> The older geometers regarded a theorem as directed to proving what is proposed, a problem as directed to constructing what is proposed, and finally a porism as directed to finding what is proposed.

Got it?

Gábor Damásdi has a fun page on Tumblr:

- Gábor Damásdi, Symmetry.

He writes:

> Hi there!
>
> I am a Hungarian math student, currently doing my master degree at Eötvös Loránd University in Budapest. In my free time I like to draw mathematical stuff like fractals, tillings, tessellations, polyhedrons and so on.
>
> I usually use the following programs to create them: Processing, Geogebra, Gimp, Inksckape. If you want to do similar pictures this is a good place to start: processing.org.
>
> I also organize math camps and math competitions in Hungary. I usually work with a really good foundation called the "The joy of thinking foundation".If you are interested you can find more information here: http://agondolkodasorome.hu/en/.

Jakob Steiner did a lot of fundamental work in projective geometry in the 1800s. A contemporary described him this:

> He is a middle-aged man, of pretty stout proportions, has a long intellectual face, with beard and moustache and a fine prominent forehead, hair dark rather inclining to turn grey. The first thing that strikes you on his face is a dash of care and anxiety, almost pain, as if arising from physical suffering - he has rheumatism. He never prepares his lectures beforehand. He thus often stumbles or fails to prove what he wishes at the moment, and at every such failure he is sure to make some characteristic remark.

Here is some information about his porism:

- Steiner chain, Wikipedia.

A **Steiner chain** is a ring of circles, all touching, that fit snugly inside one circle and outside another.

May 22, 2017

In the news I heard about a book of photos by Marcel Heinjen called *Hong Kong Shop Cats*. Here's one of those photos:

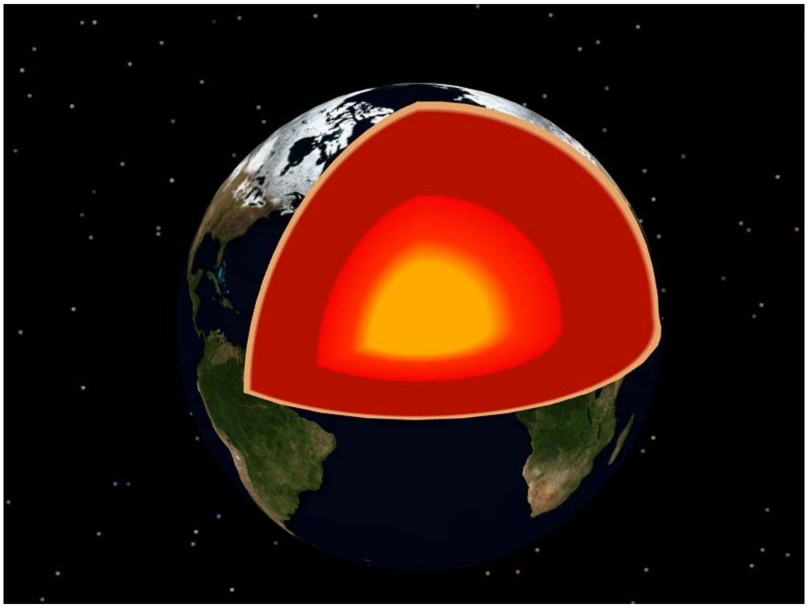
It captures the quaintness of some older Hong Kong shops as well as the charm of the cat. A lot of older neighborhoods are getting displaced by fancy skyscrapers, but Lisa and I spent a lot of time in Ya Mau Tei, an area which retains its charm. That's where Lisa visits the jade market. And that's where I saw this cat in a store window, sleeping next to a statue of a cat:

We also visited a lot of temples. Here is Lisa in the Nan Lian Garden, near a Buddhist nunnery:

and here's another view of that garden:

May 29, 2017

Set-theoretic geology



Set theory starts out as a very simple way of organizing our thoughtsb something every student should learn. But it gets more tricky when we start pondering infinite sets. And when we start pondering the universe — the collection of all sets — it gets a lot harder. Mathematicians have learned that there are obstacles to fully understanding the universe.

The collection of all sets can't be a set — Bertrand Russell and other logicians discovered this over a century ago. But more importantly, Gödel's theorem puts limits on how well any axioms can pin down the properties of the universe. Most mathematicians like to use the Zermelo-Fraenkel axioms together with the axiom of choice. But there are many questions left unsettled by these axioms.

Knowing this, you might give up on trying to fully understand the universe. That's actually what most mathematicians do. Frankly, the questions left unsettled by the ZFC axioms don't seem very urgent to most of us!

But set theorists don't give up. They've developed a lot of fascinating ways to make progress despite the obstacles.

In the 1960s, Paul Cohen introduced forcing. This is a way to make the universe larger, by making up a bunch of new sets, without violating the axioms you're using.

If I think the universe is U, you can use forcing to say "fine, but it's equally consistent to assume the universe is some larger collection V". Cohen used this to show the axiom of choice couldn't be proved from the other axioms in ZFC. Given a universe U where the Zermelo-Fraenkel axioms hold, he used forcing to build a bigger universe V where those axioms still hold, but the axiom of choice does not!

As an undergrad, I gave up my studies of set theory before I learned forcing. It was too hard to understand, and probably too badly explained: I don't think anyone even said what I just told you! I moved on to other things - there's a lot of fun stuff to learn. But for modern set theorists, forcing is utterly basic.

So what's new?

One new thing is 'set-theoretic geology':

- Joel David Hamkins, Set-theoretic geology and the downward-directed grounds hypothesis, CUNY Set Theory seminar, September 2016.

In this approach to set theory, instead of making the universe larger, you make it smaller. You try to 'dig down' and find the smallest possible universe!

So, starting with some universe V, we look for a smaller universe U that can give rise to V by forcing. If this is true, we call U a **ground** for V.

There can be lots of grounds for a universe V. This raises a big question: if we have two grounds for V, is there a ground that's contained in both?

In 2015, Toshimichi Usuba showed this is true! In fact he showed that for any set of grounds of V, there's a ground contained in all of these.

This raises another big question: is there a smallest ground, a ground contained in all other grounds? If so, this is called the **bedrock** of our universe V.

Usuba showed that the bedrock exists if a certain kind of infinite number exists! There are different sizes of infinity, and this particular kind is called 'hyper-huge'. It's so huge that it's not even explained in the Wikipedia article on huge cardinals. So, I can't explain it to you, or even to myself.

But still, I think I get the basic idea: if we have a large enough infinity, digging down infinitely far that much will get us down to the bedrock of the universe.

Naively, I tend to favor small universes. So, the bedrock appeals to me. However, you need a big universe to have large infinities like 'hyper-huge cardinals'. So, my minimalist philosophy runs into a problem, because your universe needs to contain big infinities for you to 'have time' to dig deep enough to hit bedrock!

Is this a paradox? Certainly not in the literal sense of a logical contradiction. But how about in the sense of something bizarre that makes no sense?

Probably not. There's a way to take the universe and divide into 'levels', called the von Neumann hierarchy. If you assert the existence of large cardinals, you're making the universe 'taller' — you're adding extra levels. But if you stick in extra sets by forcing, you might be making the universe 'wider' — that is, adding more sets at existing levels. So, you may need a super-huge cardinal to have enough time to chip away at the stuff in all these levels until you hit bedrock.

This is just my guess; I'm no expert. For more information from an actual expert try Joel David Hamkins' article.

He talks about a concept called the 'mantle', without explaining it. But he explains it in a comment to his post: the mantle of the universe is the intersection of all grounds. If there's a hyper-huge cardinal, this must be the bedrock. If not, other things can happen.

For my June 2017 diary, go here.

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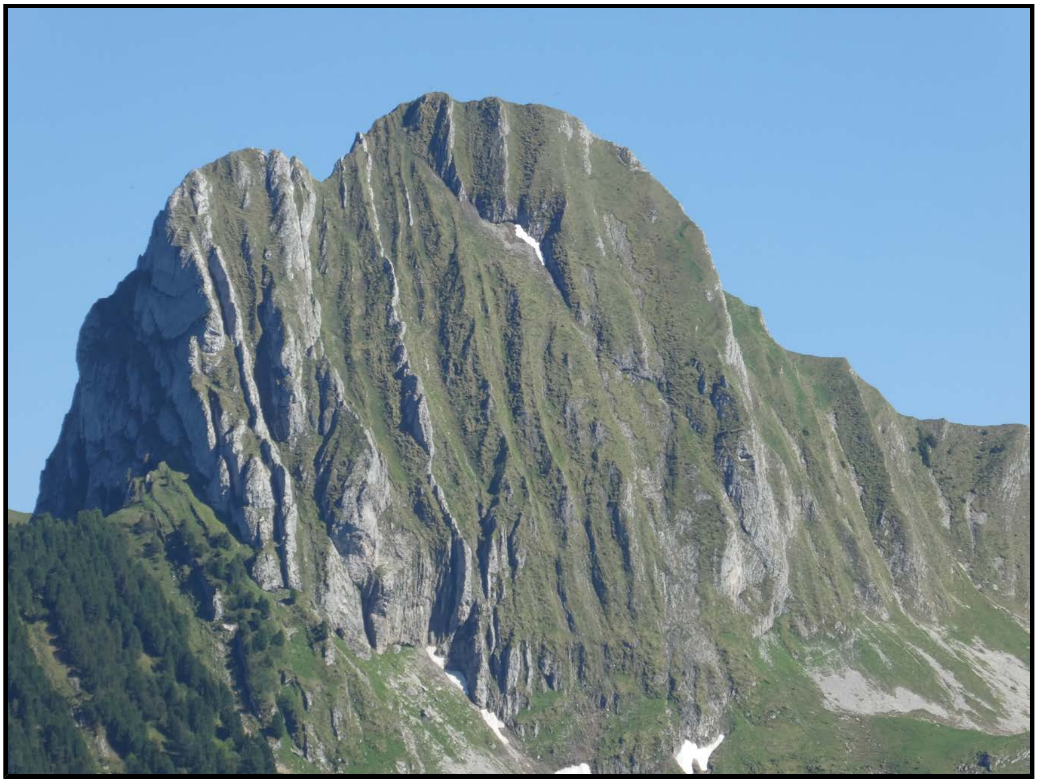
Diary — June 2017

John Baez

June 8, 2017



Yesterday I took a hike in the Alps. This was my favorite Alp. It's not so tall as Alps go, but it's quite steep and remarkably green.

Lisa was attending a conference near Bern. It was called The Art of Feeling. That sounds strange, but it was about classical Greek and Chinese philosophy, and how they understood the role of the emotions in the good life. Since Lisa knows both classical Greek and classical Chinese, and not too many people do, I've gotten to know most of the people who do, and I wind up hanging out with them in unusual places. This particular conference was held in the countryside, near the town of Rubigen, in a place that's a kind of graduate school for farmers. It had cows and pigs and gardens but also classrooms and dorm rooms, and for some reason they let a bunch of philosophers pay to stay there for a few days.

One important feature of academic conferences is the 'excursion'. If you're not an academic you may not know what this is, but if you are you surely do. About halfway through the conference, when people are getting sick of spending 8 hours a day cooped up in a room listening to talks, the organizers take people out to one of the beautiful nearby places where everyone would rather have been all along. Then people have fun, stay up too late talking and drinking, and come in bleary-eyed and grumpy to the next day's talks.

This particular excursion was especially fun: a hike through the low Alps near Gurnigel Pass, about 35 kilometers south of Bern. It was a beautiful day, and we had a nice view of the more serious Alps further south: Eiger, Mönch and Jungfrau. They were distant, snowy, forbidding yet alluring. We didn't her mountain! But it was fun to see. It may be called Nünenenfluh.

June 15, 2017

Far above a thunderstorm in the English Channel, red sprites are dancing in the upper atmosphere.

You can't usually see them from the ground — they happen 50 to 90 kilometers up. People usually photograph them from satellites or high-flying planes. But this particular bunch was videotaped from a distant mountain range in France by Stephane Vetter, on May 28th.

Sprites are quite different from lightning. They're not electric discharges moving through hot plasma. They involve *cold* plasma — more like a fluorescent light.

They're quite mysterious. People with high speed cameras have found that a sprite consists of *balls of cold plasma, 10 to 100 meters across*, shooting downward at speeds up to *10% the speed of light*... followed a few milliseconds later by a separate set of upward moving balls!

Sprites usually happen shortly after a lightning bolt. And about 1 millisecond before a sprite, people often see a 'sprite halo': a faint pancake-shaped burst of light approximately 50 kilometres across 10 kilometres thick.

Don't mix up sprites and ELVES — those are something else, for another day:

Wikipedia, Sprite.
- Wikipedia, Upper-atmospheric lightning: ELVES.

You also shouldn't confuse sprites with terrestrial gamma-ray flashes. Those are also associated to thunderstorms, but they actually involve antimatter::

- Wikipedia, Terrestrial gamma-ray flash.

A lot of weird stuff is happening up there!

The photo is from here:

- Astronomy Picture of the Day, Red sprites over the Channel, June 15, 2017.

June 25, 2017

My real name is Cleo, I'm female.
I have a medical condition that makes it very difficult for me to engage in conversations, or post long answers, sorry for that.
I like math and do my best to be useful at this site,
although I realize my answers might be not useful for everyone.

There's a website called Math StackExchange where people ask and answer questions. When hard integrals come up, Cleo often does them — with no explanation! She has a lot of fans now.

Oksana Gimmel: Please help me to find a closed form for this integral:

$$I = \int_0^1 \frac{\ln^3(1+x)\ln x}{x}\,dx$$

Cleo: Indeed, there is a closed form for this integral:

$$I = \frac{\pi^2}{3}\ln^3 2 - \frac{2}{5}\ln^5 2 + \frac{\pi^2}{2}\zeta(3) + \frac{99}{16}\zeta(5) - \frac{21}{4}\zeta(3)\ln^2 2 - 12\operatorname{Li}_4\left(\frac{1}{2}\right)\ln 2 - 12\operatorname{Li}_5\left(\frac{1}{2}\right)$$

The integral here is a good example. When you replace $\ln^3(1+x)$ by $\ln^2(1+x)$ or just $\ln(1+x)$, the answers were known. The answers involve the third Riemann zeta value:

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} +$$

They also involve the fourth polylogarithm function:

$$\operatorname{Li}_4(x) = \frac{x}{1^4} + \frac{x^2}{2^4} + \frac{x^3}{3^4} +$$

Cleo found that the integral including $\ln^3(1+x)$ can be done in a similar way — but it's much more complicated. She didn't explain her answer... but someone checked it with a computer and showed it was right to 1000 decimal places. Then someone gave a proof.

The number

$$\zeta(3) = 1.2020569031595942853997381615114499907649862923...$$

is famous because it was proved to be irrational only after a lot of struggle. Apéry found a proof in 1979. Even now, nobody is sure that the similar numbers $\zeta(5), \zeta(7), \zeta(9), \ldots$ are irrational, though most of us believe it. The numbers $\zeta(2), \zeta(4), \zeta(6), \ldots$ are much easier to handle. Euler figured out formulas for them involving powers of π, and they're all irrational.

But here's a wonderful bit of progress: in 2001, Wadim Zudilin proved that *at least one* of the numbers $\zeta(5), \zeta(7), \zeta(9),$ and $\zeta(11)$ must be irrational. Sometimes we can only snatch tiny crumbs of knowledge from the math gods, but they're still precious.

For Cleo's posts, go here:

- Math StackExchange, Cleo.
- Integrals and Series, CleoMSE — Posts.

For more on $\zeta(3)$, go here:

- Wikipedia, Apéry's constant.

This number shows up in some physics problems, like computing the magnetic field produced by an electron! And that's just the tip of the iceberg: there are deep connections between Feynman diagrams, the numbers $\zeta(n)$, and mysterious mathematical entities glimpsed by Grothendieck, called 'motives'. Very roughly, a motive is what's left of a space if all you care about are the results of integrals over surfaces in this space.

The world record for computing digits of $\zeta(3)$ is currently held by Dipanjan Nag: in 2015 he computed 400,000,000,000 digits. But here's something cooler: David Broadhurst, who works on Feynman diagrams and numbers like $\zeta(n)$, has shown that there's a linear-time algorithm to compute the nth binary digit of $\zeta(3)$:

- David Broadhurst, Polylogarithmic ladders, hypergeometric series and the ten millionth digits of $\zeta(3)$ and $\zeta(5)$.

He exploits how Riemann zeta values $\zeta(n)$ are connected to polylogarithms... it's easy to see that

$$\mathrm{Li}_n(1) = \zeta(n)$$

but at a deeper level this connection involves motives. For more on polylogarithms, go here:

- Wikipedia, Polylogarithm.

Thanks to David Roberts for pointing out Cleo's posts on Math StackExchange!

June 27, 2017

How did the publisher Elsevier get profit margins of 37% last year - higher than almost any other business? Simple: get people to work for free, then sell their product at high prices!

But how do you do that? Over on Google+, Richard Poynder pointed out this great article which explains the history:

- Stephen Buranyi, Is the staggeringly profitable business of scientific publishing bad for science?, *The Guardian*, June 27, 2017.

It started with Robert Maxwell, a clever fellow who knew that flattering top scientists would get them to publish in his journals.... making them "prestigious". He also knew the advantages of publishing lots of journals:

> Maxwell's success was built on an insight into the nature of scientific journals that would take others years to understand and replicate. While his competitors groused about him diluting the market, Maxwell knew that there was, in fact, no limit to the market. Creating *The Journal of Nuclear Energy* didn't take business away from rival publisher North Holland's journal *Nuclear Physics*. Scientific articles are about unique discoveries: one article cannot substitute for another. If a serious new journal appeared, scientists would simply request that their university library subscribe to that one as well. If Maxwell was creating three times as many journals as his competition, he would make three times more money.

Later, publishers got more systematic about making their journals "prestigious"... so scientists would want to publish in them... and get their universities to subscribe to these journals:

> "At the start of my career, nobody took much notice of where you published, and then everything changed in 1974 with *Cell*," Randy Schekman, the Berkeley molecular biologist and Nobel prize winner, told me. *Cell* (now owned by Elsevier) was a journal started by Massachusetts Institute of Technology (MIT) to showcase the newly ascendant field of molecular biology. It was edited a young biologist named Ben Lewin, who approached his work with an intense, almost literary bent. Lewin prized long, rigorous papers that answered big questions — often representing years of research that would have yielded multiple papers in other venues — and, breaking with the idea that journals were passive instruments to communicate science, he rejected far more papers than he published.
>
> What he created was a venue for scientific blockbusters, and scientists began shaping their work on his terms. "Lewin was clever. He realised scientists are very vain, and wanted to be part of this selective members club; *Cell* was 'it', and you had to get your paper in there," Schekman

said. "I was subject to this kind of pressure, too." He ended up publishing some of his Nobel-cited work in *Cell*.

> Suddenly, where you published became immensely important.

Read the whole story! It's depressing, but we need to understand why we're in this mess to get out of it.

Also, read [Richard Poynder's posts on Google+](), to keep track of the scholarly publishing world and attempts to fix it.

June 30, 2017

Today Sabine Hossenfelder wrote a nice attack on 'naturalness' in physics:

- Sabine Hossenfelder, The understand the foundations of physics, study numerology, *Backreaction*, June 30, 2017.

There's a particle called the muon that's almost like the electron, except it's about 206.768 times heavier. Nobody knows why. The number 206.768 is something we measure experimentally, with no explanation so far. Theories of physics tend to involve a bunch of unexplained numbers like this. If you combine general relativity with Standard Model of particle physics, there are about 25 of these constants.

Many particle physicists prefer theories where these constants are not incredibly huge and not incredibly tiny. They call such theories 'natural'. Naturalness sounds good — just like whole wheat bread. But there's no solid evidence that this particular kind of naturalness is really a good thing. Why should the universe prefer numbers that aren't huge and aren't tiny? Nobody knows.

For example, many particle physicists get upset that the density of the vacuum is about

0.00000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000001

Planck masses per Planck volume. They find it 'unnatural' that this number is so tiny. They think it requires 'fine-tuning', which is supposed to be bad.

I agree that it would be nice to explain this number. But it would also be nice to explain the mass of the muon. Is it really more urgent to explain a tiny number than a number like 206.768, which is neither tiny nor huge?

Sabine Hossenfelder say no, and I tend to agree. More precisely: I see no *a priori* reason why naturalness should be a feature of fundamental physics. If for some mysterious reason the quest for naturalness always, or often, led to good discoveries, I would support it. In science, it makes sense to do things because they tend to work, even if we're not sure why. But in fact, the quest for naturalness has not always been fruitful. Sometimes it seems to lead us into dead ends.

Besides the cosmological constant, another thing physicists worry about is the Higgs mass. Avoiding the 'unnaturalness' of this mass is a popular argument for supersymmetry... but so far that's not working so well. Hossenfelder writes:

> Here is a different example for this idiocy. High energy physicists think it's a problem that the mass of the Higgs is 15 orders of magnitude smaller than the Planck mass because that means you'd need two constants to cancel each other for 15 digits. That's supposedly unlikely, but please don't ask anyone according to which probability distribution it's unlikely. Because they can't answer that question. Indeed, depending on character, they'll either walk off or talk down to you. Guess how I know.
>
> Now consider for a moment that the mass of the Higgs was actually about as large as the Planck mass. To be precise, let's say it's 1.1370982612166126 times the Planck mass. Now you'd again have to explain how you get exactly those 16 digits. But that is, according to current lore, not a finetuning problem. So, erm, what was the problem again?

She explains things in such down-to-earth terms, with so few of the esoteric technicalities that usually grace discussions of naturalness, that it may be worth reading a more typical discussion of naturalness just to imbibe some of the lore.

This one is quite good, because it includes a lot of lore but doesn't try too hard to intimidate you into believing in the virtue of naturalness:

- G. F. Giudice, Naturally speaking: the naturalness criterion and physics at the LHC.

For my July 2017 diary, go here.

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home

[For my June 2017 diary, go here.](#)

Diary — July 2017

John Baez

July 1, 2017

A BLACK HOLE MOVING AT THE SPEED OF LIGHT:

$$ds^2 = -8m\,\delta(u)\,\log r\,du^2 + 2\,du\,dv + dr^2 + r^2\,d\theta^2$$

$$-\infty < u < \infty,\ 0 < r < \infty,\ -\infty < v < \infty, -\pi < \theta < \pi$$

What happens if a black hole moves at the speed of light?

Well, an ordinary black hole *can't*, because only things with no mass can move at the speed of light. If a heavy thing zips past you, its gravity will yank at you for a short time. The faster it goes, the stronger this effect will be. The yank will last for a shorter time — but its total effect on you will be bigger. If the thing moved at the speed of light, this effect would be infinite. That makes no sense.

But we can do this. Take lighter and lighter black holes and make them move faster and faster, closer to the speed of light. In the limit we have a *massless* black hole moving at the speed of light! And it's not nothing — like a photon, which is also massless and moving at the speed of light, it has *energy* and *momentum*.

We can't do this in the lab — not yet, anyway. But we can work it out mathematically. We get a solution of Einstein's equations — the equations that describe gravity. This solution has a wonderful name: it's called the **Aichelburg–Sexl ultraboost**.

When something moves near the speed of light, it actually gets thinner - this is called a **Lorentz contraction**. So, the Aichelburg–Sexl ultraboost is a pulse of gravity that's infinitely thin, moving at the speed of light, strong near the center and weaker far away.

We can also do this trick with a spinning black hole. We get a solution of Einstein's equations that describes the gravitational field of a spinning massless particle.

Okay, that was the fun part for ordinary people. Now comes the math. In spacetime without any gravity messing things up, distances and times are measured by the **Minkowski metric**:

$$-dt^2 + dx^2 + dy^2 + dz^2$$

We can write this down using other coordinates, like

$$u = \frac{x + t}{\sqrt{2}}$$

and

$$v = \frac{x - t}{\sqrt{2}}$$

in units where the speed of light is 1. These coordinates are called **lightcone coordinates**. They're nice because the surface $u = 0$ is a plane moving forwards at the speed of light — just right for what we want. We get

$$-dt^2 + dx^2 + dy^2 + dz^2 = 2dudv + dy^2 + dz^2$$

Using polar coordinates in the yz plane, so that $r^2 = y^2 + z^2$, this becomes

$$2dudv + dr^2 + r^2 d\theta^2$$

If we now include a black hole moving at the speed of light, we get an extra term, and get the formula I showed above:

$$ds^2 = -8m\delta(u)\log r du^2 + 2dudv + dr^2 + r^2 d\theta^2$$

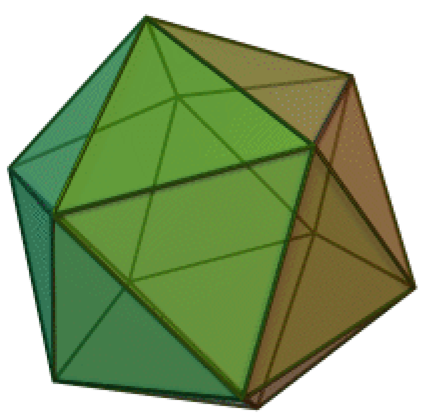
The interesting thing is the first term. This describes a *shock wave* moving at the speed of light, which becomes infinitely strong at the center of the black hole! For more, see:

- Wikipedia, Aichelburg–Sexl ultraboost.

The Aichelburg–Sexl ultraboost is just one special case of a 'pp-wave spacetime':

- Wikipedia, pp-wave spacetime.

July 3, 2017



My latest quest is to find a really simple, clear way to get E_8 from the icosahedron. These are two of my favorite things, and they're connected.

The icosahedron is a Platonic solid with 120 = 1 × 2 × 3 × 4 × 5 symmetries. Just for fun, the picture here shows a 'stellated' icosahedron with sharper points. But it has all the same symmetries, and that's all that matters to me now.

E_8 is an 8-dimensional lattice: a periodic pattern of points in 8 dimensions. This pattern gives the densest way to pack spheres in 8 dimensions: center a sphere at each lattice point, and make them big enough to just touch each other. Each sphere touches 240 others. That's the maximum possible in 8 dimensions. And in fact, if you pack spheres in 8 dimensions and get each to touch 240 others, you've got E_8. This pattern shows up all over math, in cool and mysterious ways.

E_8 has two little brothers. If you take a well-chosen slice of E_8 you get a lattice called E_8. This gives the densest known way to pack spheres in 7 dimensions. Similarly, if you take the right slice of E_8 you get a lattice called E_6, which gives the densest known way to pack spheres in 6 dimensions.

The McKay correspondence is a way to get E_6, E_8 and E_8 from the tetrahedron, the octahedron and the icosahedron! This is one of nature's true marvels. It's yet more evidence that

In mathematics, everything sufficiently beautiful is connected.

There are actually several versions of the McKay correspondence. I'm interested in one called the 'geometric' McKay correspondence. Experts already understand it, but I want to bring it down to earth a bit... and I want to go for the jugular and focus on the icosahedron and E_8.

My plan is to look at the space of all ways you can place an icosahedron of any size centered at the origin in 3d space. This space is 4 dimensional, since it takes 3 numbers to say how the icosahedron is rotated, and 1 more to say its size. And this 4-dimensional space has a singularity where the icosahedron shrinks down to zero size!

It reminds me ever so slightly of the Big Bang, where we have a 4-dimensional spacetime with a singularity where the universe shrinks down to zero size (roughly speaking). But this is just a cute analogy, the sort science journalists use to attract and confuse readers. The lazy readers only look at the headline, and come away with weird ideas. Don't be one of them.

The serious business here is seeing how E_8 is lurking in the space of all possible icosahedra centered at the origin. Where is it?

It's sitting right at the singularity!

How? How is it sitting there, you ask?

I could tell you, but then I'd have to...

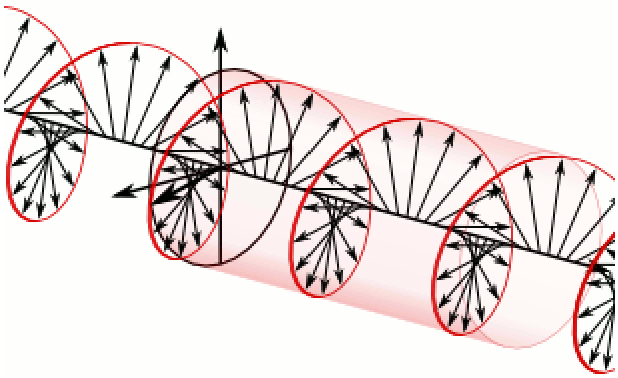
On second thought, I'll tell you here:

- John Baez, The geometric McKay correspondence (part 1), June 19, 2017.

However, you'll need to know some math to follow this. The basic idea is that if you 'smooth out' or 'resolve' the singularity, it gets replaced by 8 spheres that intersect in a pattern governed by E_8.

This is just the first part of a series, since there's a lot I still need to figure out! I want to see very concretely how these 8 spheres show up. I'm hoping some math friends of mine will help me. With luck, if we figure enough out, I can write a more polished article about it.

July 17, 2017



This is a wave of **circularly polarized light**. As the wave moves forwards, but you stand in the same place and measure the electric field there, the electric field goes round and round. The magnetic field, not shown here, also goes round and round, at right angles to the electric field.

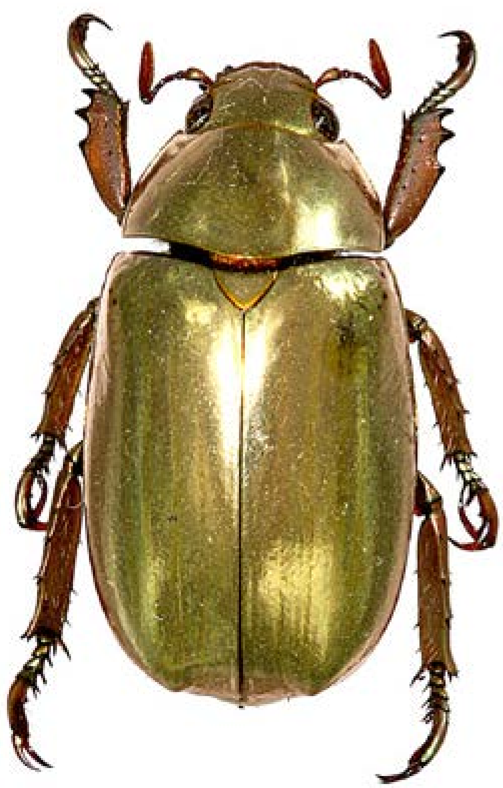
This movie shows **right** circularly polarized light. If you take your right hand, make a fist, and point your thumb in the direction the light is moving, the electric field will rotate in the direction your fingers are curling. There's also **left** circularly polarized light, where the electric field turns around in the other direction.

All this stuff can be figured out mathematically by solving the vacuum Maxwell equations, which describe light with no matter around.

But where can you see circularly polarized light in nature?

Albert Michelson found some back in 1911!

You may know this guy: he won the Nobel prize with Robert Morley for discovering that light moves past you at the same speed no matter how you're moving. But he also discovered something else. Light reflected from a certain kind of beetle called a golden scarab tends to be left circularly polarized! The reason was discovered much later: at the microscopic level, the shells of these beetles are made of spiral-shaped molecules!



Light from certain firefly larvae is also circularly polarized, but nobody knows why yet.

And sometimes starlight is circularly polarized... slightly. It's actually a messy mix of different kinds of light. Sometimes it's 'linearly' polarized — the electric field wiggles back and forth rather than round and round. This is because it scatters from elongated interstellar dust grains whose long axes tend to be oriented at right angles to the galactic magnetic field. But these grains spin rapidly, with their rotation axis along the magnetic field. This winds up creating a bit of circular polarization. The effect is tiny but measurable.

I was going to talk more about the math of circularly polarized light, but I got distracted. I wanted to explain how the polarization of light involves complex numbers. This is easier to talk about using quantum mechanics. To describe a photon with a certain energy in a certain direction, we need to use two complex numbers! A photon like

$$(1, 0)$$

is linearly polarized in one direction: say, its electric field wiggles back and forth. A photon like

$$(0, 1)$$

is linearly polarized in the other direction: say, its electric field wiggles up and down. So, a photon like

$$(1, 1)$$

would be linearly polarized in a diagonal way. But less obviously, a photon like

$$(1, i)$$

is right circularly polarized, and one like

$$(1, -i)$$

is left circularly polarized.

How did the complex numbers get into the game? We use them in quantum mechanics, but polarization of light is also there in the vacuum Maxwell equations, which were known before quantum mechanics. So the complex numbers should be lurking in the vacuum Maxwell equations!

They are. Mathematically, photons are solutions of the vacuum Maxwell equations. While these solutions involve two *real* vector fields, the electric and magnetic field, the space of solutions is a *complex* Hilbert space. To multiply a solution by i you multiply its positive-frequency part by i and its negative-frequency part by $-i$.

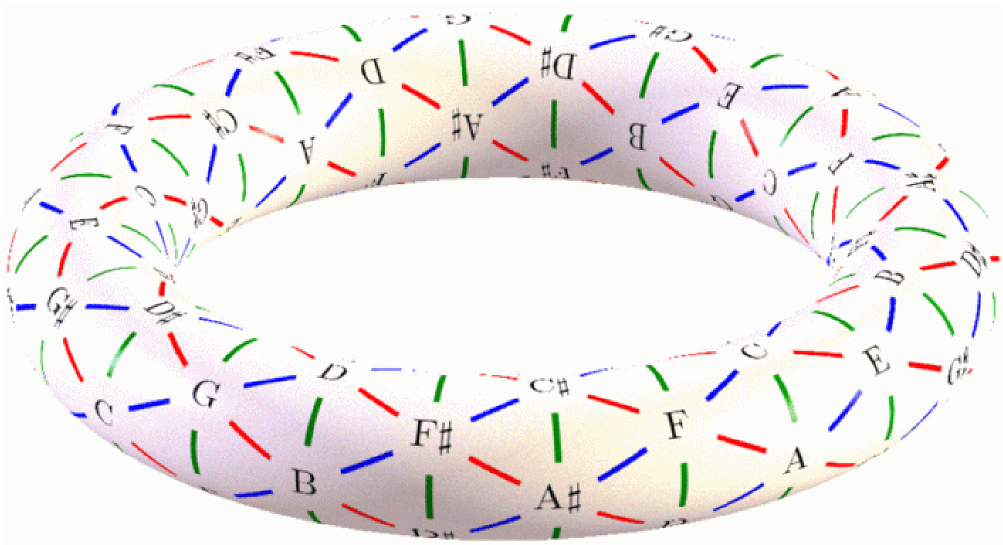
In short: to fully understand light bouncing off a scarab beetle, you need to understand how the complex numbers are lurking in Maxwell's equations. The universe is cool. Let's be kind to our planet, so our civilization can stick around long enough to learn more. We're just getting started!

For more, try these:

- Wikipedia, [Circular polarization].
- Wikipedia, [Photon polarization].

I got the animation from [Wikicommons]. You can see some interesting discussion on my [G+ post].

July 29, 2017



The geometry of music revealed! The red lines connect notes that are a **[major third]** apart. The green lines connect notes that are a **[minor third]** apart. The blue lines connect notes that are a **[perfect fifth]** apart.

Each triangle is a chord with three notes, called a **[triad]**. These are the most basic chords in Western music. There are two kinds:

A **major triad** sounds happy. The major triads are the triangles whose edges go red-green-blue as you go around clockwise.

A **minor triad** sounds sad. The minor triads are the triangles whose edges go green-red-blue as you go around clockwise.

This pattern is called a **tone net**, and this one was created by David W. Bulger. There's a lot more to say about it, and you can read more in this Wikipedia article:

- Wikipedia, Neo-Riemannian theory.

and this great post by Richard Green:

- Richard Green, Group theory in music.

The symmetry group of this tone net is important in music theory, and if you read these you'll know why!

For my August 2017 diary, go here.

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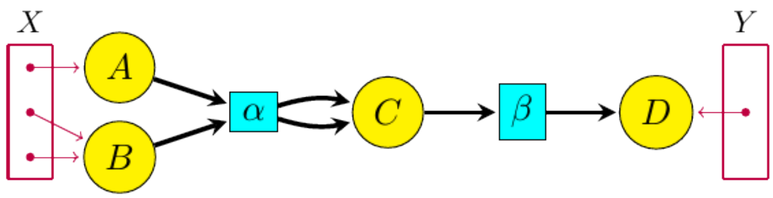
For my July 2017 diary, go here.

Diary — August 2017

John Baez

August 1, 2017



My working hypothesis is that living systems seem 'messy' to physicists because they operate at a higher level of abstraction than physicists are used to. That's what I'm trying to explore these days.

Back in 1963, Bill Lawvere had the idea that the process of assigning 'meaning' to expressions could be seen as a functor from one category to another. This idea has caught on in theoretical computer science: it's called **functorial semantics**.

The basic idea is that a program is a morphism in a category, and what it computes is a morphism in another category, and there's a functor from the first category to the second. Some programming languages like Haskell, Scheme and Scala have been designed to explicitly take advantage of this point of view.

What I want to to do is apply functorial semantics to biology. I expect that in biology there are many ways to view the 'meaning' of what's going on — there's no one best answer; instead, there are many different levels of abstraction at which we can usefully view things. Life somehow manages to exploit this.

This is hard to think about: biology is much more tricky than computer programming! So I've been starting with simpler things, like chemistry.

Blake Pollard and I have been working on **open reaction networks**: that is, networks of chemical reactions where some chemicals can flow in from an outside source, or flow out. The picture to keep in mind is shown above.

The yellow circles are different kinds of chemicals. The aqua boxes are different reactions. The purple dots in the sets X and Y are 'inputs' and 'outputs', where certain kinds of chemicals can flow in or out.

There's no serious difference between 'inputs' and 'outputs': chemical can flow in or out at any of these points. The only reason for segregating inputs and outputs is to make it easy to stick together two open reaction networks: we attach the outputs of the first to the inputs of the second.

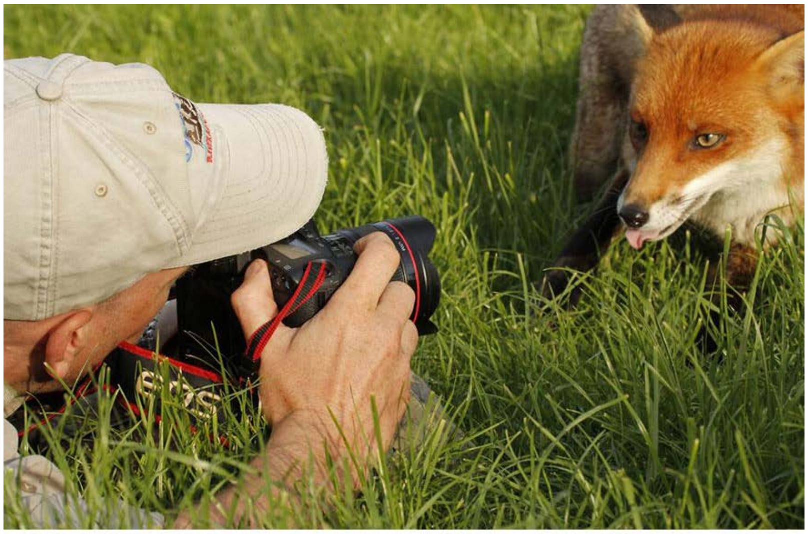
This makes open reaction networks into the morphisms of a category. The main thing you do with morphisms is compose them, and here that means attaching the outputs of one open reaction network to the inputs of another.

Blake and I figured out how to first 'gray-box' an open reaction network, converting it into an open dynamical system, and then 'black-box' it, obtaining the relation between input and output flows and concentrations that holds in steady state. The first step extracts the dynamical behavior of an open reaction network; the second extracts its static behavior. And both these steps are functors between categories!

For a more detailed story about this, go here:

 vork for reaction networks, *Azimuth*, July 30, 2017.

August 6, 2017



Forests in the east coast of the US are increasingly dangerous: there are more ticks that carry Lyme disease. My wife has a friend who got it after taking walks in the woods. But we can fight this disease — with foxes!

When you get Lyme disease, it starts with a circular rash near the tick bite. Then you may get chills and fever, a headache, fatigue, muscle and joint pain, and swollen glands. As the disease progresses you may experience severe fatigue, a stiff aching neck, and tingling or numbness in the arms and legs. Part of your face may become paralyzed. The most severe symptoms of Lyme disease may take weeks, months or *years* to appear. These can include severe headaches, painful arthritis, swelling of the joints, and problems with your heart and brain. Nasty!

Lyme disease is caused by bacteria that infect certain kinds of ticks. Why is this disease more common now?

I used to think it was the *deer*. With the lack of predators, and rules against hunting in many areas, deer populations have exploded, limited only by starvation when they eat everything they can find. The forests near my mother's home have been devastated by deer. The trees still look good, but there's no green undergrowth, so no *new* trees — because the deer eat everything. And deer carry the kind of ticks that cause Lyme disease.

But now I hear *mice* are also to blame. Mice also get infected by ticks. This year in New York there's been a big rise in ticks. And the cause is mice:

> Everybody knows about Lyme disease. But experts say the Northern United States may be in for a bad tick season this summer, raising concerns about Lyme and other scary tick-borne diseases, including the

Powassan virus, which causes encephalitis and can leave people with permanent neurological damage.

"This spring definitely seems worse than others I remember," said Dr. Catherine Wiley, chief of general pediatrics at Connecticut Children's Medical Center. "People are coming in from the yard with numerous ticks on them."

When we think of ticks, we tend to think of deer, but Richard S. Ostfeld, a senior scientist at the Cary Institute of Ecosystem Studies in Millbrook, N.Y., said it's really all about mice. He has been studying white-footed mouse population ecology for the past 25 years. Every four or five years, he said, there's a bumper acorn crop, so more mice survive the following winter, breed and reach what he called "mouse plague levels" in the summer.

These mice will be the main source of infection for the tiny larval ticks that hatch in August and can attach to many mammals and birds, which will try to groom them off. Mice "are just not fastidious groomers," Dr. Ostfeld said, so their ticks tend to survive. Those larval ticks then morph into the nymph stage and stay dormant through the following winter. And then, in late spring through early summer, the nymphs begin to feed. It's those nymphs, infected in the larval stage by mice, that transmit the infections to humans.

How can you control this? It turns out *foxes* do the job quite nicely! And they do it remarkably well: not just by *eating* mice, but by *scaring* mice so they spend more time in their burrows!

It is August, the month when a new generation of black-legged ticks that transmit Lyme and other diseases are hatching. On forest floors, suburban estates and urban parks, they are looking for their first blood meal. And very often, in the large swaths of North America and Europe where tick-borne disease is on the rise, they are feeding on the ubiquitous white-footed mice and other small mammals notorious for harboring pathogens that sicken humans.

But it doesn't have to be that way. A new study suggests that the rise in tick-borne disease may be tied to a dearth of traditional mouse predators, whose presence might otherwise send mice scurrying into their burrows. If mice were scarcer, larval ticks, which are always born uninfected, might feed on other mammals and bird species that do not carry germs harmful to humans. Or they could simply fail to find that first meal. Ticks need three meals to reproduce; humans are at risk of contracting diseases only from ticks that have previously fed on infected hosts.

For the study, Tim R. Hofmeester, then a graduate student at Wageningen University in the Netherlands and the lead researcher of the study, placed cameras in 20 plots across the Dutch countryside to measure the activity of foxes and stone martens, key predators of mice. Some were in protected areas, others were in places where foxes are heavily hunted.

Over two years, he also trapped hundreds of mice — and voles, another small mammal — in the same plots, counted how many ticks were on them, and tested the ticks for infection with Lyme and two other disease-causing bacteria. To capture additional ticks, he dragged a blanket across the ground.

In the plots where predator activity was higher, he found only 10 to 20 percent as many newly hatched ticks on the mice. Thus, there would be fewer ticks to pass along pathogens to next generation of mice. In the study, the density of infected "nymphs," as the adolescent ticks are called, was at 15 percent of levels in areas where foxes and stone martens were less active.

"The predators appear to break the cycle of infection," said Dr. Hofmeester, who earned his Ph.D. after the study.

Despite stuffing his pant legs into his socks and using permethrin, a tick repellent, he said he removed more than 100 ticks from his own body.

Interestingly, the predator activity in Dr. Hofmeester's plots did not decrease the density of the mouse

> population itself, as some ecologists had theorized it might. Instead, the lower rates of infected ticks, Dr. Hofmeester suggested in the paper, published in *Proceedings of the Royal Society B*, may be the result of small mammals curtailing their own movement when predators are around.
>
> "This is the first paper to empirically show that predators are good for your health with respect to tick-borne pathogens," said Dr. Taal Levi, an ecologist at Oregon State University who was not involved in the study. "We've had the theory but this kind of field work is really hard and takes years." He also said of Dr. Hofmeester, "Wow, I have to send him an email."
>
> Habitat fragmentation, hunting and the removal of larger predators like cougars may all figure into the dwindling of small mammal predators like foxes, weasels, fishers and martens, Dr. Levi said. If the study's results are borne out by more research, public health officials might be moved to try interventions like protecting foxes or factoring the habitat needs of particular predators into land-use decisions to foster their population size. Nothing else — like culling deer or spraying lawns with tick-killing pesticide — has worked so far to stem the incidence of tick-borne disease, which is spreading in the Midwestern United States, in parts of Canada and at higher altitudes across Europe.
>
> "The takeaway is, we shouldn't underestimate the role predators can play in reducing Lyme disease risk," said Richard S. Ostfeld, a senior scientist at the Cary Institute of Ecosystem Studies, who originally speculated on the importance of small mammal predators in a 2004 paper. "Let's not discount these cryptic interactions that we don't see very often unless we put camera traps in the woods."

So, let's try to bring back foxes to the forests of the US! Besides, they're cool in their own right.

The first quote was from here:

- Peri Klass, M.D., With a tick boom, it's not just Lyme disease you have to fear, *New York Times*, July 3, 2017.

The second is from here:

- Amy Harmon, Lyme disease's worst enemy? It might be foxes, *New York Times*, August 2, 2017.

August 16, 2017

Math made difficult: multiplying numbers using trigonometry

Back in the 1500's, people on long sea journeys navigated using the stars. They needed big tables of trig functions to do this!

These tables were made by astronomers. Those folks did thousands of calculations. Often they needed to multiply large numbers! That was tiring... but around 1580, they figured out a clever way to approximately multiply large numbers using tables of trig functions.

Here's an example:

Say you want to multiply 105 and 720. You do this:

- Shift the decimal point in each one to get numbers less than 1. You get 0.105 and 0.720.
- Look up angles whose cosines are these numbers. Use a table! The cosine of 84° is about 0.105, and the cosine of 44° is about 0.720
- Add and subtract these angles: 84° + 44° = 128° and 84° - 44° = 40°.
- Use a table to look up the cosines of these new angles: -0.616 and 0.766.
- Take their average, which is 0.075.
- Scale it back up. At the beginning of this game you took 105 and 720 and shifted the decimal point 3 places to the left in each. So now, shift the decimal point 3+3 = 6 places to the right! The answer is 75,000.

It's not exactly right, but it's pretty close!

Puzzle. Why is it close?

This wacky-sounding method has a wacky-sounding name: it's called **prosthaphaeresis**.

Tables of logarithms are easier. To multiply two numbers you just look up their logs, add them, and then look up the number whose log is that! But logs were invented only later, in 1614.

So for a while, prosthaphaeresis was the way to go!

And Napier, the guy who invented logs, did it after studying this earlier method.

It goes to show: a clunky way of doing something is often the first step toward something less clunky. You can't be slick right away!

For a nice answer to the puzzle, see Chris Greene's comment on my G+ post:

> So, let's call our original numbers x and y. And let's reduce both of them by a scale factor s to get them in the range $(0, 1)$. Then
>
> $$x/s = \cos\theta_1$$
>
> $$y/s = \cos\theta_1$$
>
> $$xy/s^2 = \cos\theta_1\cos\theta_1$$
>
> $$xy = s^2\cos\theta_1\cos\theta_1$$
>
> So if we evaluate $\cos\theta_1\cos\theta_1$ (using whatever magical method we desire) and multiply it by s^2 we're good (of mild interest, we can actually scale the numbers by different amounts and nothing changes). Noting that
>
> $$\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$
>
> we can get
>
> $$\begin{aligned}\cos\theta_1\cos\theta_1 &= \frac{1}{4}(e^{i\theta_1} + e^{-i\theta_1})(e^{i\theta_2} + e^{-i\theta_2}) \\ &= \frac{1}{4}(e^{i(\theta_1+\theta_2)} + e^{i(\theta_1-\theta_2)} + e^{i(-\theta_1+\theta_2)} + e^{i(-\theta_1-\theta_2)}) \\ &= \frac{1}{4}(e^{i(\theta_1+\theta_2)} + e^{-i(\theta_1+\theta_2)}) + \frac{1}{4}(e^{i(\theta_1-\theta_2)} + e^{-i(\theta_1-\theta_2)}) \\ &= \frac{1}{2}(\cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2))\end{aligned}$$
>
> The average of the complementary cosines! Of course, Napier took one look at that and said "That's insanely complicated! You don't need all those complex numbers and averaging! eá¶¿ is all you need! It works the exact same way, and none of this averaging the sum and and difference of angles nonsense!"
>
> **cough**
>
> Well perhaps that was a slightly anachronistic approach. Most likely, some bright person noticed (without the aid of 18th century complex math) that
>
> $$\begin{aligned}\cos(\theta_1 + \theta_1) &= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 \\ \cos(\theta_1 - \theta_1) &= \cos\theta_1\cos\theta_2 + \sin(\theta_1\sin\theta_2\end{aligned}$$
>
> and added them together and saw that
>
> $$2\cos\theta_1\cos\theta_1 = \cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)$$
>
> thus obtaining

$$\cos\theta_1 \cos\theta_1 = \frac{1}{2}(\cos(\theta_1 + \theta_2) + \cos(\theta_1 - \theta_2))$$

For my September 2017 diary, go here.

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baez@math.removethis.ucr.andthis.edu

home

[For my August 2017 diary, go here.]

Diary — September 2017

John Baez

September 1, 2017

Lisa and I spent last week in Bali. We're thinking about spending more time there sometime... maybe a month, to dip our toes a bit deeper into the water. We'd like to rent a little apartment with a kitchen, but it's a bit tough to find one.

Here are some pictures, which might help explain why we like Bali so much. Click on them for larger versions.

We stayed in Ubud, the 'cultural capital'. But one day we went to the nearby town of Batu Balan for a 'barong dance'. Here's a dancer from the opening act:

The gamelan, an orchestra of mainly percussive instruments, played throughout:

The star of the show is 'Barong', a huge lion-like beast with googly eyes played by two men in a costume:

Barong looks scary, but he's actually good: he's the king of the spirits and the mortal enemy of the demon queen 'Rangda'. The barong dance tells the story of a battle between Barong and Rangda, which represents the eternal battle between good and evil.

Here is a poor man's motorbike, decorated with coconut shells, on a trail between rice fields north of Ubud:

Here's a view from the trail:

Here's a view of the distant Mount Agung, also from this trail:

It's the largest mountain on the island, and its name literally means Mount Big.

There are sculptures everywhere, and many restaurants have gardens and rice fields in the back. Here's the view behind the Tropical View Cafe on Monkey Forest Road in Ubud:

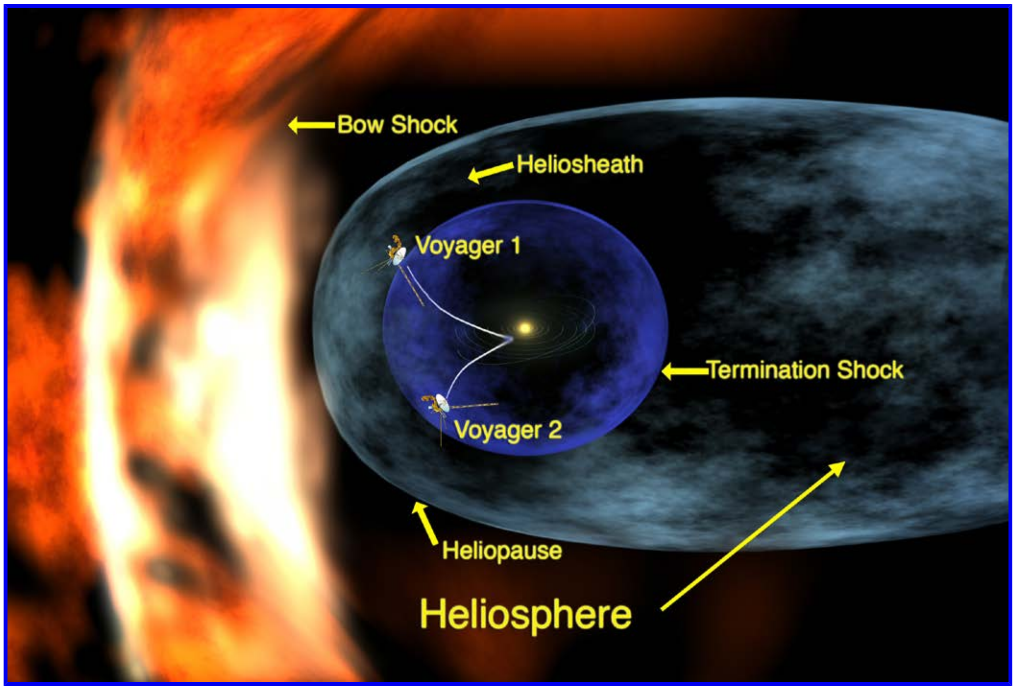
Monkey Forest Road? Yes, here's someone on that road:

Here's a sculpture behind another restaurant in Ubud, called Bebek Bengil (the 'Dirty Duck diner'):

And finally, here are some more views behind that restaurant:

It's hot on the road, but so much cooler and more breezy in these restaurant gardens!

September 2, 2017



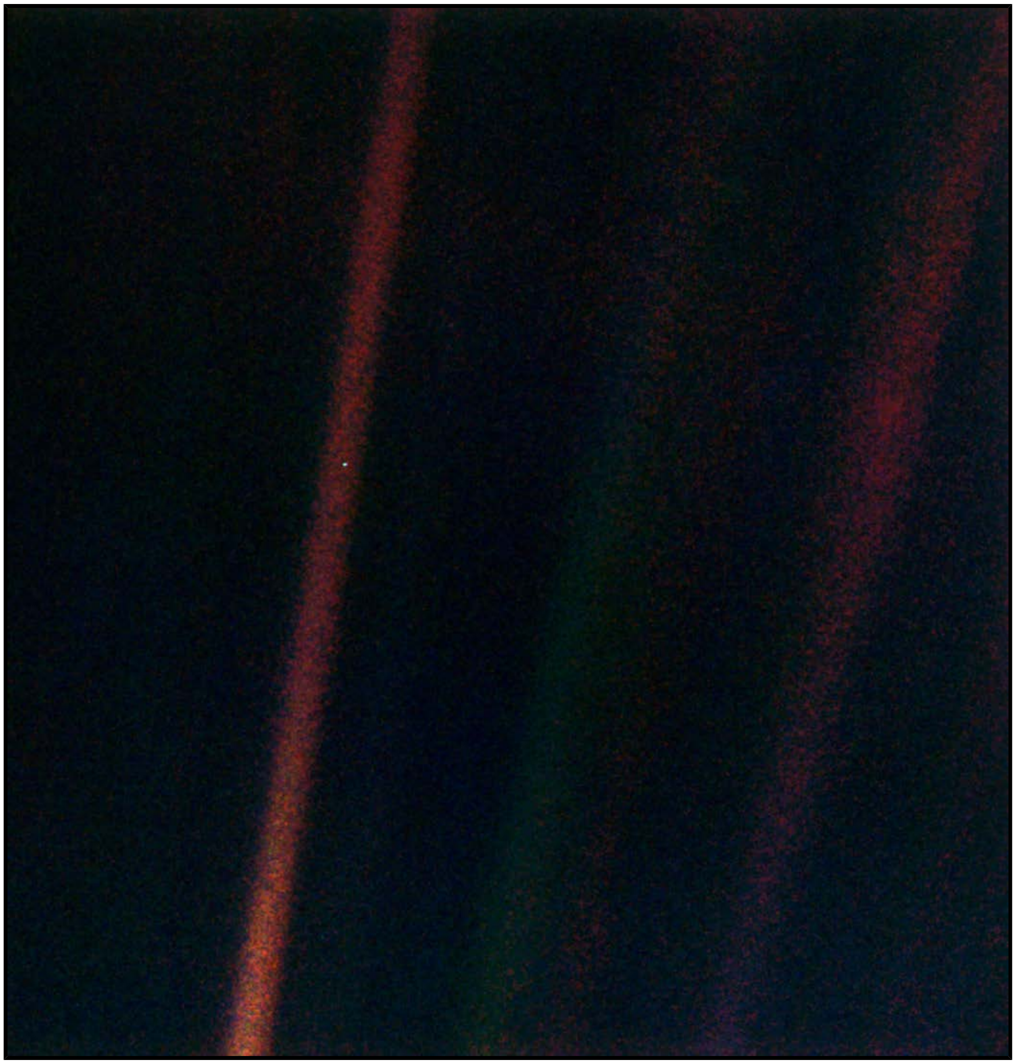
Launched 40 years ago, the Voyagers are our longest-lived and most distant spacecraft. Voyager 2 has reached the edge of the **heliosphere**, the realm where the solar wind and the Sun's magnetic field live. Voyager 1 has already left the heliosphere and entered interstellar space! A new movie, *The Farthest*, celebrates the Voyagers' journey toward the stars:

- *The Farthest*, trailer.

What has Voyager 1 been doing lately? I'll skip its amazing exploration of the Solar System....

Leaving the realm of planets

On February 14, 1990, Voyager 1 took the first ever 'family portrait' of the Solar System as seen from outside. This includes the famous image of planet Earth known as the **Pale Blue Dot**:



On February 17, 1998, Voyager 1 reached a distance of 69 AU from the Sun b 69 times farther from the Sun than we are. At this moment it overtook Pioneer 10 as the most distant spacecraft from Earth! Traveling at about 17 kilometers per second, it was moving away from the Sun faster than any other spacecraft. It still is.

That's 520 million kilometers per year — hard to comprehend. I find it easier to think about this way: it's 3.6 AU per year. That's really fast... but not if you're trying to reach other stars. It will take 20,000 years just to go one light-year.

Termination shock

As Voyager 1 headed for interstellar space, its instruments continued to study the Solar System. Scientists at the Johns Hopkins University said that Voyager 1 entered the **termination shock** in February 2003. This is a bit like a 'sonic boom', but in reverse: it's the place where the solar wind drops to below the speed of sound. Yes, sound can move through the solar wind, but only sound with extremely long wavelengths — nothing humans can hear.

Some other scientists expressed doubt about this, and the issue wasn't resolved until other data became available, since Voyager 1's solar-wind detector had stopped working in 1990. This failure meant that termination shock detection had to be inferred from the other instruments on board. We now think that Voyager 1 reached the termination shock on

December 15, 2004 — at a distance of 94 AU from the Sun.

Heliosheath

In May 2005, a NASA press release said that Voyager 1 had reached the **heliosheath**. This is a bubble of stagnant solar wind, moving below the speed of sound. It's outside the termination shock but inside the **heliopause**, where the interstelllar wind crashes against the solar wind.

On March 31, 2006, amateur radio operators in Germany tracked and received radio waves from Voyager 1 using a 20-meter dish. They checked their data against data from the Deep Space Network station in Madrid, Spain and yes — it matched. This was the first amateur tracking of Voyager 1!

On December 13, 2010, the the Low Energy Charged Particle device aboard Voyager 1 showed that it passed the point where the solar wind flows away from the Sun. At this point the solar wind seems to turn sideways, due to the push of the interstellar wind. On this date, the spacecraft was approximately 17.3 billion kilometers from the Sun, or 116 AU.

In March 2011, Voyager 1 was commanded to change its orientation to measure the sideways motion of the solar wind. How? I don't know. Its solar wind detector was broken.

But anyway, a test roll done in February had confirmed the spacecraft's ability to maneuver and reorient itself. So, in March it rotated 70 degrees counterclockwise with respect to Earth to detect the solar wind. This was the first time the spacecraft had done any major maneuvering since the family portrait photograph of the planets was taken in 1990.

After the first roll the spacecraft had no problem in reorienting itself with Alpha Centauri, Voyager 1's guide star, and it resumed sending transmissions back to Earth.

On December 1, 2011, it was announced that Voyager 1 had detected the first Lyman-alpha radiation originating from the Milky Way galaxy. Lyman-alpha radiation had previously been detected from other galaxies, but because of interference from the Sun, the radiation from the Milky Way was not detectable.

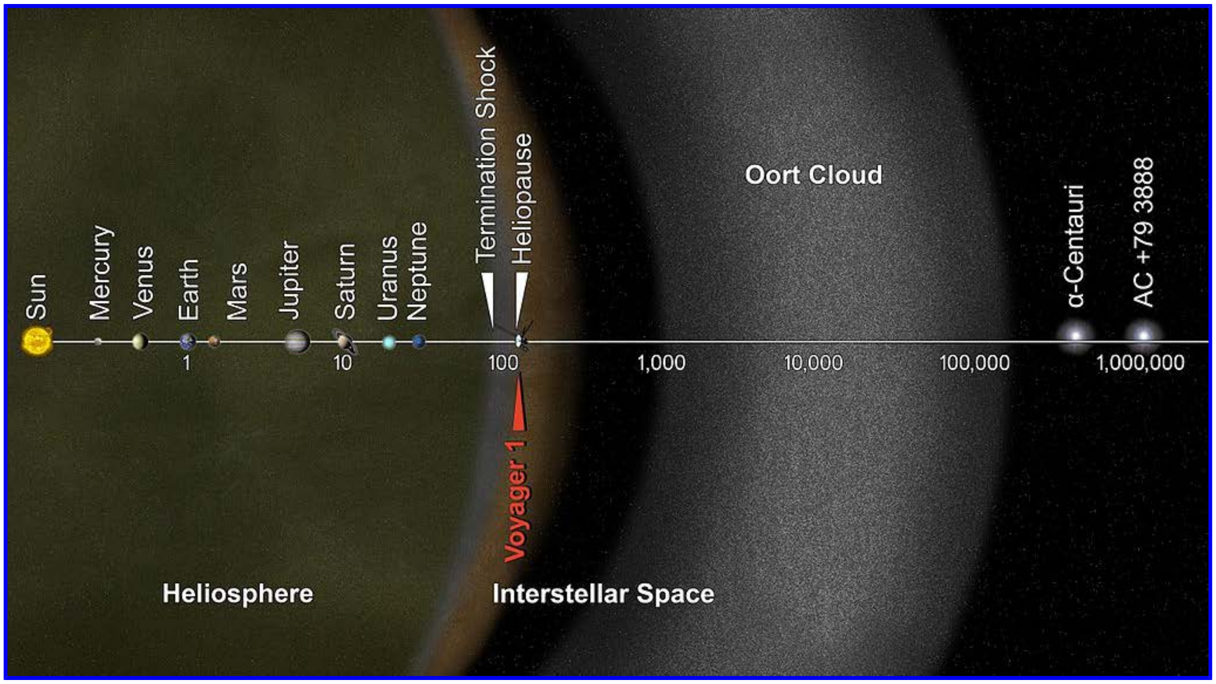
Puzzle. What the heck is Lyman-alpha radiation?

On December 5, 2011, Voyager 1 saw that the Solar System's magnetic field had doubled in strength, basically because it was getting compressed by the pressure of the interstellar wind. Energetic particles originating in the Solar System declined by nearly half, while the detection of high-energy electrons from outside increased 100-fold. At this point Voyager 1 was 113 AU from the Sun.

Heliopause and beyond

In June 2012, NASA announced that the probe was detecting even more charged particles from interstellar space. This meant that it was getting close to the **heliopause**: the place where the gas of interstellar space crashes into the solar wind.

Voyager 1 actually crossed the heliopause in August 2012, although it took another year to confirm this. It was 121 AU from the Sun.



In about 300 years Voyager 1 will reach the **Oort cloud**, the region of frozen comets. It will take 30,000 years to pass through the Oort cloud. Though it is not heading towards any particular star, in about 40,000 years it will pass within 1.6 light-years of the star Gliese 445.

NASA says that

> *The Voyagers are destined — perhaps eternally — to wander the Milky Way.*

That's an exaggeration. The Milky Way will not last forever. In just 3.85 billion years, before our Sun becomes a red giant, the Andromeda galaxy will collide with the Milky Way. In just 100 trillion years, all the stars in the Milky Way will burn out. And in just 10 quintillion years, the Milky Way will have disintegrated, with all stars either falling into black holes or being flung off into intergalactic space.

But still: the Voyagers' journeys are just beginning. Let's wish them a happy 40th birthday!

My story here is adapted from this Wikipedia article:

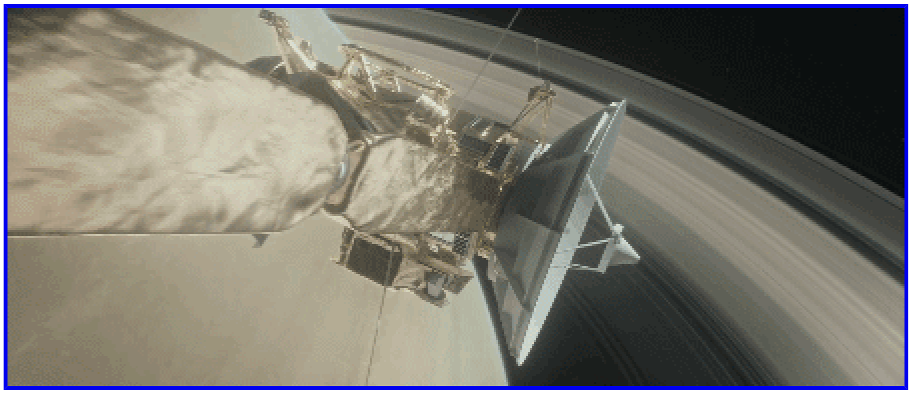
- Wikipedia, Voyager 1.

You can download PDFs of posters commemorating the Voyagers here:

- NASA, NASA and iconic museum honor Voyager spacecraft 40th anniversary, August 30, 2017.


VOYAGER
Voyager 1
Voyager 2
1977
Io Volcanoes | Titan Atmosphere | Triton Geysers | Pale Blue Dot | Interstellar Space
2017
40th ANNIVERSARY

September 10, 2017



On Friday, NASA will crash the Cassini spacecraft into Saturn! You can watch:

- September 14th, 11 pm EDT (Sept. 15, 0300 GMT): Final downlink of Cassini images starts. These images will be streamed online.
- September 15th, 7:00-8:30 am EDT (1100 to 1230 GMT): Live commentary about Cassini's plunge into Saturn, with an uninterrupted camera feed from Mission Control.
- About 8:00 am EDT (1200 GMT): Cassini's last science data, and final signal, should reach Earth.
- 9:30 am EDT (1330 GMT): Post-mission news conference.

Go here to watch:

- https://www.nasa.gov/live
- http://www.youtube.com/nasajpl/live

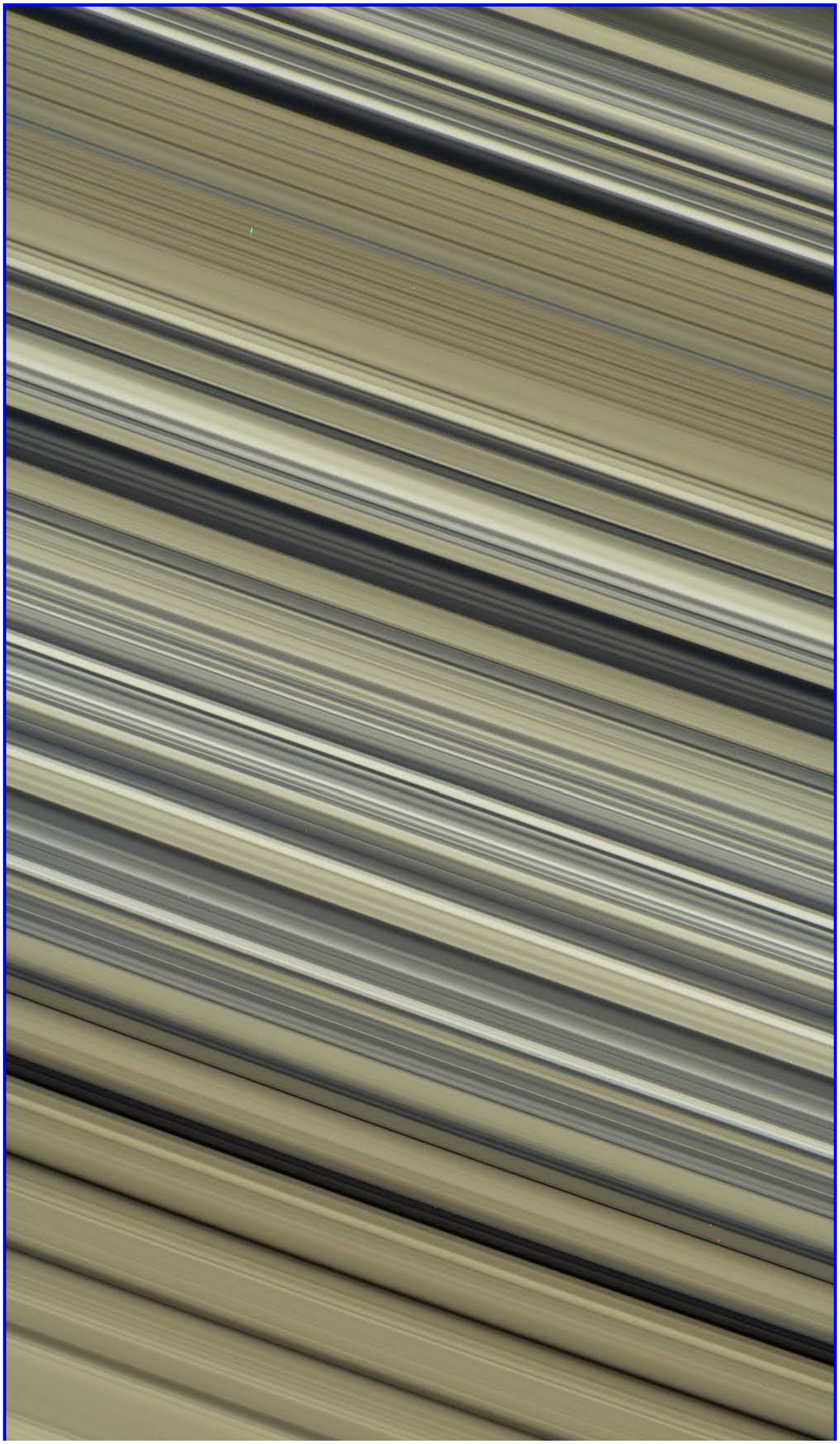
If you're impatient, watch this now:

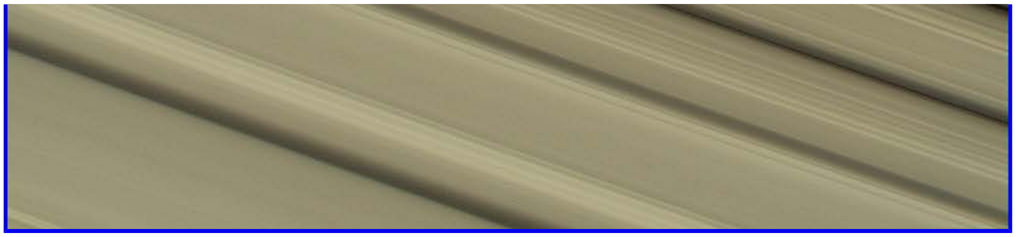
If this doesn't make you shed a tear, you've got no heart. Since 2004, Cassini has been taking magnificent photos of Saturn, its moons, and its rings. It successfully sent the Huygens probe down to the methane oceans of Titan; it swooped past the steam plumes of the geysers on Enceladus, it discovered the huge hexagon on Saturn's north pole, and more!

But now it is running out of propellant and losing its ability to manuever. To prevent it from crashing into the moon Enceladus and perhaps infecting its ice-covered ocean, NASA wants Cassini to burn up and fall into Saturn. So in April

2017 they put it on an impact course: **The Grand Finale**.

They shot Cassini past Titan and used the giant moon's gravity to fling the spacecraft toward Saturn. Since then it's made 22 daring dives between the Saturn and its rings — one each week! I hope you saw this wonderful image from its latest plunge:



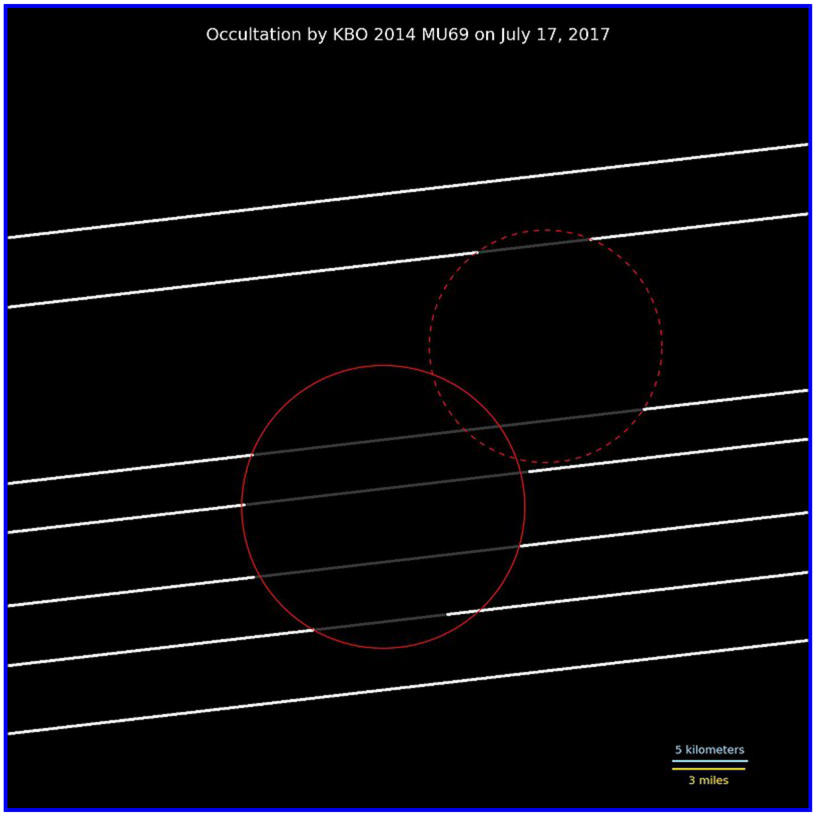


Over millennia, gravity organizes chunks of ice floating in space into amazingly delicate structures... mathematics in action!

Soon Cassini will fall into Saturn. Its final images will have been sent to us several hours before... but even as makes its fatal dive, it will be sending new data in real time. Its mass spectrometer will sample Saturn's atmosphere until Cassini loses contact and burns up like a meteor, finally becoming part of the planet it has been circling for years.

September 11, 2017

The dreamer awakens



After shooting past Pluto, the New Horizons spacecraft went to sleep. But on September 11 it woke up. It's preparing to visit something in the outskirts of the Solar System.

This thing is called **2014 MU_{69}**. It was discovered just 3 years ago. The picture shows what we saw as stars moved behind it in July... and a guess of its outline. It could be two things, or one shaped like a dumbbell.

What is it? It's called a **cubewano**. There are lots of them in the Kuiper Belt, but we don't know much about them... which is why we're taking a look.

Maybe cubewanos are made of ice, but in early July we took a good look at 2014 MU_{69} with the Hubble telescope, and we know it's *red*. That's actually not surprising. Lots of things out there are covered with reddish organic compounds called **tholins**. I wish I understood that better. But anyway, cubewanos could be balls of ice, or ice and rock, covered with tholins.

Back to New Horizons:

> NASA's New Horizons spacecraft — which visited Pluto in July, 2015 — was placed in hibernation on April 7, 2017. The craft is set to be awoken today (September 11, 2017). In the meantime, the science and mission operations teams have been developing detailed command loads for New Horizon's next encounter, a nine-day flyby of the Kuiper Belt object 2014 MU69 on New Year's Day, 2019. Among other things, the mission has now set the flight plan and the distance for closest approach, aiming to come three times closer to MU69 than it famously flew past Pluto in 2015.
>
> Hibernation reduced wear and tear on the spacecraft's electronics, lowered operations costs and freed up NASA Deep Space Network tracking and communication resources for other missions. But New Horizons mission activity didn't entirely stop during the hibernation period. While much of the craft is unpowered during hibernation, the onboard flight computer has continued to monitor system health and to broadcast a weekly beacon-status tone back to Earth. About once a month, the craft has sent home data on spacecraft health and safety. Onboard sequences sent in advance by mission controllers will eventually wake New Horizons to check out critical systems, gather new Kuiper Belt science data, and perform any necessary course corrections.

I don't know exactly why they woke it up just now. Can you find out?

And back to 2014 MU_{69}:

> Its orbital period is slightly more than 295 years and it has a low inclination and low eccentricity compared to other objects in the Kuiper belt. These orbital properties mean that it is a cold classical Kuiper belt object which is unlikely to have undergone significant perturbations. Observations in May and July 2015 as well as in July and October 2016 greatly reduced the uncertainties in the orbit. The updated orbit parameters are available in the MPC database.
>
> 2014 MU_{69} has a red spectrum, making it the smallest Kuiper belt object to have its color measured.
>
> Between 25 June and 4 July 2017, the Hubble Space Telescope spent 24 orbits observing 2014 MU_{69}, in an effort to determine its rotation period and further reduce the orbit uncertainty. First results show that the brightness of 2014 MU_{69} varies by less than 20 percent as it rotates. This places significant constraints on the axis ratio of 2014 MU_{69} to <1.14 assuming an equatorial view. Together with the fact that its shape has been shown to be very irregular, the small amplitude indicates that its pole is pointed towards Earth. This means that the timing of the New Horizons fly-by does not need to be adjusted to look at the "larger" axis of the object, simplifying the engineering of the fly-by significantly. The small amplitude makes it difficult to uniquely identify the rotation period at this time. Distant satellites of 2014 MU_{69} have been excluded to a depth of >29th magnitude.

Stay tuned! Make sure you wake up in 2019 and read what happens when New Horizons flies past this cubewano.

The first quote is from here:

- Deborah Byrd, Pluto craft wakes from hibernation today, *EarthSky*, September 11, 2017.

The second is from here:

- Wikipedia, (486958) 2014 MU_{69}.

For more on cubewanos, go here:

- Wikipedia, Classical Kuiper belt object.

September 14, 2017

It's beautiful!

This is a **hellbender** — a salamander, and the biggest amphibian in North America. Some people call them 'snot otters'. They're up to 2 feet long, they're slimy, and they look a bit like turds. But I think they're beautiful, a marvel of nature! They've lived on Earth for 65 million years. We've been here for only about 2 million.

Who will last longer? The hellbender is threatened — but some people are helping it out! They're cleaning up streams and repopulating them with hellbenders. Check out this fun video:

Hellbenders live in many eastern states of the USA, and are especially common in Missouri, Pennsylvania, and Tennessee — but mining and other human activities have silted up many of the fast-moving streams that they like. Already by 1981, hellbenders were extinct or endangered in Illinois, Indiana, Iowa, and Maryland, decreasing in Arkansas and Kentucky, and generally threatened.

So, restoring hellbenders must go hand in hand with restoring streams. But that's a good thing in itself!

The hellbender is a 'habitat specialist': it's adapted to fill a specific niche within a very specific environment. They like streams with large, irregularly shaped rocks and swiftly moving water. They avoid wider, slow-moving waters with muddy banks or slab rock bottoms. They love to hide next to a big rock, where they can hunt crayfish and small fish. Unfortunately, this helps amphibian collectors easily find them - another reason for their decline.

They start out with gills, but when they're a year and a half old they lose these gills and develop toes on their front and hind feet. After this metamorphosis they can only absorb oxygen through the folds in their skin. And that's another problem: they can only live in fast-moving, oxygenated water! If they get stuck in slow-moving water, they can't breathe.

Now people are trying to help hellbenders by breeding them in zoos and releasing them in clean streams. They can survive out in the wild *if* they don't get the fungal disease that's wiping out amphibians around the world: the **chytrid disease**. We really need good biologists to tackle that disease!

People are also creating artificial structures for hellbenders to hide in:

- Robert Whitmore, Saving the Eastern hellbender salamander, *Precast Inc. Magazine*, April 17, 2013.

Read more about hellbenders at *National Geographic*, where this picture came from:

- Jane J. Lee, U.S. giant salamanders slipping away: inside the fight to save the hellbender, *National Geographic*, December 22, 2013.

The hellbender's real name is *Cryptobranchus alleganiensis*, and it has two subspecies: the Eastern hellbender *Cryptobranchus alleganiensis alleganiensis*, and the Ozark hellbender *Cryptobranchus alleganiensis bishopi*. It's the only species in its genus, and its family contains the only two salamanders that are even larger: the Japanese and Chinese giant salamanders.

September 17, 2017

Mars is full of mysterious, intriguing landscapes. The south pole of Mars is covered with 'dry ice': frozen carbon dioxide. There's a lot of Swiss cheese terrain, where this layer of ice is full of holes. But the big pit in this picture is something else! It could be an impact crater.

This observation from NASA's Mars Reconnaissance Orbiter show it is late summer in the Southern hemisphere, so the Sun is low in the sky and subtle topography is accentuated in orbital images.

We see many shallow pits in the bright residual cap of carbon dioxide ice (also called 'Swiss cheese terrain'). There is also a deeper, circular formation that penetrates through the ice and dust. This might be an impact crater or it could be a collapse pit.

What causes the Swiss cheese terrain? The holes in the Swiss cheese are usually a few hundred meters across and 8 meters deep, with a flat base and steep sides. Here the holes seem to go all the way to the ground, but often they just go down to a layer of water ice.

We can learn how these holes form by actually watching them form. They start as small cracks. Once they have a steep wall at least 10 centimeters tall and at least 5 meters long, they start growing fast.

Remember, this is near the south pole, so at some times of year the Sun goes around very near the horizon. So, the walls of these holes catch more sunlight than the flat bottom. The holes grow as the dry ice in the walls evaporates.

This doesn't answer a bigger puzzle:

Puzzle. If the holes keep growing, why isn't all the dry ice gone by now?

The picture, and the quote, is from here:

- NASA, Mars Reconnaisance Orbiter, A south polar pit or an impact crater?, June 2, 2017.

In my version of this picture, north is to the left. You can see the Sun is shining from that direction. The full, unshrunken version of this picture is magnificently detailed: 50 centimeters per pixel!

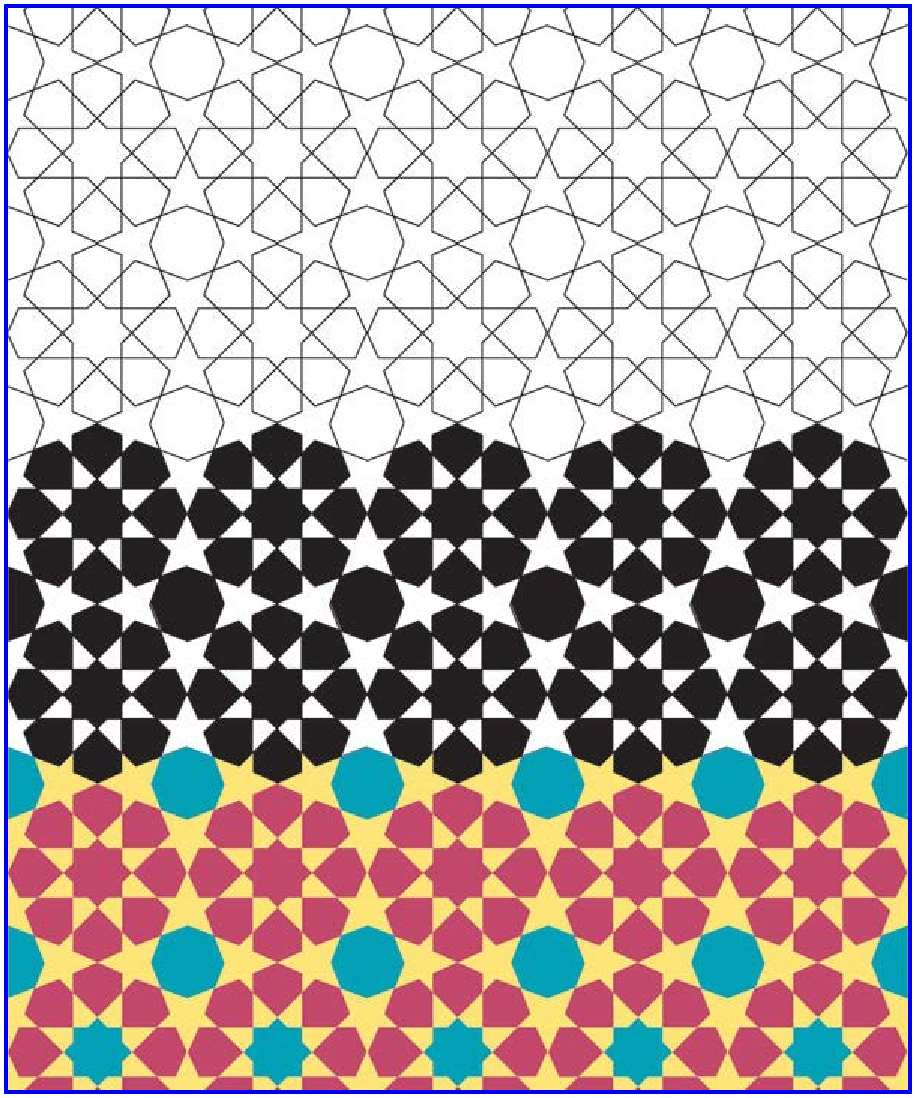
For more on Swiss cheese:

- Wikipedia, Swiss cheese features.

Also see the comments on my G+ post.

September 23, 2017

The 8-fold rosette



Islam forbids alcohol — but, except for the more extreme sects, it allows the subtler intoxication of geometry. The **8-fold rosette**, found on many a tiled wall, comes alive as you scan from top to bottom in this picture by Joumana Medlej. Learn to draw it here:

- Joumana Medlej, [Geometric design: two variations on an Islamic tiling pattern](), *Envato*, July 13, 2015.

You can do it with a ruler and compass!

First draw a grid of squares. Then inscribe a circle in each square. Then divide each circle in 16 equal parts.

It already gets fun at this stage! It's easy to find the 4 points at the top, bottom, left and right of each circle, because that's where it touches the square. It's also easy to construct 4 more: just use your ruler to draw diagonal lines between opposite corners of the square.

At that point you've divided your circle into 8 equals parts. How do you get 16?

If you remember your high-school geometry you can bisect angles with a ruler and compass, so you can do it that way.

But Joumana Medlej does a different way, which is more efficient and less messy. I'm betting this is the traditional method. Can you guess it? If not, take a look at his website.

I had trouble understanding why her method works... until I used a calculator to check that

$$\tan(\pi/8) = 0.41421356237\ldots$$

which confirmed my guess that

$$\tan(\pi/8) = \sqrt{2} - 1$$

a fact I'd never known.

How is this relevant?

When you divide a circle into 16 parts, it's like slicing a pie into slices with angles of $\pi/8$. You can do this if you can draw a line whose slope is $\tan(\pi/8)$. But it's easy to draw a line of slope $\sqrt{2} - 1$ if you happen to have a grid of squares, a compass and a ruler.

Okay, now figure out how &mddash; or see how Joumana Medlej does it! And there's more to drawing the 8-fold rosette that just this — it's fun to see the whole process.

It's easy to check that

$$\tan(\pi/8) = \sqrt{2} - 1$$

if you remember your half-angle formulas:

$$\sin(\theta/2) = \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos(\theta/2) = \sqrt{\frac{1 + \cos\theta}{2}}$$

From these it follows that

$$\tan(\theta/2) = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

and a little algebra and trig give

$$\tan(\theta/2) = \frac{1 - \cos\theta}{\sin\theta}$$

which I suppose I should have remembered, but didn't. When you take $\theta = \pi/4$ this gives

$$\begin{aligned} \tan(\pi/8) &= \frac{1 - \cos(\theta/4)}{\sin(\theta/4)} \\ &= \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \end{aligned}$$

$-$

$$= \sqrt{2} - 1$$

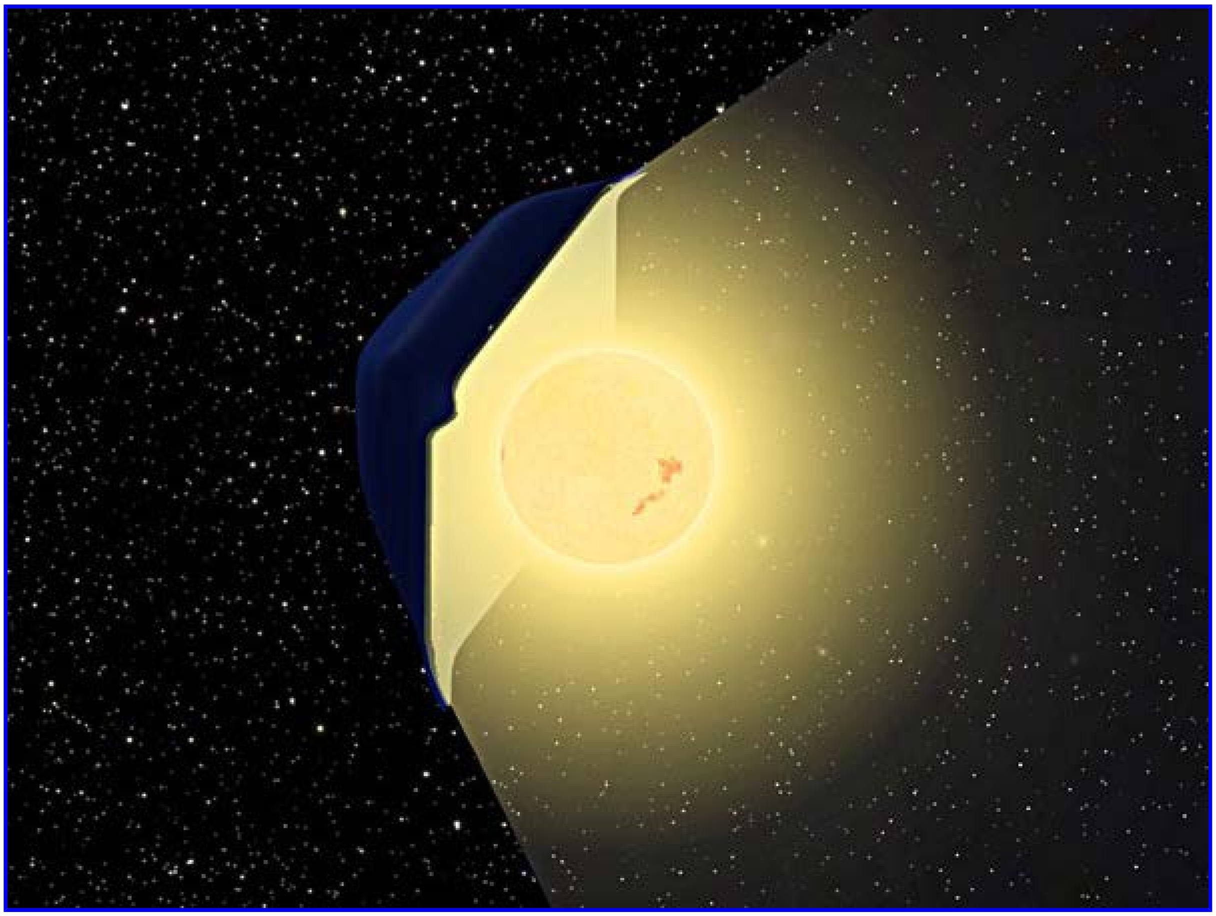
Puzzle. what's a more efficient way to see that $\tan(\theta/8) = \sqrt{2} - 1$?

Before we were distracted by the dazzling delights of modern mathematics, mathematicians knew Euclidean geometry inside and out in ways most of us can scarcely imagine now. Tiling patterns like the 8-fold rosette are a little taste of that bygone age.

Ken Smith gave a very nice answer to the puzzle on my G+ post. It boils down to a single picture, whose contemplation also leads to a proof that $\sqrt{2}$ is irrational.

September 29, 2017

How to move the Sun



Suppose we wanted to move the Solar System. How could we do it?

Okay, first things first: *why would we want to?*

Well, our Sun will eventually become a red giant. In just about 1.1 billion years it will become 10% brighter — enough to boil the Earth's oceans and create a runaway greenhouse effect. If we could move the Earth farther from the Sun, that would buy us time. But it would be even cooler to carry the Earth to a brand new star. And to keep it from freezing en route, we could try to move the whole Solar System.

Of course this seems like a wacky idea. But a billion years ago, the whole concept of intelligent life was a wacky idea. Heck, back then they didn't even have the idea of an *'idea'*. So a lot can happen in a billion years.

One way to move the Solar System is a Shkadov thruster. The Russian physicist Leonid Shkadov came up with this idea in 1987. Russian physicists have had some impressively bold thoughts, and this is a great example.

The idea is to build an enormous mirror or 'light sail'. If you did it right, the push of sunlight would balance the pull of gravity towards the Sun, so it wouldn't fall in and it wouldn't fly away.

With this mirror in place, more sunlight would shine out into space in one direction than another! This would push the Sun, which would drag the Solar System with it.

The acceleration would be very tiny. At best, after a million years the Sun would be moving at just 20 meters per second... and it would have moved 0.03 light-years. That's a respectable distance, but nowhere near the closest star.

But with a constant acceleration, the distance traveled grows as the *square* of the time (at least until special relativity kicks in). So, after a billion years, the speed would be 20 kilometers per second... and the Sun would have moved 34,000 light-years! That's a third the diameter of the Milky Way!

Of course, a billion years would be pushing it, since we're expecting the oceans to boil away just 100 million years after that. You don't want a last-minute rush to hand off the Earth to a new star! Luckily, we won't need to go nearly this far to reach a nice new star.

Building a Shkadov thruster won't be easy.

For starters, it will take a *lot* of material! Viorel Badescu, a physicist at the Polytechnic University of Bucharest in Romania, estimated the mirror would have to weigh 1/10,000th of the Earth's mass. That's 600,000,000,000,000,000 tonnes. The easiest way to get this stuff might be to mine the planet Mercury.

Hey, I've got an idea! Let's start with an easier project, as a kind of warmup. Let's *stop global warming*.

What really pisses me off about modern politics is that we're spending so much energy fighting about stupid stuff instead of thinking big.

For more on the Shkadov thruster:

- The Shkadov thruster, or how to move an entire solar system...

The Shkadov thruster is just one kind of **stellar engine**. For others, try this:

- Wikipedia, Stellar engine.

Also try this:

- V. Badescu and R. B. Cathcart, Use of class A and class C stellar engines to control Sun movement in the galaxy, *Acta Astronautica* **58** (2006), 119–129.

Ah, those Eastern Europeans, with their big ideas and their disdain for little words like 'the'.

Also check out the discussion on my G+ post.

For my October 2017 diary, go here.

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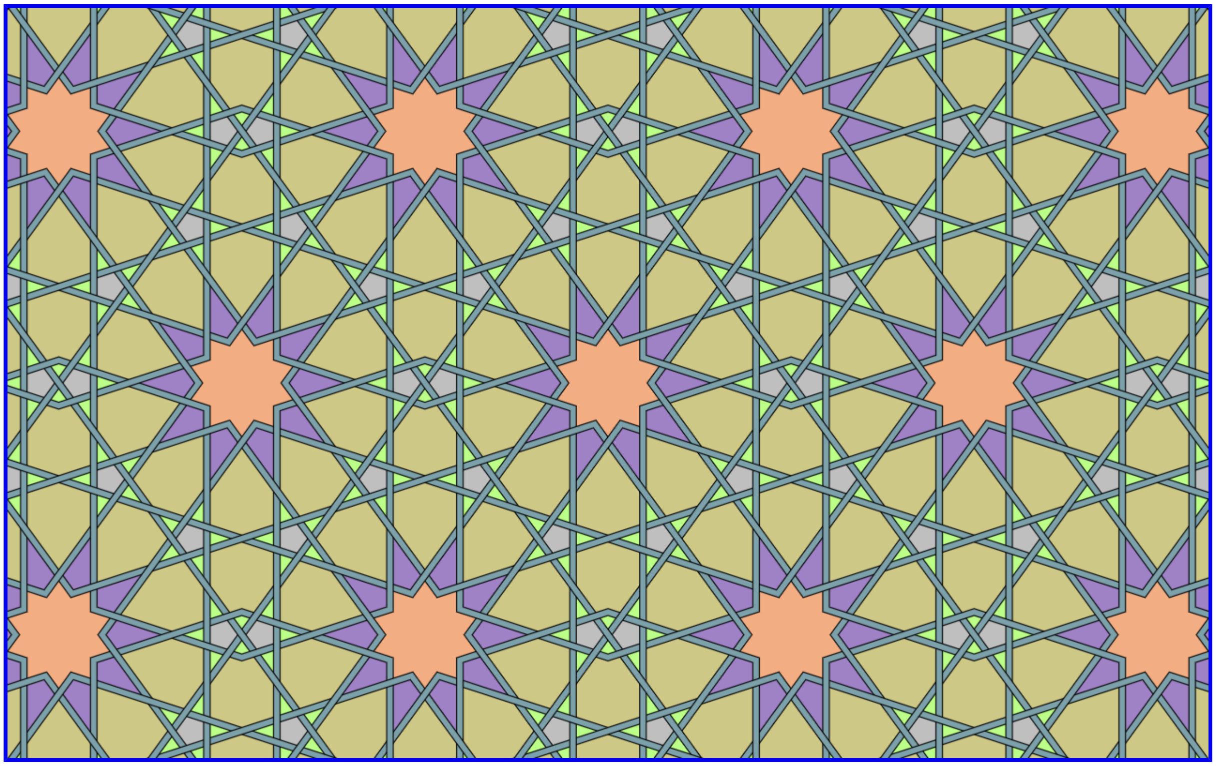
For my September 2017 diary, go here.

Diary — October 2017

John Baez

October 1, 2017

Mystery of the gray ribbons



Abdelaziz Nait Merzouk has done it yet again: he's created a mathematical work of art! This one is a traditional Islamic tiling pattern that flirts with the impossible... namely, 5-fold symmetry. See all the small green 5-pointed stars?

The most exciting feature is one you might not notice at first. It's the gray ribbons! Follow one with your eye and see where it goes. What does it do?

If you followed it forever, would it loop around back to where it started?

I don't know, so this makes a nice puzzle. Let's do it systematically.

In this picture you can see a lot of **purple stars**.

Puzzle 1. How many points does each purple star have?

Next to each purple star are a bunch of 5-pointed stars with light green points. I'll call these **green stars**.

There are also some more complicated things where two green stars overlap, sharing 2 points. I'll call these **twin stars**.

Puzzle 2. How many points of each purple star end in a green star?

Puzzle 3. How many points of each purple star end in a twin star?

If you look carefully, all the designs are formed by **gray ribbons**. And that's where things get really interesting. What happens to a gray ribbon as you follow it along? It's hard to say because the picture isn't big enough to see. But you can figure it out anyway.

When a gray ribbon goes through a green star an into a purple star, it turns either left or right and pops out.

Then the gray ribbon continues until it hits another purple star, and the story goes on. So we can keep track of its progress like this:

LRLRLLRLR....

...*unless* it hits a twin star!

When hits a twin star, it makes a *slight* turn either left or right. In this case let's write a lower-case "l" or "r". It then quickly reaches a purple star. It goes in, and as usual it turns either left or right and pops out.

So, we get a sequence sort of like this:

RRLRlRLRLLLRrRRLl....

I'm just making this one up, it probably ain't exactly right.

Puzzle 4. What's the pattern of this sequence?

I believe it's the same for every gray ribbon that hits a purple star. Some gray ribbons just go along straight lines, minding their own business. But let's ignore these.

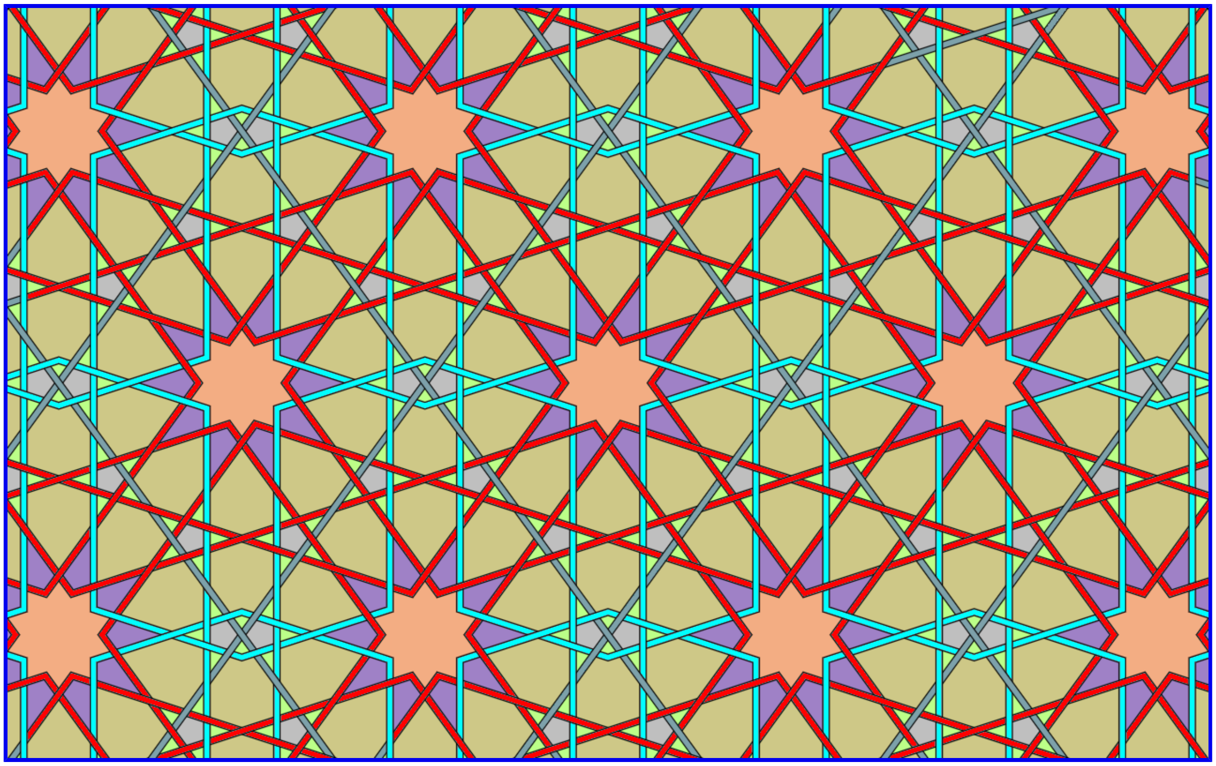
Puzzle 5. If we follow a gray ribbon that hits a purple star for long enough, do we get back where we started? Is the answer the same for every gray ribbon?

For more of Abdelaziz Nait Merzouk's tiling patterns, go here.

The twin stars look like 'defects', but they're inevitable. Greg Egan and I explained the math here:

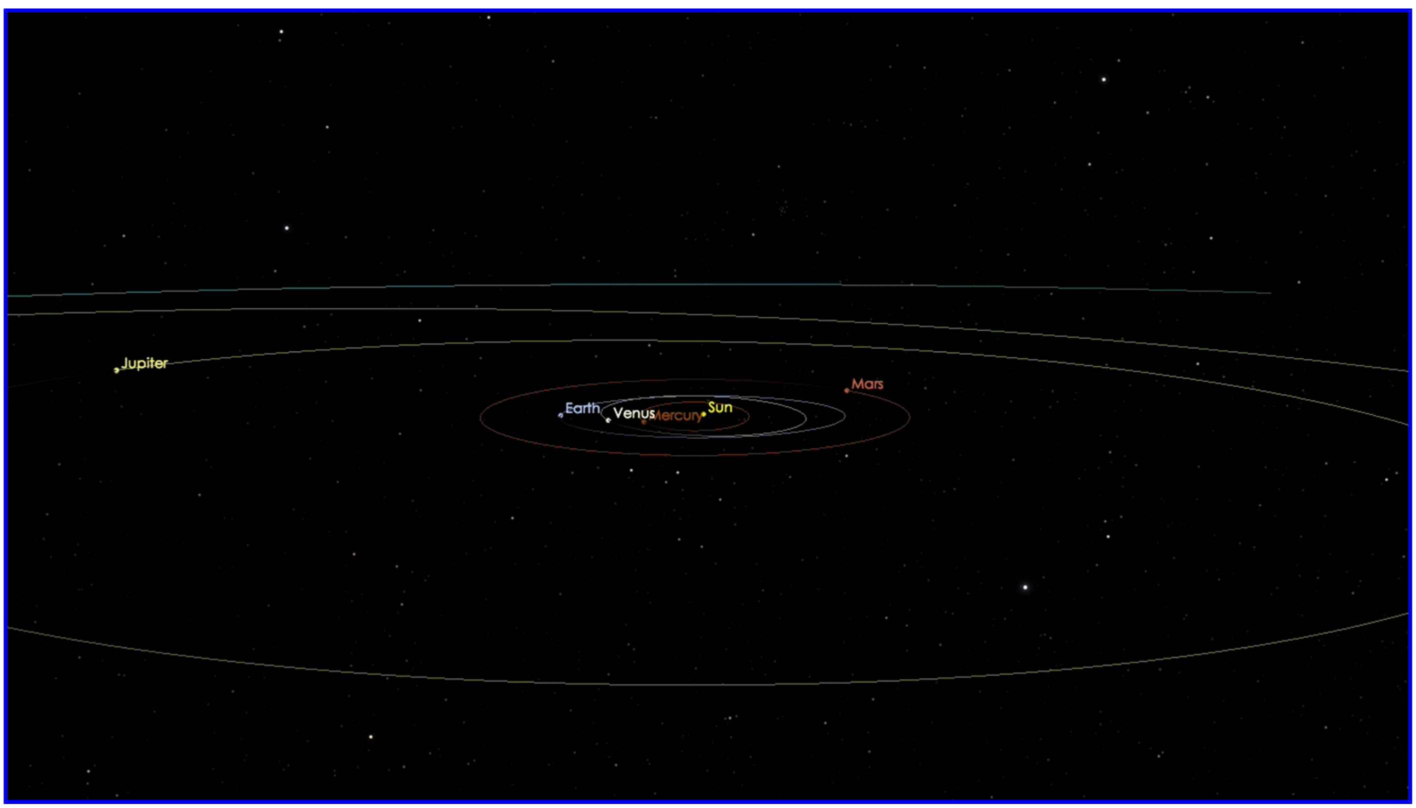
- Pentagon-decagon packing, *Visual Insight*, February 1, 2015.

You can see answers to the puzzles in the comments on my G+ post. Xah Lee colored the ribbons in a way that shows the different kinds:



October 30, 2017

A visitor from outside the Solar System!



It came from the direction of the star Vega in the constellation Lyra. It shot toward us at 26 kilometers per second. That's much faster than the escape velocity of the Solar System. So it wasn't orbiting the Sun. It's an interloper from interstellar space! We've never seen such a thing in our Solar System before.

As it fell toward the Sun it picked up speed. It shot past the Sun at 88 kilometers per second. It took a sharp turn... and now it's leaving.

It soon received the name 1I/2017 U1. It was discovered on October 19th. Rob Weryk, a postdoc at the University of Hawaii Institute for Astronomy, was the lucky fellow. He spotted it using a telescope at the University of Hawaii. Every night this telescope helps NASA search for potentially dangerous near-Earth objects. This was his lucky night.

It came fairly close to Earth: 24 million kilometers, about 60 times the distance to the Moon. It was never a threat. It's an intriguing puzzle!

Weryk immediately realized this was an unusual object. "Its motion could not be explained using either a normal solar system asteroid or comet orbit," he said. Weryk contacted Institute for Astronomy graduate Marco Micheli, who had the same realization using his own follow-up images taken at the European Space Agency's telescope on Tenerife in the Canary Islands. But with the combined data, everything made sense. Said Weryk, "This object came from outside our solar system."

"This is the most extreme orbit I have ever seen," said Davide Farnocchia, a scientist at NASA's Center for Near-Earth Object Studies (CNEOS) at the agency's Jet Propulsion Laboratory in Pasadena, California. "It is going extremely fast and on such a trajectory that we can say with confidence that this object is on its way out of the solar system and not coming back."

What is it? At first people thought it was a comet and called it C/2017 U1. But on October 25, incredibly detailed photos taken at the Very Large Telescope in the deserts of Chile showed it had no tail. So, it's probably made of rock. It was renamed A/2017 U1, becoming the first comet to be reclassified as an asteroid. But it's not a normal asteroid, so it was later called I1/2017 U1.

If it's a rock that reflect 10% of the light that hits it, it would be roughly 160 meters in diameter.

On October 25th another telescope, the William Herschel Telescope, saw that it's *red*. This is a big clue, because objects way out in the Kuiper belt, beyond Pluto, tend to be red. That's because they're covered with tholins — a messy and mysterious mix of complex organic chemicals formed by billion-year-long exposure to radiation.

It's on its way out now, and astronomers are watching it carefully, desperately trying to squeeze a bit more information out of this encounter. How does a rock escape another solar system? How long has this object been shooting through the icy depths of interstellar space before it reached us? How many of these things are there?

"We have been waiting for this day for decades," said CNEOS Manager Paul Chodas. "It's long been theorized that such objects exist — asteroids or comets moving around between the stars and occasionally passing through our solar system — but this is the first such detection. So far, everything indicates this is likely an interstellar object, but more data would help to confirm it."

The quotes are from NASA's webpage:

- NASA, Small asteroid or comet 'visits' from beyond the Solar System, Oct. 26, 2017.

and so is the animated gif. Later this object was named **'Oumuamua**, a Hawaiian word that means 'scout' or 'messenger'. So, you can learn more here:

- Wikipedia, 'Oumuamua.

For more, see the comments on my G+ post.

For my November 2017 diary, go here.

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[For my October 2017 diary, go here.](#)

Diary — November 2017

John Baez

November 11, 2017

A big machine to weigh a tiny particle

This is a huge vacuum chamber, bigger than a blue whale, being carried through the streets of a German town. By now it's been buried underground as part of the **Karlsruhe Tritium Neutrino Experiment**, or **KATRIN**. It aims to measure the mass of a very light particle!

There are 3 kinds of neutrinos, each with their own antiparticle. Amazingly, they're all so light that we don't know how heavy they are. The KATRIN experiment is trying to measure the mass of the electron antineutrino, which is formed whenever a neutron decays into a proton and electron.

Around 2000, another German experiment showed that the mass of this particle is *no more than* 0.0000043 times the mass of an electron. That's the mass equivalent of 2.2 electron volts, or eV. The new experiment should be able to measure the electron antineutrino's mass if it's more than 0.2 eV. It works the same basic way: let tritium, with one proton and two neutrons in its nucleus, decay to an element with two protons and one neutron. In this process a neutron turns into a proton, releasing an electron and an electron antineutrino. By very carefully measuring all the stuff you *can* see, you can estimate the mass of the electron antineutrino... even though that particle is almost impossible to see.

There's a chance the experiment will only put a better upper bound on the mass of the electron antineutrino. To see why, take a look at this article:

- [Neutrino mass](#), Wikipedia.

Wikipedia says that the difference in the squares of the masses between neutrino mass eigenstates 1 and 2 is about 0.000079 eV^2, while the difference in the squares of the masses between eigenstates 2 and 3 is about 0.0027 eV^2.

None of this says anything about the actual masses. But then, they say:

> In 2009, lensing data of a galaxy cluster were analyzed to predict a neutrino mass of about 1.5 eV. This surprisingly high value requires that the three neutrino masses be nearly equal, with neutrino oscillations on the order of milli electron-Volts. In 2016 this was updated to a mass of 1.85 eV.

But then, they say:

> In July 2010, the 3-D MegaZ DR7 galaxy survey reported that they had measured a limit of the combined mass of the three neutrino varieties to be less than 0.28 eV. A tighter upper bound yet for this sum of masses, 0.23 eV, was reported in March 2013 by the Planck collaboration, whereas a February 2014 result estimates the sum as 0.320 B1 0.081 eV based on discrepancies between the cosmological consequences implied by Planck's detailed measurements of the cosmic microwave background and predictions arising from observing other phenomena, combined with the assumption that neutrinos are responsible for the observed weaker gravitational lensing than would be expected from massless neutrinos.

So, while the astronomical estimates are quite different from each other, and some of them must be wrong, they seem to point to neutrino masses that are considerably larger than the neutrino mass *differences*.

Unfortunately, if the sum of all 3 neutrino masses is about 0.3 eV, and their masses are close, each individual mass is about 0.1 eV, which is below the 0.2 eV that the Karlsruhe Tritium Neutrino Experiment can measure.

This article is quite good, so go here for more:

- Robin McKie, In search of the neutrino, ghost particle of the universe, *The Guardian*, November 4, 2017,

However, it's gotta be wrong where it says this:

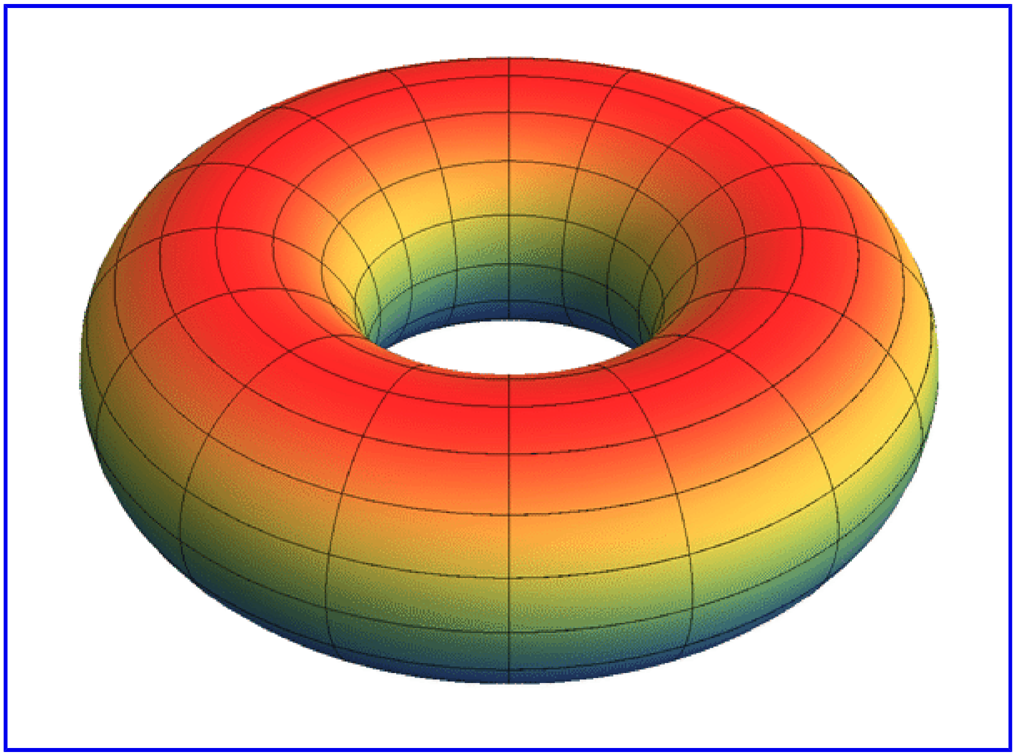
> First efforts, made after the second world war, placed an upper limit on its mass at around 500 electron volts (eV). This figure is about 1/500th of the mass of the electron, itself a relatively tiny particle. (Using a unit of energy to describe the mass of an object may seem strange but all subatomic particles are measured in electron volts, which can also be used as a unit of mass because energy and mass are convertible concepts according to Einstein's $E = mc^2$.

The problem is that the electron's mass is about 511,000 eV, so 500 eV would be 1/1000th of that. Math.

For more, see the discussion on my G+ post.

November 12, 2017

How to make a zero-calorie doughnut



Take a doughnut. Remove most of it, leaving a thin tube of dough that wraps twice around the original doughnut hole.

Then remove most of what's left, leaving a thinner tube of dough that wraps *4 times* around the original hole.

Then remove most of what's left, leaving an even thinner tube of dough that wraps *8 times* around the original hole.

Keep repeating this forever... and then stop. Now you have a zero-calorie doughnut!

They'll never become popular at bakeries, but mathematicians love 'em. You can't really make one out of dough, but it's a perfectly fine mathematical set of points in 3-dimensional space. When I said it has no calories, what I really mean is that it has zero volume . Mathematicians call it a solenoid.

The solenoid was first invented by the topologist Vietoris in 1927. But it also showed up as an attractor in a certain dynamical systems studied by the mathematician Smale.

One cool thing about the solenoid is that it's connected, but not 'path-connected'. In other words, given two different points in the solenoid, you can't connect them with a path that stays in the solenoid. At any finite stage of its construction you could... but as you continue the construction, after a while the path you'd need to take typically keeps roughly doubling in length!

Yet another cool thing about the solenoid is that you can give it the structure of an abelian group. For this it's better to use a more abstract construction — so, get ready for some more serious math. Take the circle, which is an abelian group. Mathematicians call it S^1. There's a function

$$f: S^1 \to S^1$$

that doubles angles, and wraps the circle around itself twice. Take the set of infinite sequences of points in the circle where each point is f of the next point. You can make this set into a group where

$$(x_1, x_2, x_3, \ldots) + (y_1, y_2, y_3, \ldots) = (x_1 + y_1, x_2 + y_2, x_3 + y_3, \ldots)$$

This is the solenoid!

To make this more precise we need to get topology into the game. The circle is not only an abelian group, it's also a compact topological space — and the group operations in the circle are continuous, so we call it a 'compact abelian group'. And the solenoid, being built from the circle as above, is also a compact abelian group! Mathematicians call it the 'limit' of the sequence

$$S^1 \;{}^{f}\; S^1 \;{}^{f}\; S^1 \;{}^{f}\; S^1 \;{}^{f}$$

where f, the map vaguely described above, is a homomorphism of compact abelian groups. The idea is simply that a point in the limit is a sequence of points, one in each copy of S^1, such that each point is f of the next.

In fact, the limit of *any* sequence of compact abelian groups is again a compact abelian group: this is a spinoff of Tychnonoff's theorem, which says that any product of compact topological spaces is again compact. So, we can build lots of compact abelian groups as limits, for example of

$$S^1 \;{}^{f}\; S^1 \;{}^{g}\; S^1 \;{}^{f}\; S^1 \;{}^{h}$$

where each map f, g, h etc. could wrap the circle around itself any number of times. All these are called 'solenoids'.

Puzzle. When are two of these solenoids isomorphic as compact abelian groups?

The answer involves prime numbers! For clues, see this:

- Encyclopedia of Mathematics, Solenoid.

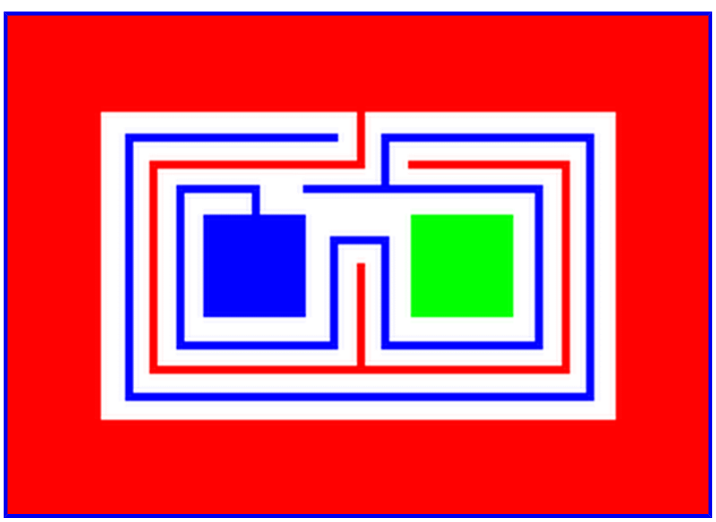
and also the comments on my G+ post.

The above animated gif of the solenoid was created by Jim Belk, and it's part of the Wikipedia article:

- Solenoid, Wikipedia

November 16, 2017

The Lakes of Wada



Wada lived on a white island in a red sea. On the island there was a blue lake and a green lake.

Wada led a peaceful life. Some days he would sit by the red sea. Some days he would sit by the blue lake. And some days he would sit by the green lake.

But eventually he became bored of watching just one color of water at a time. So he decided to dig canals.

On the first day, Wada dug a canal from the red sea so that every piece of land was within 1 mile of some red water.

In the next 1/2 day, Wada dug a canal from the blue lake so that every piece of land was within 1/2 mile of some blue water. The picture here shows what his island looked like then.

In the next 1/4 day, Wada dug a canal from the green lake so that every piece of land was within 1/4 mile of some green water. Can you draw it?

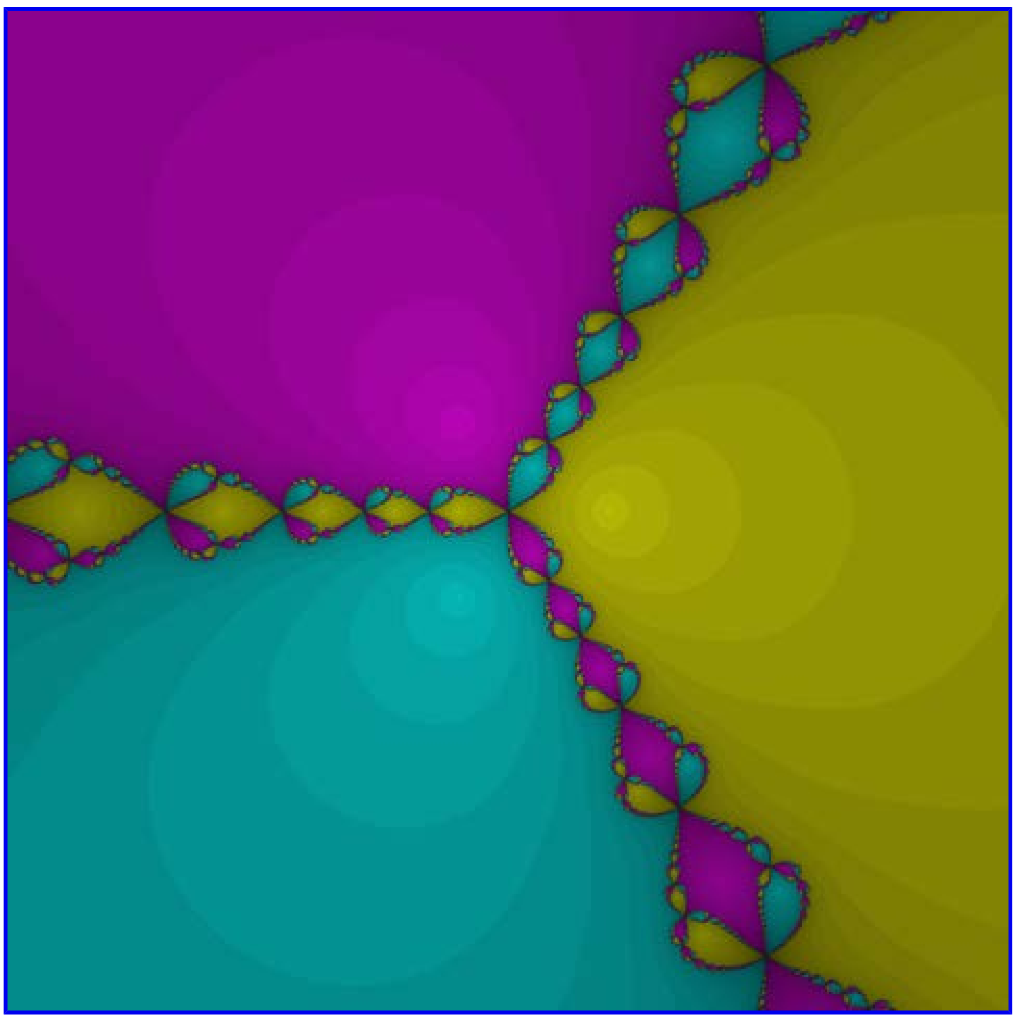
Wada continued this way, digging more and more canals. They got thinner and thinner, so there was always plenty of land left.

By the end of the second day, every piece of land touched the red sea, the blue lake and the green lake! Now he could watch all three bodies of water at once.

He had built the famous Lakes of Wada, which people visit even today.

You can create the Lakes of Wada effect by looking at the reflections in three mirrored spheres that touch each other.

You can also create this effect by applying Newton's method to a cubic polynomial with 3 distinct roots in the complex plane, such as $z^3 - 1$:



Newton's method is a simple way of solving equations like $f(z) = 0$. You start by making a guess for z, you figure out

$f(z)$ and its derivative $f'(z)$ at your guess, and you use those to figure out where $f(z)$ would equal zero if f were a linear function. This is your new guess for z. Then you repeat this. In good situations your guesses will quickly converge to a value of z with $f(z) = 0$. In worse situations your guesses will hop around in a complicated way. If you take $f(z) = z^3 - 1$, there are 3 solutions. If you start near one of those solutions, your guess will converge to that solution. This defines three basins of attraction. But these basins of attraction are not connected, and each touches the other two at every point of its boundary!

I got the picture in my post from here:

- Yale Mathematics Department, [Julia sets and the Mandelbrot set: Newton's method basins of attraction: Wada basins]().

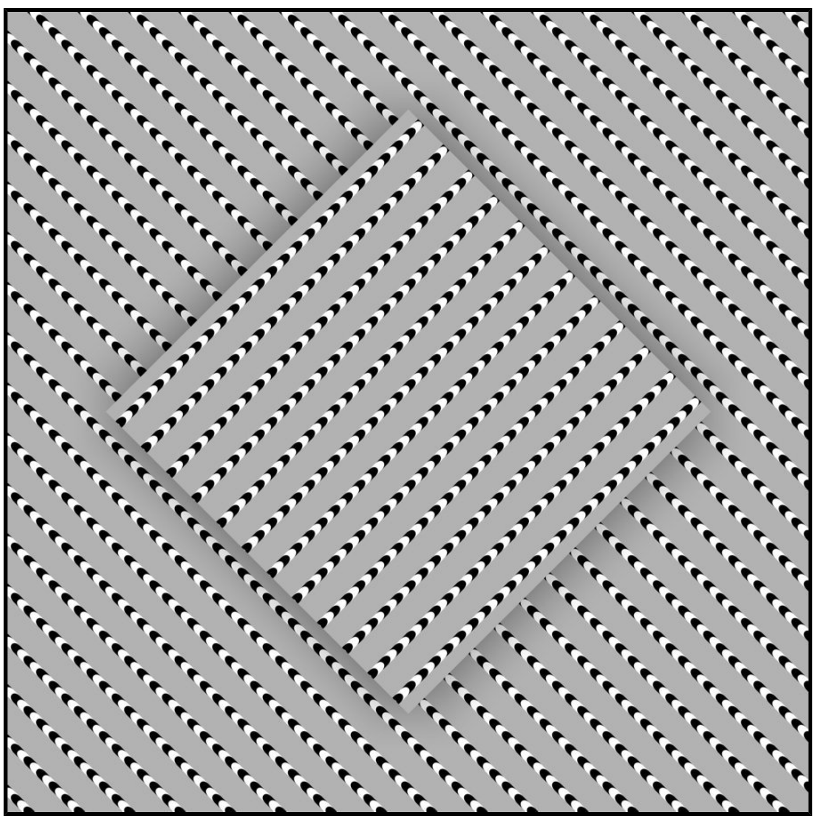
and clicking the link at the bottom of this page will take you to more information on the Lakes of Wada. See also the discussion on my [G+ post]() and this Wikipedia article:

- [Lakes of Wada](), Wikipedia.

The Lakes of Wada were actually discovered by the Japanese mathematicia Takeo Wada, who lived from 1882 to 1944 and worked on analysis and topology at Kyoto University.

November 19, 2017

This is not an animated gif



This is Akiyoshi Kitaoka messing with your brain. He's a professor of psychology at Ritsumeikan University in Kyoto. He's spent a long time collecting and perfecting illusions. You can see them on his website:

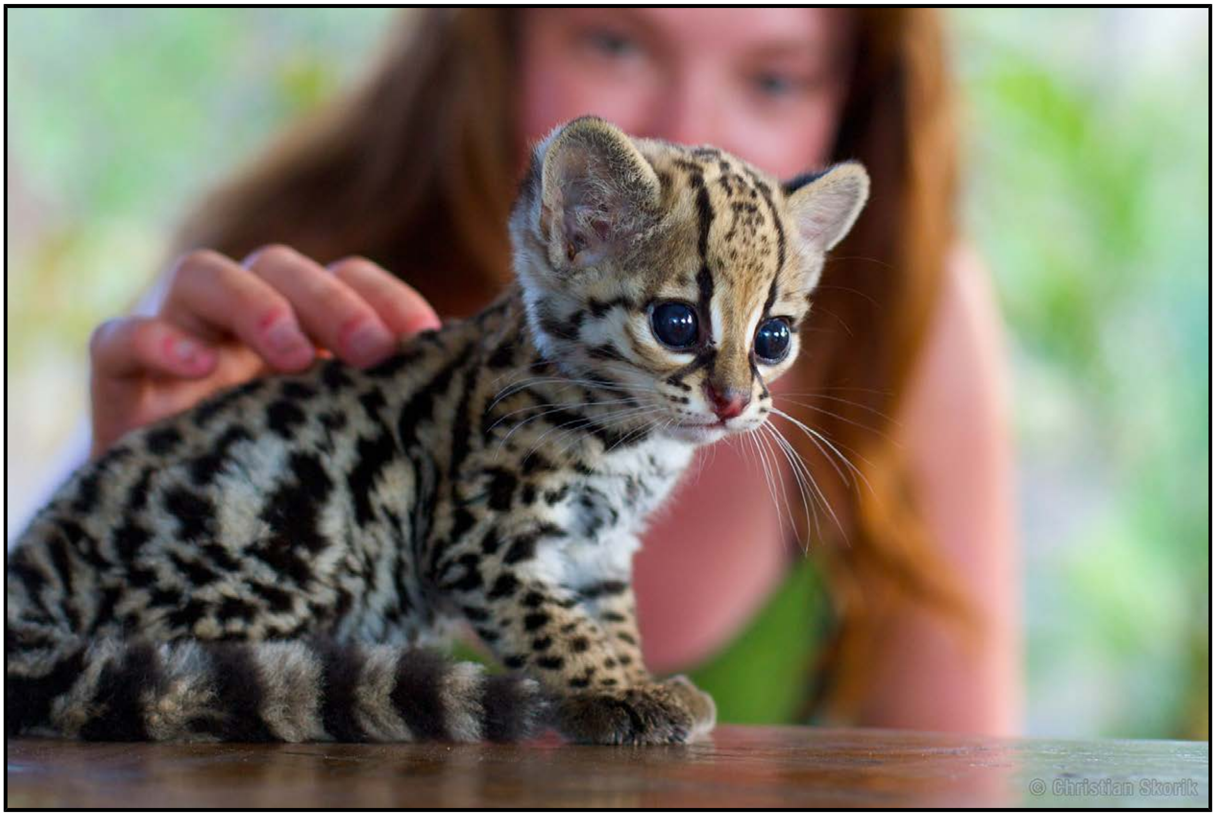
- Akiyoshi Kitaoka, [Akiyoshi's illusion pages]().

where he writes

Should you feel dizzy, you had better leave this page immediately.

You can also see them on Twitter, where there is no such warning. And he has a book, *The Oxford Compendium of Illusions*.

November 29, 2017



Criminally cute! **Ocelots** are small wild cats that live in Mexico, Central America and South America. There are even a few in the southwestern United States. This one is just a kitten.

Hilary Swarts has been involved in setting up an ocelot reserve in south Texas, the Laguna Atascosa National Wildlife Refuge. Here's some of a story about her:

> Survival can be a real cat fight when you get squeezed out of your rightful home. When your food supply dwindles. When you are small and cute and easy to run down. Even though you are standoffish and try to keep to yourself.
>
> In 22 countries, from Uruguay to south Texas, the ocelot (*Leopardus pardalis*), one of smallest and most secretive of all wild cat species, is facing this sad plight. Its habitat — thorn scrub, coastal marshes, tropical and pine-oak forests — has shrunk alarmingly, swaths destroyed by building and farming and other human activity. With diminished space in which to establish territories, find secure denning sites and and forage for rodents, birds, snakes, lizards and other prey — plus the increased threat of becoming road kill as highway construction boomed in the 20th century — the ocelot has been in the fight of its life.
>
> Back in the 1960s and early '70s, ocelots were nearly loved to death. Laws then did not prohibit taking them for exotic pets or hunting them for their beautiful, dramatically marked fur. Babou, Salvador Dali's frequent sidekick, may have been the most famous of captive ocelots.
>
> In the U.S., as the wild population of these little cats became depleted under development pressures, the fashion industry turned to import, reaching a peak of 140,000 pelts from Central and South American countries in 1970. Toward the end of the century, all these human endeavors had chipped away at the

> historic U.S. ocelot range — which once stretched from Louisiana to Arizona — cornering the few known remaining individuals in the Lower Rio Grande Valley, where Texas meets the Mexican border and the Gulf of Mexico. Wildlife biologists, scientists, researchers, conservationists and other experts started running the numbers and saw that time was running out. Now, even after several decades of legal protection and some active conservation projects, only 55 or so known individual ocelots remain in the U.S.
>
> There are few rays of sunshine in this grim picture, but one of the brightest landed at Laguna Atascosa National Wildlife Refuge a little over three years ago in the form of wildlife biologist Hilary Swarts '94.

Swarts is a graduate of Pomona College, whose magazine I got this story from. The picture for the ocelot is not quite as grim as this story makes it sound: there are about 40,000 mature ocelots in the world, their population is considered stable, and they're listed as being of Least Concern by the IUCN Red List of Threatened Species. In the US, ocelots are indeed endangered, especially since with the new border walls they're getting cut off from the larger population in Mexico. But maybe we don't need ocelots. Maybe we don't deserve ocelots.

Swarts has a different opinion:

> Entering the ecology program at the University of California, Davis, she earned a Ph.D. in ecology with an emphasis on conservation. Then, shrugging off that "never working for the government" notion, she took a job with the U.S. Fish and Wildlife Service, working on regulatory projects involving endangered species. "Regulatory work is so important," she emphasizes. But after a while, the day-to-day responsibilities of what she terms "desk biology" began to wear. "It's soul-crushing work," she explains. "You know exactly what each day, a month ahead, will be."
>
> So, when a job opening in the wilds of south Texas popped up in her email for a wildlife biologist charged with leading the hands-on effort to save the ocelot in the U.S., she leapt at the challenge.
>
> The Laguna Atacosa National Wildlife Refuge is a flat, sunbaked remnant of coastal prairie mixed with thorn bush, bordering on a vast hypersaline lagoon across from South Padre Island. Its dense thicket of low scrub is home to — at last count — 15 of the remaining ocelots still living in the U.S., and for Swarts, it's where the fight to save them from extinction is being waged.
>
> Meeting with her here can feel like a bracing seminar in All Things Ocelot. For starters, she'll whip her refuge pickup into her driveway (on Ocelot Road, of course) and say, pointing at the license plate on her 2000 Buick LeSabre, "Look!" The plate says "OCELOT" (of course), and the vanity fee collected by the State of Texas goes to Friends of Laguna Atascosa for outreach programs.
>
> More important, it quickly becomes clear that she's a walking compendium of information about the species she's working to rescue. "We think that these Texas ocelots may have developed great fidelity to thick underbrush because of pursuit by hunters back in the 1960s," she explains. More facts come tumbling out: Two-thirds of births are single, after a gestation of 79 to 82 days. Kittens stay with their mothers, to learn survival and hunting skills, for up to two years. "Although," she adds, "I'm beginning to think it may be closer to a year and a half, if the teaching goes well and there is a reliable prey base. And the past two winters have been super wet, so there's been prey out the wazoo."
>
> Working with ocelots, because they stay so well hidden, is different from her previous fieldwork, when she could watch the animals she was studying in their own environment (such as following gorillas around as they nosed about on their daily routines, which she describes as "total soap opera"). In fact, the only time Swarts and her small staff of interns actually see ocelots in the flesh is during trapping season, from October to May, when the little cats are lured by caged pigeons posing as an easy meal, then sedated long enough for blood and genetic samples to be taken. After a quick exam and insertion of a microchip, they are photographed, fitted with a GPS collar, given reversal drugs and released.
>
> "With the ocelots, I'm essentially doing detective work," she explains. Across the refuge, there are more than 50 cameras tucked into the thorn scrub, monitoring animal activity night and day. Using cameras and

> GPS collars may not be as immediately satisfying as shadowing gorillas, but it's the only way she can keep tabs on the elusive little creatures she's trying to save.
>
> For instance, last year, on March 25, 2016, a heavily pregnant female was captured for routine data collection and then released. On the following two days, GPS signals from her collar indicated that she was staying put, likely in a den. After a few weeks, GPS showed more activity — she was almost certainly leaving the den for water, repeat behavior that is usual for a lactating female. "On April 15, when we knew she was away and couldn't detect us, we found the little kitten, tucked under some Spartina. A male, healthy, weighing less than a pound, with his eyes just opened." Swarts, who took hair samples, DNA swabs and his baby picture, was ecstatic to document and report this first confirmed ocelot den at the refuge in 20 years.
>
> "From my perspective they are doing their job — reproducing," she says. "And ecologically we are in great shape." However, she has grave concerns that the confirmed refuge population of 15, including kittens, may be approaching capacity. Home range for a female varies from one to nine square miles, depending on the availability of water and prey. For a male, figure four to 25 square miles.
>
> That brings us to exhibit one for the three top threats to survival of the species — habitat loss. Hemmed in by agriculture, highways and industry, the refuge itself can't be greatly expanded. The other Texas ocelots, about 40 individuals, live on limited private lands in neighboring Willacy County, with no safe passage connecting the populations.
>
> And that leads directly to the second threat — vehicular mortality, which stands at an astounding 40 percent. Swarts cites the ugly statistics that piled up between June 2015 and April 2016, when seven ocelots, including six males, were killed by vehicles on roads adjacent to fragile ocelot territory.
>
> Which brings us to the third item on Swarts' list of top threats to the ocelot's long-term survival: in-breeding, which occurs when populations are so isolated that no new genes can get into the mix. Even before her arrival in Texas, efforts to freshen the gene pool by bringing in a female ocelot from Tamaulipas, Mexico, had started and stopped several times, partly due to cartel violence. Still, she remains optimistic that, with research and negotiation, a female from Mexico will eventually be allowed to cross the border.
>
> Progress is agonizingly slow — as Swarts stoically puts it, "Conservation is often two steps forward and one step back." However, she has begun to see encouraging signs. The refuge has cranked up an aggressive habitat restoration project — planting ocelot corridors, extensions of the habitat that ocelots are known to use, with the low-growing, bushy native species they prefer. As a precaution against vehicular mortality, the refuge has closed some of its roads and plans to relocate its entrance. Most heartening, the Texas Department of Transportation is installing 12 new underpasses specifically designed for ocelots at known hot spots on two highways where there have been multiple incidents of road kill. "And now it seems likely they will put wildlife crossings into new road design from the start," she adds. "This is a sea change — and for this state agency to come around bodes so well for the state and its environmental future."

One can wonder if it's really worth such a fuss for a few dozen cats, especially when there are many more outside the US. I like to think of it this way: the ocelot is the charismatic representative of a certain ecosystem. The ecosystem, bristling with complex information evolved over millions of years, is valuable in ways we're just beginning to understand. Saving the ocelot is an easily understood stand-in for saving the ecosystem.

We're like kids in a grand library, kids who can barely read. We notice that some of the books have pretty pictures. The ocelot is one of those pretty pictures.

The article I quoted was written by Shakespeare. Margaret Shakespeare, that is. Check out the rest, and the pictures, here:

- Margaret Shakespeare, Ocelot country, *Pomona College Magazine*, April 10, 2017.

Watch someone play with a young semi-domesticated ocelot in the jungle of Costa Rica:

Learn more about ocelots here:

- Wikipedia, Ocelot.

Their closest relative is the margay... but that's another cat for another day.

For my December 2017 diary, go here.

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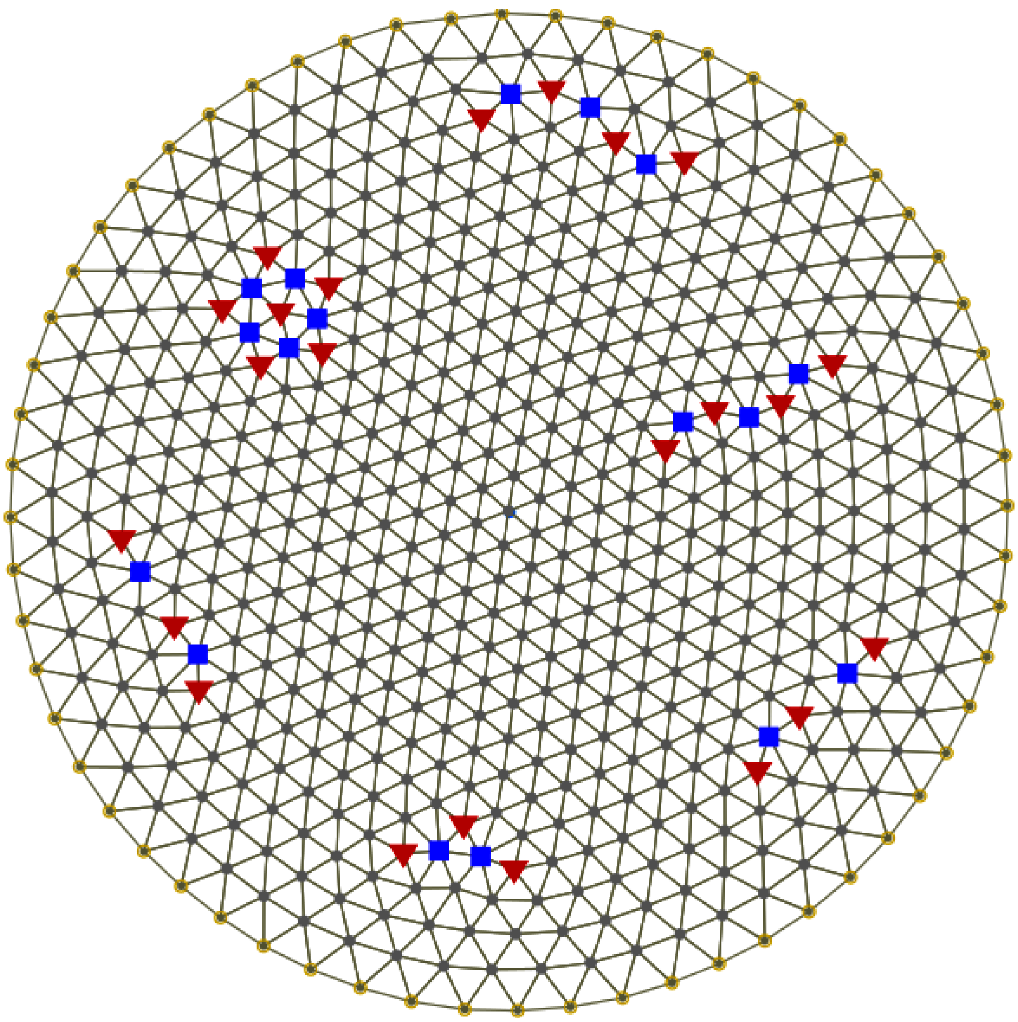
For my November 2017 diary, go here.

Diary — December 2017

John Baez

December 6, 2017

A crystal made of electrons



Electrons repel each other, so they don't usually form crystals. But if you trap a bunch of electrons in a small space, and cool them down a lot, they will try to get as far away from each other as possible — and they can do this by forming a crystal!

This is sometimes called an **electron crystal**. It's also called a **Wigner crystal**, because the great physicist Eugene Wigner predicted in 1934 that this would happen.

Only since the late 1980s have we been able to make Wigner crystals in the lab. A crystal can only form if the electron density is low enough. This is due to the uncertainty principle of quantum mechanics, which implies that even at absolute zero, electrons wiggle around — and they do this more when they're densely packed! When the density is low, they settle down and form a crystal.

But when an electron gas is *rapidly* cooled, sometimes it doesn't manage to form a perfect crystal. It can form a *glass!*

This is called a **Coulomb glass**.

It's an amazing world we live in, where people can study a *glass made of electrons*.

We can do other cool stuff, like create *electron crystals in 2 dimensions* using electrons trapped on a thin film of metal. That's what this picture shows. It's a theoretical picture, but you can trust it, since we understand the laws of physics needed to figure out what electrons do when trapped in a disk. The density here is low enough that the uncertainty principle doesn't play a significant role — so we can visualize the electrons as dots with a well-defined position.

The lines between the dots are just to help you see what's going on. In general, a 2-dimensional electron crystal wants to form a triangular lattice. But a triangular lattice doesn't fit neatly into a disk, so there are **defects** — places where things go wrong.

Puzzle 1. What is happening at the blue defects?

Puzzle 2. What is happening at the red defects?

Puzzle 3. What can you say about the number of blue defects and the number of red defects? Do these numbers obey some rule?

Puzzle 3 has some very interesting answers: see the comments on my G+ post and my somewhat more detailed blog article on this topic.

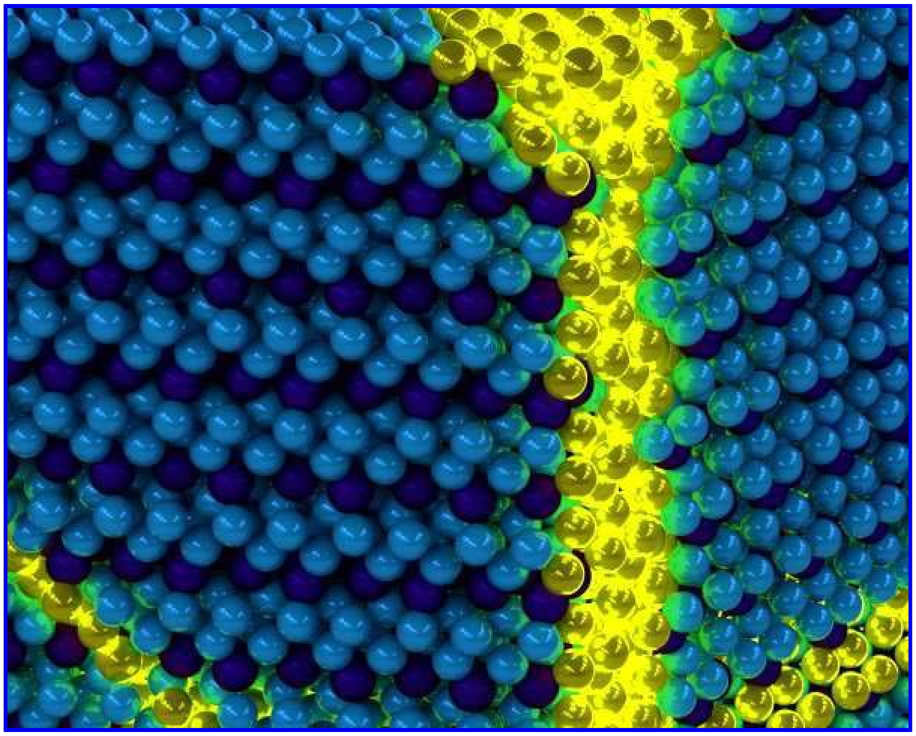
To know if a uniform electron gas at zero temperature forms a crystal, you need to work out its so-called **Wigner–Seitz radius**. This is the average inter-particle spacing measured in units of the Bohr radius. The **Bohr radius** is the unit of length you can cook up from the electron mass, the electron charge and Planck's constant. (It's also the average distance between the electron and a proton in a hydrogen atom in its lowest energy state.)

Simulations show that a 3-dimensional uniform electron gas crystallizes when the Wigner–Seitz radius is at least 106. In 2 dimensions, it happens when it's at least 31.

The picture above was drawn by Arunas.rv and placed on Wikicommons on a Creative Commons Attribution-Share Alike 3.0 Unported license.

December 9, 2017

Excitonium



In certain crystals you can knock an electron out of its favorite place and leave a **hole**: a *place with a missing electron*. Sometimes these holes can move around like particles. And naturally these holes attract electrons, since they are *places an electron would want to be*.

Since an electron and a hole attract each other, they can orbit each other. An orbiting electron-hole pair is a bit like a hydrogen atom, where an electron orbits a proton.

An orbiting electron-hole pair is called an **exciton**, because it's really just a special kind of 'excited' electron — an electron with extra energy, not in its lowest energy state where it wants to be.

An exciton usually doesn't last long: the orbiting electron and hole spiral towards each other, the electron finds the hole it's been seeking, and it settles down. Typical lifetimes range from picoseconds to nanoseconds.

But excitons can last long enough to do interesting things. In 1978 the Russian physicist Abrikosov wrote a short and very creative paper in which he raised the possibility that *excitons could form a crystal in their own right!* He called this new state of matter **excitonium**.

In fact his reasoning was very simple.

Just as electrons have a mass, so do holes. That sounds odd, since a hole is just a vacant spot where an electron would like to be. But such a hole can move around, and it takes force to accelerate it, so it acts just like it has a mass! The precise mass of a hole depends on the nature of the substance we're dealing with.

Now imagine a substance with very heavy holes.

When a hole is much heavier than an electron, it will stand almost still when an electron orbits it. So, they form an exciton that's *very* similar to a hydrogen atom, where we have an electron orbiting a much heavier proton.

Hydrogen comes in different forms: gas, liquid, solid... and at extreme pressures, like in the core of Jupiter, hydrogen becomes *metallic*. So, we should expect that excitons can come in all these different forms too!

We should be able to create an exciton gas... an exciton liquid... an exciton solid.... and under certain circumstances, a *metallic crystal of excitons*. Abrikosov called this **metallic excitonium**.

People have been trying to create this stuff for a long time. Some claim to have succeeded. But a new paper claims to have found something else: a Bose-Einstein condensate of excitons:

- Anshul Kogar, Melinda S. Rak, Sean Vig, Ali A. Husain, Felix Flicker, Young Il Joe, Luc Venema, Greg J. MacDougall, Tai C. Chiang, Eduardo Fradkin, Jasper van Wezel and Peter Abbamonte, Signatures of exciton condensation in a transition metal dichalcogenide, *Science* **358** (2017), 1314–1317.

There's a pretty good simplified explanation at the University of Illinois website:

- Siv Schwink, Physicists excited by discovery of new form of matter, excitonium, 7 December 2017.

However, the picture here shows domain walls moving through crystallized excitonium — I think that's different than a Bose-Einstein condensate! I'm a bit confused.

I urge you to look at Abrikosov's paper. It's two pages long and beautiful:

- A. A. Abrikosov, A possible mechanism of high temperature superconductivity, *Journal of the Less Common Metals* **62** (1978), 451–455.

He points out that previous authors had the idea of metallic excitonium. Maybe his new idea was that this might be a superconductor — and that this might explain high-temperature superconductivity. The reason for his guess is that metallic hydrogen, too, is widely suspected to be a superconductor.

Later Abrikosov won the Nobel prize for some other ideas about superconductors. I think I should read more of his papers.

Puzzle 1. If a crystal of excitons conducts electricity, what is actually going on? That is, which electrons are moving around, and how?

This is a fun puzzle because an exciton crystal is a kind of *abstract* crystal created by the motion of electrons in another, ordinary, crystal.

Puzzle 2. Is it possible to create a hole in excitonium? If so, it possible to create an exciton in excitonium? If so, is it possible to create **meta-excitonium**: an crystal of excitons in excitonium?

December 18, 2017

What's cooler than a superfluid?

When you cool helium enough, it becomes a superfluid. It can then do amazing things like climb out of a cup, as shown here. What could be cooler than that? A *supersolid*.

In a superfluid, the atoms become exactly the same in every way. They're not even in different places: they're all spread out everywhere. This lets them move in lock step. The viscosity drops to zero. So a superfluid can do things like climb out of a cup thanks to the tiny attraction it feels to the walls of the cup, and the force of gravity pulling down. Each atom is both inside the cup and outside getting pulled down!

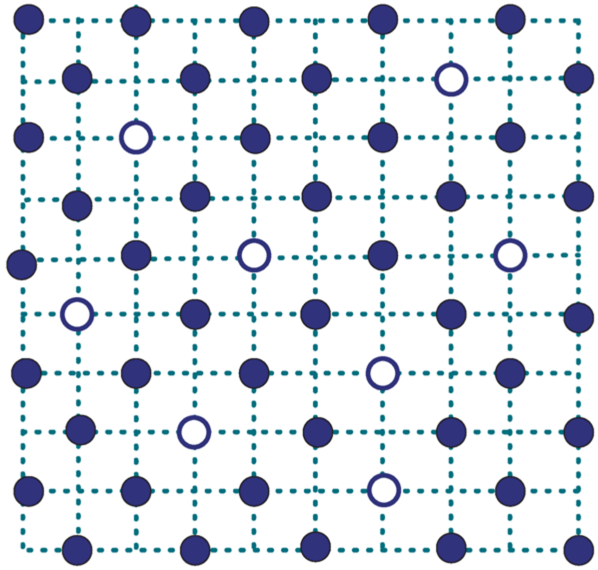
This is only possible thanks to quantum mechanics — and only because the most common form of helium is a boson. Every particle in nature is either a boson or fermion. Two particles that are fermions can't be in the same state. But bosons can. And at low temperatures, identical bosons like to be in the exact same state. This is called a **Bose–Einstein condensate**.

So what's a supersolid?

When you compress liquid helium enough, it becomes a crystal. But as with many crystals, there will be vacancies: places where an atom is missing.

In an ordinary crystal, the vacancies can move around. But in solid helium, the vacancies are bosons, so it's possible that at low temperatures they will form a superfluid! The result is called a 'supersolid'.

In short: a **supersolid** is a crystal where *vacancies* form a Bose–Einstein condensate, allowing them to flow through the crystal with no viscosity. It's like a superfluid formed by the *absence* of particles, moving like ghosts through a crystal!



Here the white circles represent vacancies. Click for more details!

Now for the complicated part. There have been a lot of arguments about whether helium can form a supersolid. The current consensus seems to be that it can't. However, people claim to have made supersolids using other materials. So the idea is still very interesting.

Here's the story, paraphrased from Wikipedia:

> While several experiments yielded negative results, in the 1980s, John Goodkind from UCSD discovered the first 'anomaly' in a solid by using ultrasound. Inspired by his observation, Eun-Seong Kim and Moses Chan at Pennsylvania State University saw phenomena which were interpreted as supersolid behavior. Specifically, they observed an unusual decoupling of the solid helium from a container's walls which could not be explained by classical models but which was consistent with a superfluid-like decoupling of a small percentage of the atoms from the rest of the atoms in the container. If such an interpretation is correct, it would signify the discovery of a new quantum phase of matter.
>
> The experiment of Kim and Chan looked for superflow by means of a "torsional oscillator." To achieve this, a turntable is attached tightly to a spring-loaded spindle; then, instead of rotating at constant speed, the turntable is given an initial motion in one direction. The spring causes the table to oscillate similarly to a balance wheel. A toroid filled with solid helium-4 is attached to the table. The rate of oscillation of the turntable and toroid depend on the amount of solid moving with it. If there is frictionless superfluid inside, then the mass moving with the doughnut is less, and the oscillation will occur at a faster rate. In this way, one can measure the amount of superfluid existing at various temperatures. Kim and Chan found that up to about 2% of the material in the doughnut was superfluid. (Recent experiments have increased the percentage to over 20%). Similar experiments in other laboratories have confirmed these results.

In short, without all jargon: if you have a supersolid, you can twist it back and forth and the liquid formed by the vacancies will not turn back and forth, because it can flow right through the crystal.

But then comes the controversy:

> A mysterious feature, not in agreement with the old theories, is that the transition continues to occur at high pressures. High-precision measurements of the melting pressure of helium-4 have not resulted in any observation of a phase transition in the solid.
>
> Prior to 2007, many theorists performed calculations indicating that vacancies cannot exist at zero temperature in solid helium-4. While there is some debate, it seems more doubtful that what the experiments observed was the supersolid state. Indeed, further experimentation, including that by Kim and

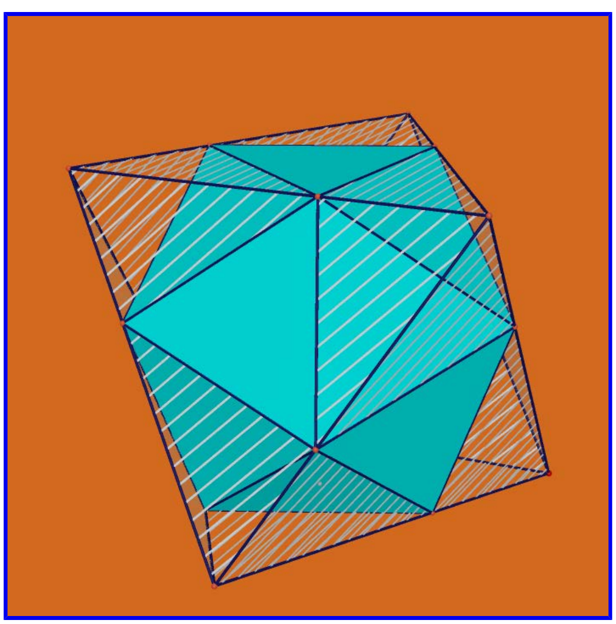
> Chan, has also cast some doubt on the existence of a true supersolid. One experiment found that repeated warming followed by slow cooling of the sample causes the effect to disappear. This annealing process removes flaws in the crystal structure.
>
> Furthermore, most samples of helium-4 contain a small amount of helium-3. When some of this helium-3 is removed, the superfluid transition occurs at a lower temperature, which suggests that the superflow is involved with actual fluid moving along imperfections in the crystal rather than a property of the perfect crystal.
>
> In 2009, it was proposed to realize a supersolid in an optical lattice. Starting from a molecular quantum crystal, supersolidity is induced dynamically as an out-of-equilibrium state. While neighboring molecular wave functions overlap, two bosonic species simultaneously exhibit quasicondensation and long-range solid order, which is stabilized by their mass imbalance. This proposal can be realized in present experiments with bosonic mixtures in an optical lattice that features simple on-site interactions.
>
> Experimental and theoretical work continues in hopes of finally settling the question of the existence of a supersolid.
>
> In 2012, Chan repeated his original experiments with a new apparatus that was designed to eliminate any contribution from elasticity of the helium. In this experiment, Chan and his coauthors found no evidence of supersolidity.

Too bad! But...

> In 2017, two research groups from ETH Zurich and from MIT reported on the first creation of a supersolid with ultracold quantum gases. The Zurich group placed a Bose–Einstein condensate inside two optical resonators, which enhanced the atomic interactions until they start to spontaneously crystallize and form a solid that maintains the inherent superfluidity of Bose–Einstein condensates. The MIT group exposed a Bose–Einstein condensate in a double-well potential to light beams that created an effective spin-orbit coupling. The interference between the atoms on the two spin-orbit coupled lattice sites gave rise to a density modulation that establishes a stripe phase with supersolid properties.

In short: there's still hope that people can create supersolids, but it will take more experiments to be sure.

December 30, 2017



In math, all sufficiently beautiful entities are connected to all others. Here's another example. A regular octahedron has 12 edges. A regular icosahedron has 12 corners. So there could be a way to draw the icosahedron with its corners on the edges of the octahedron. And yes — there, is!

But as a final twist of the knife, you don't put the corners at the middle of the edges. That wouldn't work. Instead, each of the corners divides each of the edges according to the golden ratio!

Math just *had* to do that.

I got this beautiful image here:

- Jen-chung Chuan, Quick-and-dirty constructions with Cabri 3d.

Puzzle 1. What shape has 12 corners, with one located exactly in the center of each edge of the regular octahedron?

Puzzle 2. Can you make or find an animated gif of that shape morphing into a regular icosahedron as its corners move from the midpoints of the octahedron edges to the points shown here?

Puzzle 3. How many ways are there to create a regular icosahedron whose corners lie on the edges of a given regular octahedron?

Puzzle 4. How many ways are there to create a regular octahedron edges contain the edges of a given regular icosahedron?

The answers can be found in the comments on my G+ post.

For my January 2017 diary, go here.

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