Categories: The Mathematics of Connection

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The Mathematics of Collective Intelligence
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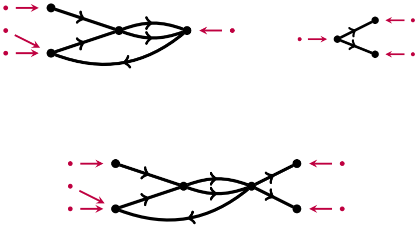
A closed system can often be seen as built from interacting open systems. Thus *composing open systems along their interfaces* becomes important.



Lessons from open systems theory

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This law says the entropy of any *closed* system must increase over time.

Lessons from open systems theory

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Unitary time evolution applies to *closed* systems; we need its generalization to open systems to understand measurement processes.

Lessons from open systems theory

3. Your cell phone is not a Turing machine.

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A Turing machine is a *closed* system. It evolves deterministically given its initial state.

A cell phone is an open system: it affects and is affected by its environment. Its behavior can only be predicted if we know its environment's behavior... and vice versa.

Lessons from open systems theory

4. Your brain is not a Turing machine.

Lessons from open systems theory

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Oddly, this is more controversial than the case of a cell phone.

I'm not attributing magical powers to the brain. A Turing machine is simply an inaccurate model of the brain, for many reasons. *One* reason is that the brain is an open system.

A possible lesson from open systems theory?

5. Intelligence is fundamentally collective.

There is no such thing as a single intelligent agent, not interacting with others.

A category has a collection of **objects**, and for any pair of objects x, y a set of **morphisms** $F\colon x \to y$.

We can **compose** morphisms $F\colon x \to y$ and $G\colon y \to z$ and get a morphism $G \circ F\colon x \to z$. Composition is associative:

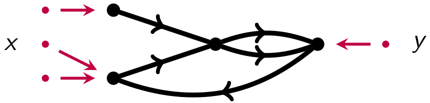
$$(H \circ G) \circ F = H \circ (G \circ F)$$

For every object y there's an **identity morphism** $1_y\colon y \to y$ such that

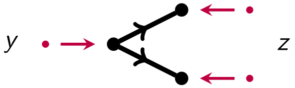
$$1_x \circ F = F \qquad H \circ 1_x = H$$

for all morphisms $F\colon x \to y$, $H\colon y \to z$.

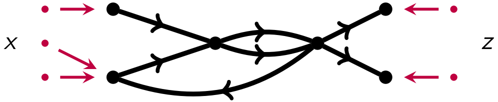
A morphism $F\colon x \to y$ can represent an open system of some sort, with interfaces x and y:



Then composing it with another open system $G\colon y \to z$

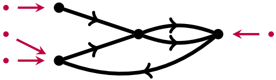


along the interface y gives a new open system $G \circ F\colon x \to z$:

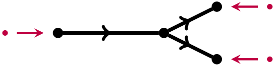


Using "monoidal" categories we can deal with open systems that have any number of interfaces, not just two. Monoidal categories also let us put open systems side by side:

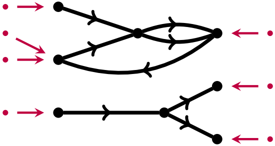
$$F\colon x \to y$$



$$G\colon x' \to y'$$



$$F \otimes G\colon x \otimes x' \to y \otimes y'$$



Applying categories to open systems

Category theory is being used to study or design open systems of many kinds. You can read about some here:

- electrical circuits
- chemical reaction networks
- Markov processes
- Petri nets
- control theory
- dynamical systems
- machine learning
- game theory

Here are two good places to learn some basics:

- Tai-Danae Bradley, *What is Applied Category Theory?*
- Brendan Fong and David Spivak, *An Invitation to Applied Category Theory: Seven Sketches in Compositionality*.

Applying categories to open systems — a case study

While a powerful tool, category theory is not a "silver bullet". Applying it requires sustained effort from people who know both category theory and the area of application quite well.

But it's getting easier!

Let me give an example.

Around 2011, I asked my student Brendan Fong to construct a category where the morphisms are open electrical circuits.

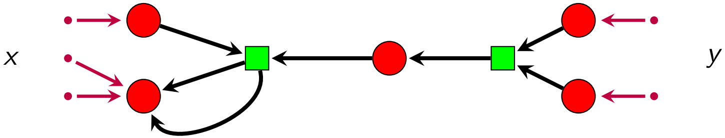
He took a long time, but he did much more: he came up with a *general theory* of how to build categories whose morphisms are open systems: the theory of "decorated cospans".

His thesis is a great introduction to categories for open systems:

- Brendan Fong, *The Algebra of Open and Interconnected Systems*, 2016.

Another student and I then applied decorated cospans to study open chemical reaction networks:

- John Baez and Blake Pollard, A compositional framework for reaction networks, 2017.



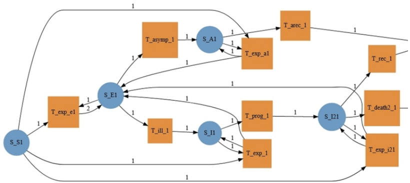
This is called an "open Petri net".

In 2020, James Fairbanks and Evan Patterson set up AlgebraicJulia, a software environment for doing scientific computing with categories.

Using AlgebraicJulia, they created a tool to build flexible models of infectious disease using open Petri nets and a variant of decorated cospans:

- Micah Halter and Evan Patterson, Compositional epidemiological modeling using structured cospans, 2020.

They illustrated it by quickly building part of the UK's model of the COVID-19 pandemic out of smaller open Petri nets.



In 2019, Brendan and David Spivak started the Topos Institute in Berkeley — a home for applied category theory. Evan Patterson works there now.

His work on open Petri nets for modeling infectious disease attracted attention from epidemiologists. The Topos Institute now holds regular meetings with a team of category theorists, computer scientists and epidemiologists including Nathaniel Osgood, who leads COVID-19 modeling for Saskatchewan.

In March 2022, Osgood plans to run a hackathon where his students build models of infectious disease using the software we're designing now in AlgebraicJulia.

So, it took 11 years to go from "let's invent math that describes open systems" to "let's use this math to write software that will help epidemiologists more efficiently build models of infectious disease".

Most of this time was spent developing the math of open systems.

Now this math is here, with computer support! It's very general, so it can be applied to many domains — maybe collective intelligence?

But I believe success requires sustained, intensive collaboration between mathematicians and domain experts.