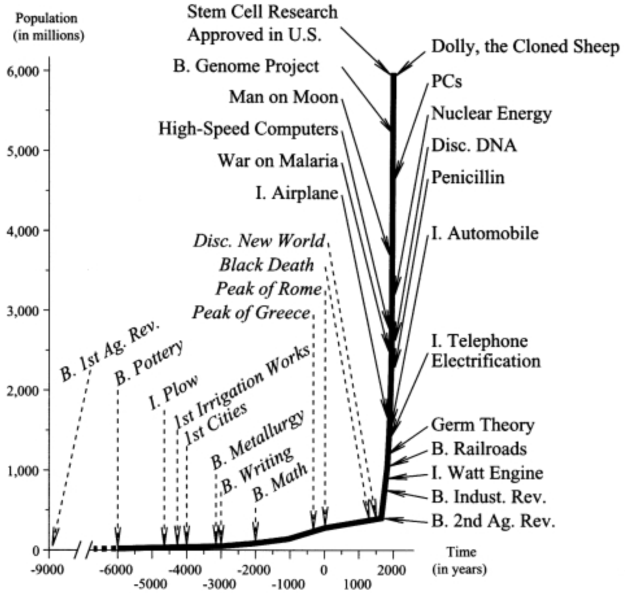
THE MATHEMATICS OF PLANET EARTH

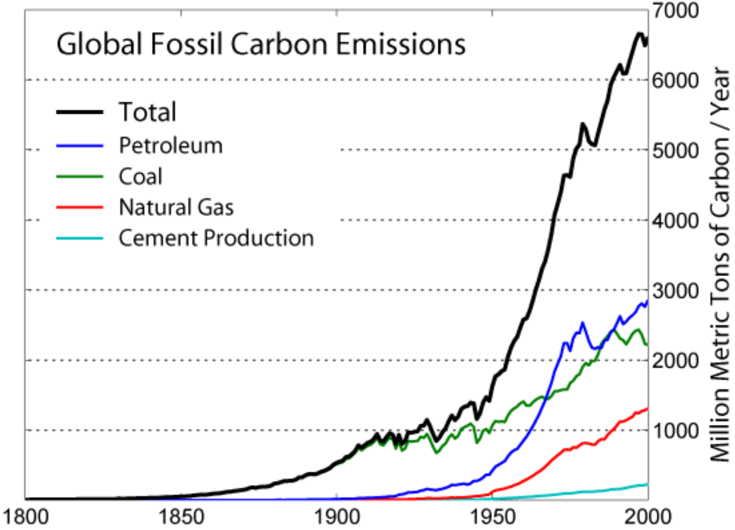
John Baez

Lang Lecture

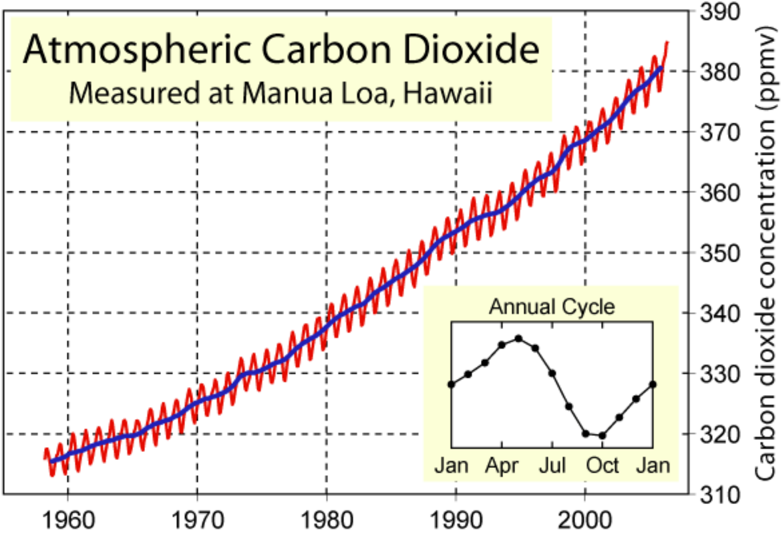
November 16, 2012



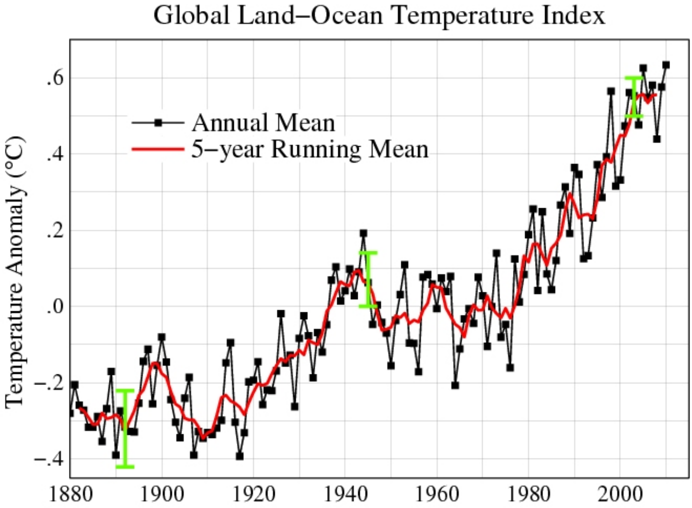
Robert Fogel - *The Escape from Hunger and Premature Death, 1700-2100*



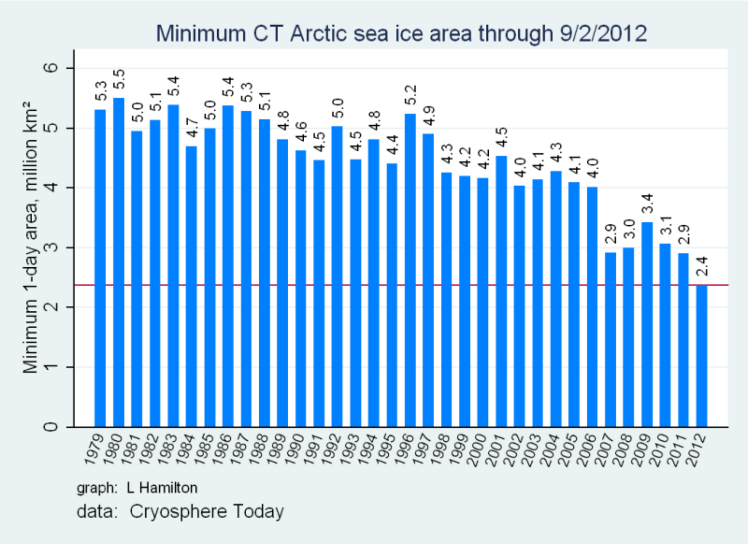
Global Warming Art



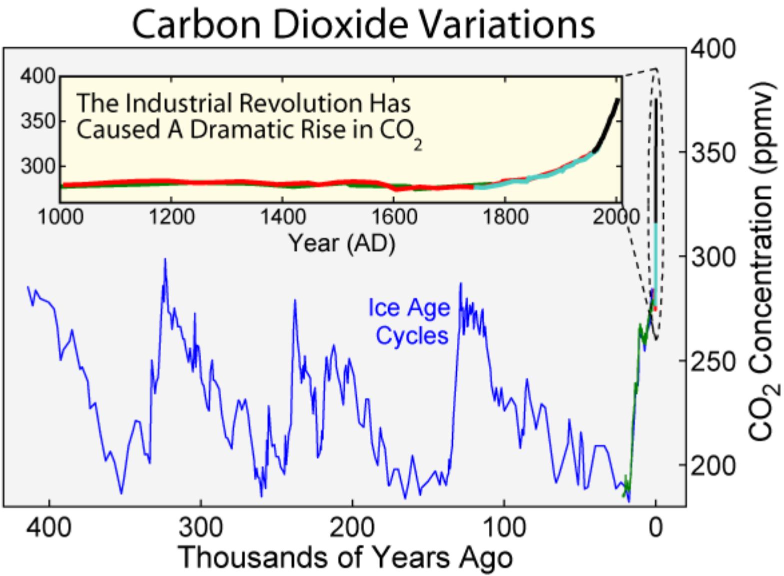
The Keeling Experiment — Global Warming Art



NASA Goddard Institute of Space Science



The Cryosphere Today



Antarctic ice cores and other data — Global Warming Art

In April 2012, International Energy Agency official Richard H. Jones said:

> *We have a responsibility and a golden opportunity to act. Energy-related CO_2 emissions are at historic highs; under current policies, we estimate that energy use and CO_2 emissions would increase by a third by 2020, and almost double by 2050. This would likely send global temperatures at least 6 °C higher. Such an outcome would confront future generations with significant economic, environmental and energy security hardships—a legacy that I know none of us wishes to leave behind.*

From ABC News, 13 November 2012:

> *A recent report by the U.S. Government Accountability Office estimated that if half of the oil bound up in the rock of the Green River Formation could be recovered it would be "equal to the entire world's proven oil reserves".*
>
> *Both the GAO and private industry estimate the amount of oil recoverable to be 3 trillion barrels.*
>
> *"In the past 100 years—in all of human history—we have consumed 1 trillion barrels of oil. There are several times that much here," said Roger Day, vice president for operations for American Shale Oil (AMSO).*

Scientist Lonnie Thompson writes:

> *Climatologists, like other scientists, tend to be a stolid group. We are not given to theatrical rantings about falling skies. [....] Why then are climatologists speaking out about the dangers of global warming? The answer is that* **virtually all of us are now convinced that global warming poses a clear and present danger to civilization***.*

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Invent the math we need for life on a finite-sized planet.

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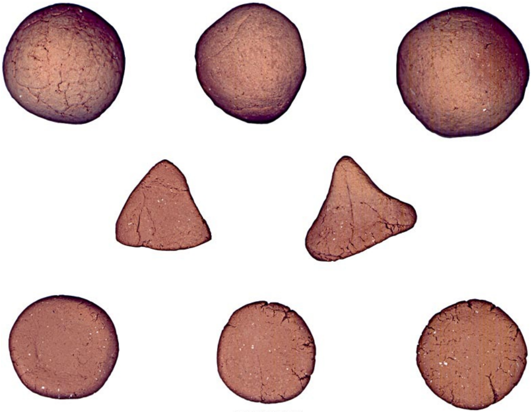
By now we use about 25% of all plant biomass grown worldwide! If this reaches 100% there will be, in some sense, no 'nature' separate from humanity.

Starting shortly after the end of the last ice age, the agricultural revolution led to:

- surplus grain production, and thus kingdoms and slavery.
- *astronomical mathematics* for social control and crop planning.
- *geometry* for measuring fields and storage containers.
- *written numbers* for commerce.

Consider the last...

Starting around 8,000 BC, in the Near East, people started using 'tokens' for contracts: little geometric clay figures that represented things like sheep, jars of oil, and amounts of grain.



MS 5067/1-8
Neolithic plain counting tokens. Near East, ca. 8000--3500 BC

The Schøyen Collection

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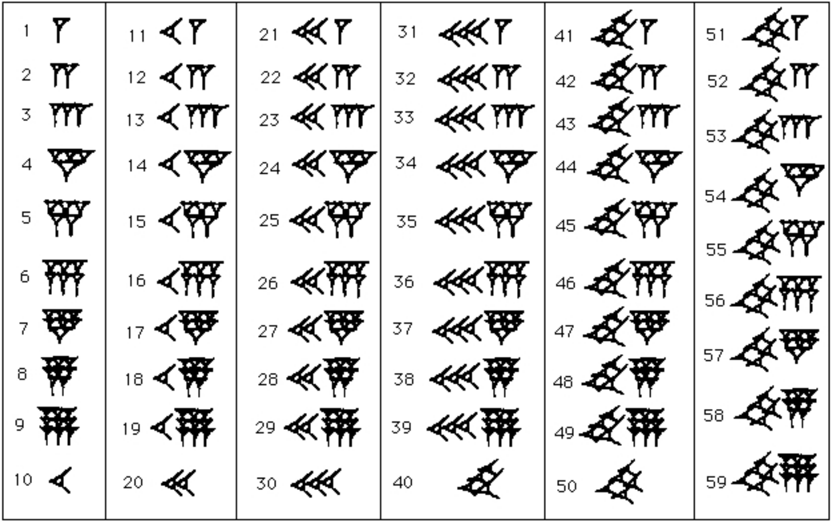
Eventually they gave up on the tokens. The marks on tablets then developed into the Babylonian number system! The transformation was complete by 3,000 BC.



J. J. O'Connor and E. F. Robertson, Babylonian Numerals

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By 1700 BC the Babylonians could compute $\sqrt{2}$ to 6 decimals:

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx 1.414213...$$

Yale Babylonian Collection, YBC7289

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Math may undergo a transformation just as big as it did in the Agricultural Revolution.

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Even better, these machines should spread without human intervention.

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For sophisticated ecotechnology we need to pay attention to what's already known—permaculture, systems ecology and so on. But better mathematics could help.

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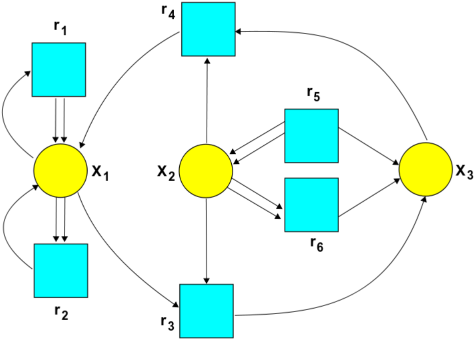
My own work on networks is rather abstract: nice math, but you might not see how it's connected to ecology.



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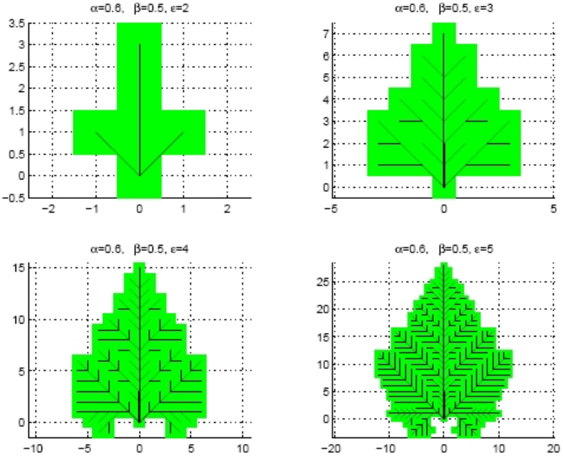
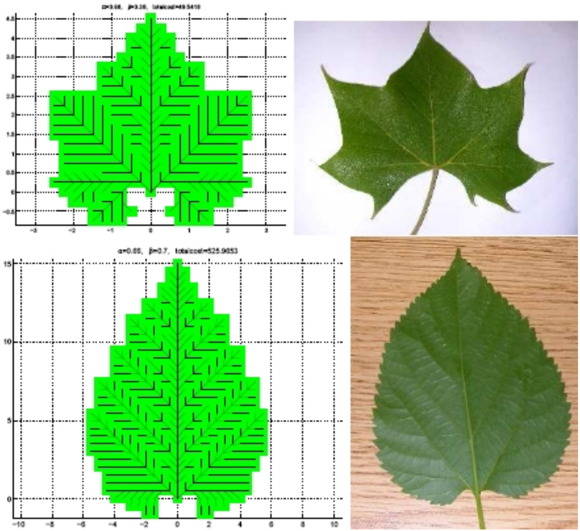
Yes! A mathematician at U.C. Davis, Qinglan Xia, has written a paper called *The Formation of a Tree Leaf*.

He models a leaf as a union of square cells centered on a grid, together with 'veins' forming a weighted directed graph from the centers of the cells to the root. The leaf grows new cells at the boundary while minimizing a certain function.



The function depends on two parameters. Changing these gives different leaf shapes:



Qinglan Xia's work is definitely math:

Lemma 3.8. *Suppose* (Ω, G) *is an* (ϵ, h) *leaf and* $(\mu, \Theta) = \phi_h(\Omega, G)$. *Then the total mass of the Radon measure is bounded above by*

$$\mathbf{M}(\mu) \leq \pi (R_\epsilon + h)^2$$

and the total variation of the vector measure Θ *is bounded by*

$$\mathbf{M}(\Theta) \leq \epsilon \pi^{2-\alpha} (R_\epsilon + h)^{4-2\alpha}.$$

Proof. Since $\Omega \subset B_{R_\epsilon}(O)$, the mass of μ is given by

$$\begin{aligned} \mathbf{M}(\mu) &= ||\Omega||\, h^2 \\ &= \text{area}\left(\bigcup_{x\in\Omega}\left\{x + \left[-\frac{h}{2}, \frac{h}{2}\right] \times \left[-\frac{h}{2}, \frac{h}{2}\right]\right\}\right) \\ &\leq \text{area}\left(B_{R_\epsilon + h}(0)\right) = \pi (R_\epsilon + h)^2. \end{aligned}$$

Also, since $w(e) \leq ||\Omega||\, h^2$ for each $e \in E(G)$, the total variation of Θ is given by

$$\begin{aligned} \mathbf{M}(\Theta) &= \sum_{e \in E(G)} w(e)\, length(e) \\ &\leq \left(||\Omega||\, h^2\right)^{1-\alpha} \sum m_\beta(e^+)\,(w(e))^\alpha\, length(e) \end{aligned}$$

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It's just beginning to be born. I hope you can help out!