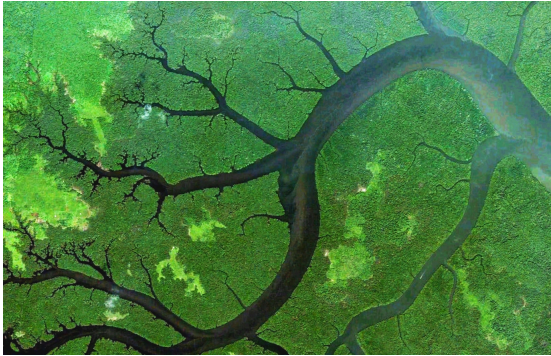
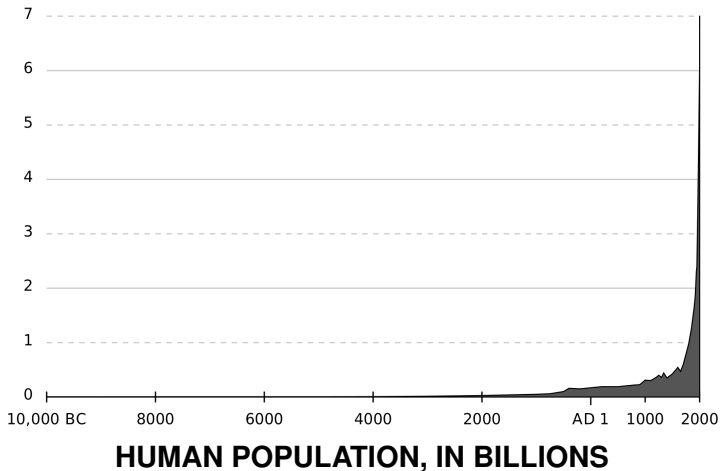


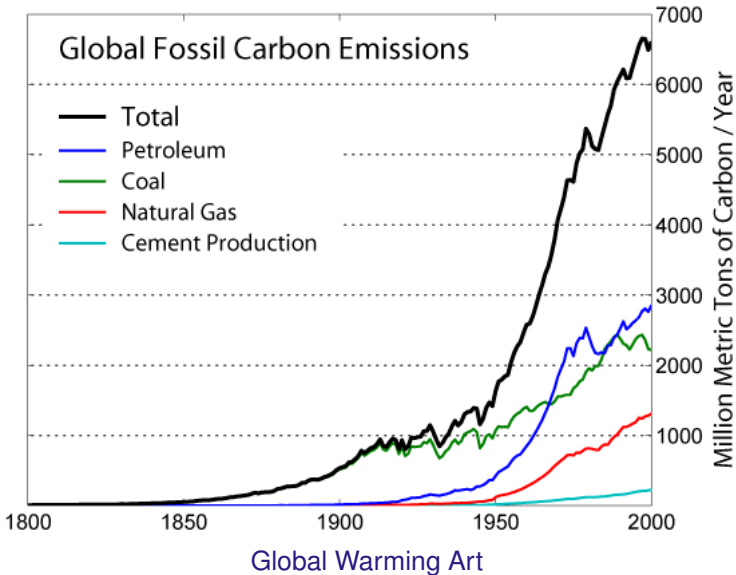
# THE MATHEMATICS OF PLANET EARTH



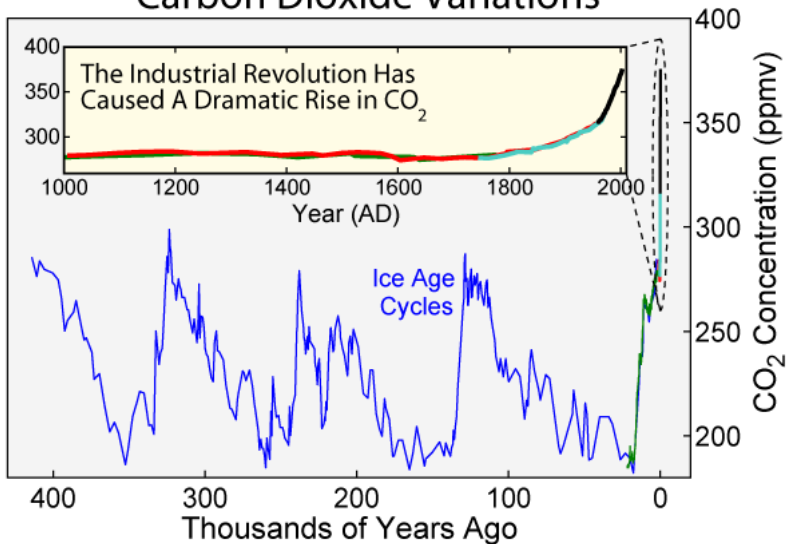
**John Baez**  
**University of Cambridge**  
**12 March 2014**

We have left the Holocene and entered a new epoch, the **Anthropocene**, when the biosphere is rapidly changing due to human activities. Global warming is just *part* of this process.

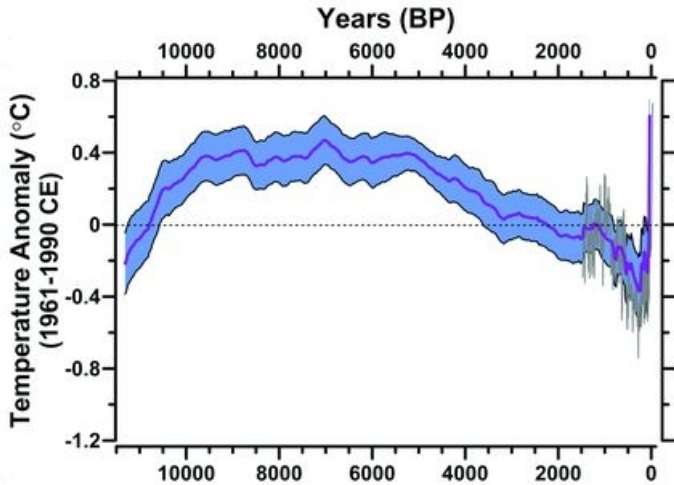




# Carbon Dioxide Variations

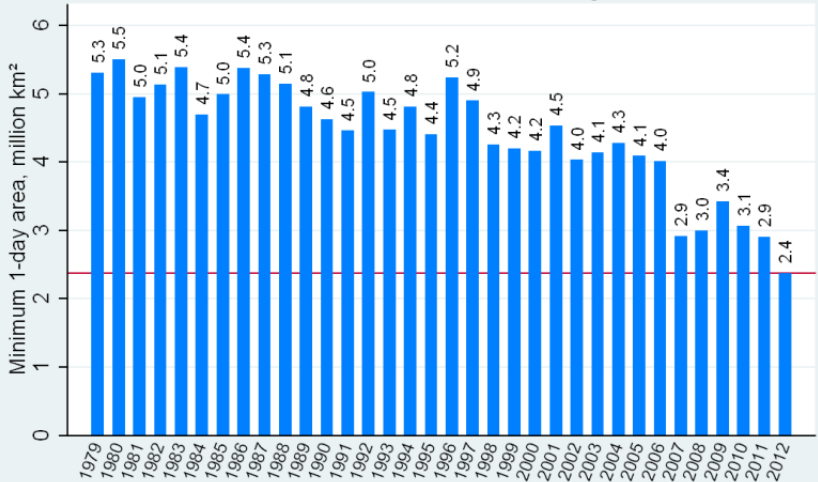


Antarctic ice cores and other data — Global Warming Art



Reconstruction of temperature from 73 different records —  
Marcott *et al.*

## Minimum CT Arctic sea ice area through 9/2/2012



graph: L Hamilton

data: Cryosphere Today

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- Populations of large ocean fish have declined 90% since 1950.
- The rate of species going extinct is 100-1000 times the usual background rate.

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- *Before*, we could treat 'nature' as distinct from 'civilization'. *Now*, there is no nature separate from civilization.
- *Before*, 'economic growth' could be our main goal, with many side-effects ignored. *Now*, many forms of growth are pushing the biosphere toward **tipping points**.

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Invent the math we need for life on a finite planet.

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During this revolution, from 10,000 to 5,000 BC, we began to systematically exploit solar power by planting crops.

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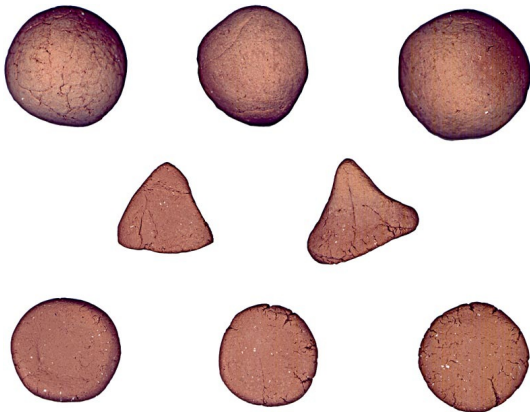
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- *astronomical mathematics* for social control and crop planning.
- *geometry* for measuring fields and storage containers.
- *written numbers* for commerce.

Consider the last...

Starting around 8000 BC, in the Near East, people started using 'tokens' for contracts: little geometric clay figures that represented things like sheep, jars of oil, and amounts of grain.



MS 5067/1-8  
Neolithic plain counting tokens. Near East, ca. 8000–3500 BC

The Schøyen Collection

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Eventually they gave up on the tokens. The marks on tablets then developed into the Babylonian number system! The transformation was complete by 3000 BC.

1		11		21		31		41		51	
2		12		22		32		42		52	
3		13		23		33		43		53	
4		14		24		34		44		54	
5		15		25		35		45		55	
6		16		26		36		46		56	
7		17		27		37		47		57	
8		18		28		38		48		58	
9		19		29		39		49		59	
10		20		30		40		50			

J. J. O'Connor and E. F. Robertson, Babylonian Numerals

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By 1700 BC the Babylonians could compute  $\sqrt{2}$  to 6 decimals:

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \approx 1.414213\dots$$



Yale Babylonian Collection, YBC7289

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Math may undergo a transformation just as big as it did in the Agricultural Revolution.

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Even better, these machines should spread without human intervention.





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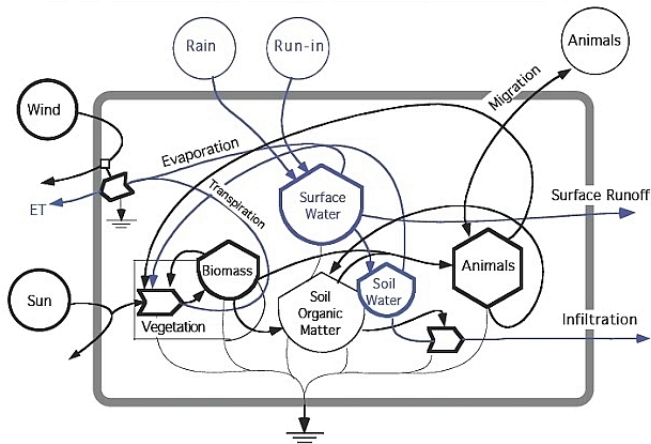
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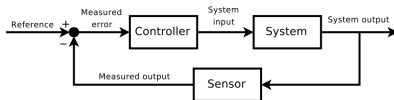
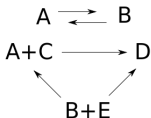
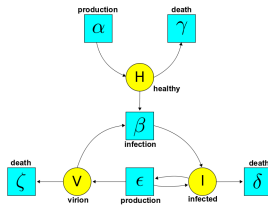
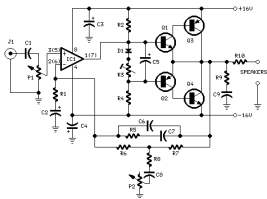
For sophisticated ecotechnology we need to pay attention to what's already known: [permaculture](#), [systems ecology](#) and so on. But better mathematics could help.

***To understand ecosystems, ultimately will be to understand networks.*** — B. C. Patten and M. Witkamp



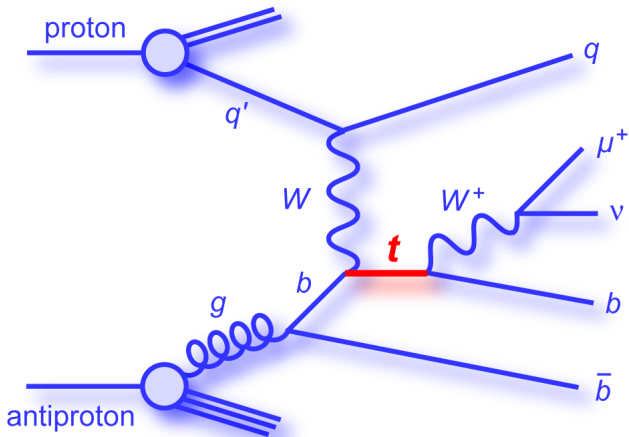
In the 1950's, Howard Odum introduced an [Energy Systems Language](#) to describe these networks.

Engineers, chemists, biologists and others now use *many* diagram languages to describe complex systems:



We need a good mathematical theory of these!

The mathematics of particle physics, and category theory, can help!



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However, I expect the mathematics of this century will draw inspiration from *biology*, *ecology* and *sociology*, much as the math of the industrial revolution was inspired by physics.

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It's just beginning to be born. I hope you can help out. Check out the [Azimuth Project](#)!