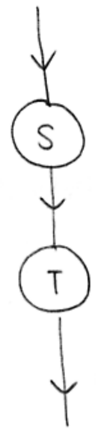
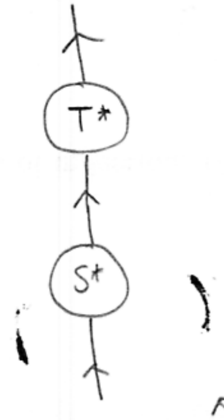
Alissa Crans

10/9/00

HW #1



$\longrightarrow$
adjoint



show these are equivalent.

vs.

compose w/

$V^* = \mathrm{Hom}(V, \mathbb{C})$

$V'^* = \mathrm{Hom}(V', \mathbb{C})$

$V''^* = \mathrm{Hom}(V'', \mathbb{C})$

$$\boxed{\underline{\text{Show}}: \quad (T \circ S)^* = S^* \circ T^*}$$

let $S: V \longrightarrow V'$ $\qquad$ $S^*: V'^* \longrightarrow V^*$

$T: V' \longrightarrow V''$ $\qquad$ $T^*: V''^* \longrightarrow V'^*$

$T \circ S: V \longrightarrow V''$ $\qquad$ $(T \circ S)^*: (V'')^* \longrightarrow V^*$

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Let $f \in (V'')^*$, $v \in V$.

$$\underbrace{(T\circ S)^*(f)}_{\in V^*}(v) = f\big((T\circ S)(v)\big) = f(T(S(v)))$$

need to feed it something in V.

$$(S^*\circ T^*)(f)(v) = S^*(\underbrace{T^*(f)}_{\in V'^*})(v) = \underbrace{T^*(f)}_{\in V'^*}(\underbrace{S(v)}_{\in V'})$$

$$= f(T(S(v)))$$

so, $(T\circ S)^* = (S^*\circ T^*)$

✓ great

Note: $T^*(f)(v) = f(Tv)$