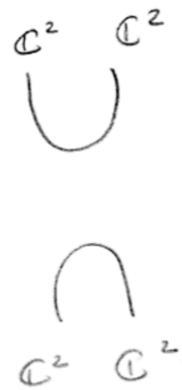
reals HW #4

Alissa Crans
MATH 260



want $\alpha : \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}$

$\beta : \mathbb{C} \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$

st

①  $= -2$ (ie multiplication by -2)

②  $= -$ 

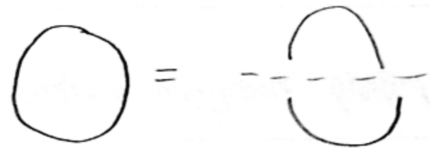
③  $= -$ $+$ 

New idea:
We let $e_{11}, e_{12}, e_{21}, e_{22}$ be basis elts. of $\mathbb{C}^2 \otimes \mathbb{C}^2$.

Define $\alpha(e_{11}) = \alpha(e_{22}) = 0.$
$\alpha(e_{12}) = 1$
$\alpha(e_{21}) = -1$

Define $\beta(1) = e_{21} - e_{12}$

HW # 4:

Check: ①  $= -2$ 

$= \alpha(\beta(1))$

$= \alpha(e_{21} - e_{12})$

$= \alpha(e_{21}) - \alpha(e_{12})$ α is linear

$= -1 - 1 = -2$ ✓

②  $= -$ 

LHS:

RHS

$e_{11} \mapsto e_{11}$ then $\alpha(e_{11}) = 0$ $\qquad -\alpha(e_{11}) = 0$

$e_{22} \mapsto e_{22}$ then $\alpha(e_{22}) = 0$ $\qquad -\alpha(e_{22}) = 0$

$e_{12} \mapsto e_{21}$ then $\alpha(e_{21}) = -1$ $\qquad -\alpha(e_{12}) = -(1) = -1$

$e_{21} \mapsto e_{12}$ then $\alpha(e_{12}) = 1$ $\qquad -\alpha(e_{21}) = -(-1) = 1$

$e_{11} = e_1 \otimes e_1$

$e_{22} = e_2 \otimes e_2$

$e_{12} = e_1 \otimes e_2$

$e_{21} = e_2 \otimes e_1$

HW #4:

(3) 

LHS:

$e_{22} \longmapsto e_{22}$

$e_{11} \longmapsto e_{11}$

$e_{12} \longmapsto e_{21}$

$e_{21} \longmapsto e_{12}$

RHS

$$e_{22} \longmapsto e_{22} + \beta(\alpha(e_{22})) = e_{22} + 0 = e_{22} \quad ✓$$

$$e_{11} \longmapsto e_{11} + \beta(\alpha(e_{11})) = e_{11} + 0 = e_{11} \quad ✓$$

$$\begin{aligned} e_{12} \longmapsto\ & e_{12} + \beta(\alpha(e_{12})) \\ &= e_{12} + \beta(1) \\ &= e_{12} + (e_{21} - e_{12}) = e_{21} \quad ✓ \end{aligned}$$

$$\begin{aligned} e_{21} \longmapsto\ & e_{21} + \beta(\alpha(e_{21})) \\ &= e_{21} + \beta(-1) \\ &= e_{21} + -\beta(1) \\ &= e_{21} + -(e_{21} - e_{12}) = e_{12} \quad ✓ \end{aligned}$$

✓ great!