

Show

$$j_1 \begin{array}{c} \diagup \\ \diagdown \\ \cup \end{array} j_2 = (-1)^{j_1+j_2-j_3} j_1 \begin{array}{c} \diagup \\ \diagdown \\ \cup \end{array} j_2$$

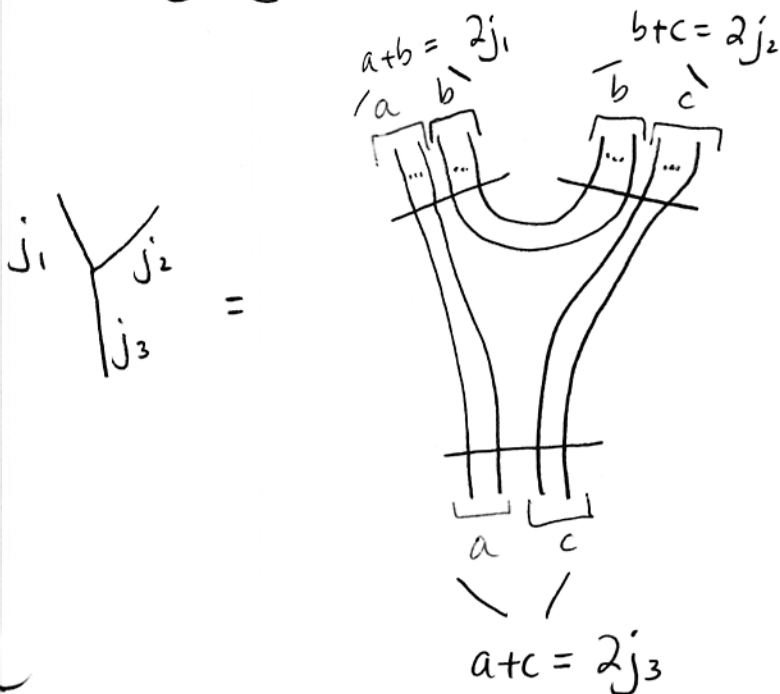
Fact ①: $\gamma = -U$

Also- notice $j_1+j_2-j_3 = b = \#$ of cups in $j_1 \begin{array}{c} \diagup \\ \diagdown \\ \cup \end{array} j_2$

$$a = j_1 + j_3 - j_2$$

$$b = j_1 + j_2 - j_3$$

$$c = j_2 + j_3 - j_1$$



Now we want to twist the cups.

$$j_1 \begin{array}{c} j_2 \\ | \\ j_3 \end{array} = (-1)^b \begin{array}{c} a \quad b \quad b \quad c \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ a \quad c \end{array}$$

by Fact ①.

$$= (-1)^b \begin{array}{c} a \quad b \quad b \quad c \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ c \quad a \end{array}$$

twist the bottom

$$= (-1)^b \begin{array}{c} a \quad b \quad b \quad c \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ a \quad c \end{array}$$

since

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} = \frac{1}{2} \left(\begin{array}{c} \diagup \quad \diagdown \\ \diagdown \quad \diagup \end{array} + \begin{array}{c} \diagdown \quad \diagup \\ \diagup \quad \diagdown \end{array} \right)$$

$$= (-1)^b \begin{array}{c} j_1 \quad j_2 \\ | \\ j_3 \end{array}$$

$$= \frac{1}{2} \left(X + \parallel \right)$$

good!

$$= \parallel$$