1.1 *Evaluate the lambda-expression*

$$\Big(\big((\lambda f.\lambda x.f(f(f(x))))\,(\lambda g.\lambda y.g(g(y)))\big)\,(\lambda z.z+1)\Big)(0).$$

Given any function f (with codomain equal to the domain, or at least contained in the domain, or at least at least equipped with a map to the domain), let f^n be the n-fold composite of f with itself. For example, $f^2(x) = f(f(x))$, so $\lambda x.f(f(x)) = f^2$ (that is $f \circ f$).

Then $\lambda x.f(f(f(x))) = f^3$, and $\lambda y.g(g(y)) = g^2$. So, $\lambda g.\lambda y.g(g(y))$ is the operation that maps g to g^2, while $\lambda f.\lambda x.f(f(f(x)))$ is the operation that maps f to f^3. Applying the latter operation to the former operation, I get the operation that maps g to $\big((g^2)^2\big)^2 = g^8$. (To generalise this, note that $8 = 2^3$.)

Applying this operation $\lambda g.g^8$ to $\lambda z.z+1$, I get $\lambda z.z+8$, so the final result is $0+8=8$.

1.2 *Let* $\omega = \lambda x.x(x)$. *What is* $\omega(\omega)$*?*

In general, $\omega(f) = f(f)$ (for any function f that belongs to its domain, or at least is equipped with an element of its domain). Therefore, $\omega(\omega) = \omega(\omega)$. The expression cannot be further evaluated.

Perhaps more interesting would be to consider $\eta = \lambda x.x(x(x))$. Then $\eta(\eta) = \eta(\eta(\eta))$; evaluating the expression just makes it more complicated!