NOV/1/2007 THU

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(1) (a) The classification of binary relationships between flags on finite sets is the same as that between such relationships between flags on finite dimensional vector spaces (eg: over finite fields).

This does NOT hold true for, say, ternary relationships, the flags' classification being harder. These things do occur, eg. in representation theory, consider $\mathrm{Hom}_G(V \otimes W, X)$.

(b) In any case from last class we obtained a (rather non-standard!) way of expressing Hecke-operators (& G-interwiners) as matrices of non-negative integers.

Here's a "down-to-earth" way of picturing these. eg. take  $= (2,1^1)$ and look at G-module self-maps between this permutation representation.

(c) Say we look at 3 values of colorings for 2 attributes.

Hats Colored R, B, G
Gloves Colored R', B', G'
Shoes Colored R'', B'', G''

(Say the Hecke operator

	R	B	G
R'	1	1	0
B'	1	0	0
G'	0	0	1

$=$ #people with R, G

What are the possible configurations of Hats vs Shoes, given

(Hats vs Gloves)

	R	B	G
R'	1	1	0
B'	1	0	0
G'	0	0	1

(Gloves vs Shoes)

	R'	B'	G'
R''	1	1	0
B''	1	0	0
G''	0	0	1

$\Rightarrow$??

Answer: There are only two choices — but yes, there may be more than one configuration, re

2	0	0
0	1	0
0	0	1

&

1	1	0
1	0	0
0	0	1

(d) Now say we just rescale to $\Sigma = 400$.

Then (note that the people are NOT interchangeable so, may occur with ~~different~~ multiplicities).

If we had

	R	B	G
R'	100	100	0
B'	100	0	0
G'	0	0	100

	R'	B'	G'
R''	100	100	0
X B''	100	0	0
G''	0	0	100

Then the possibilities and multiplicities for Hats vs. Shoes is

100 +N	100 -N	0
100 -N	N	0
0	0	100

$\cdot \binom{100}{N}$

These N out of 100 guys are the 100 ($B \to R' \to B''$) guys

So total configurations are 2^{100}

(Note that the 100 G-G'' guys are fixed.)

And where does this probability distribution peak?

Ans: It's like a Σ of iid's (100 iid (0 or 1)'s)
So it's close to the normal dist ~(50) by CLT.

Ans: $N = 50$ should be the one.

(e) ~~Also~~ Approximate Calculation of Multiplication of Hecke Operators

Say we just work w/

	R	B
R'	A	B
B'	C	D

	R'	B'
R''	E	F
B''	G	H

So, for instance, $A+B = E+G = \#R'$ $(A, \dots, H \geq 0)$.
$C+D = F+H = \#B'$

Now, ~~#R'~~ what would be the rough estimate of the "most likely" (Color vs Color") graph"?

Well, it's $\frac{R \to R' \to R''}{\#R'} + \frac{R \to B' \to R''}{\#B'} = \boxed{\frac{AE}{R'} + \frac{CF}{B'}}$

(approximate number of people in the most likely breakup)

etc.

And check that these "expected" answers do add up to $\#R''$ & $\#B''$ (as Red/column sums). etc!