10.2.07:

- Geometric Representation Theory.
  - → Representations of transformation groups.
    (rep = group + action on say vector space)
  - → Typically we mean linear groups on v. space.
  - → But we are interested in other transformation groups.
  - → we want groups ~~larger~~ that preserve structure

example:
1. Diffeomorphism group → preserve differential structure

ALSO: Every transformation group is the group of "something-o"-morphism for some unique something.
i.e. Diffeomorphism are automorphism (or symmetry) that preserve diffeomorphism.

How do we define the structure: Given trans group G & the set G is acting on, we should be able to give the structure "preserved":

Symmetry & structure is like a dual. (symmetry is like "neg" aspect of structure)

Symmetry (negative)	Structure (positive)
entropy	information
relativity	invariance

Logic: has the job of putting structure on a set.
→ Axiomatic theory puts structure on a set:
0) Types 1) Abstract predicates
2) axioms about predicates

example: Euclidean geometry: { pts, lines (0); a pt lies on a line (1); distinct pts on the line (2) }

A model of the theory is a concrete realization of abstract types, predicates & axioms.

②

transformation group ; axiomatic theory are the same thing.

To find structure we need to look at the action of G on S ; figure out what is invariant under the group. These invariants give the abstract predicates

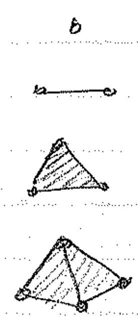
- "Orbi-simplex" picture of a transformation group. $G \leq S!$

For every axiomatic theory is there a transformation group? Sort of...
Restrict to finitary transformation group (G ; S are both finite) ; Complete axiomatic theory with an axiom stating that the "universe" of the model is bounded by N. For these types of theory, all models are isomorphic (categorical). The automorphism group of a model, is a transformation group.

this is example of Group ← Axiomatic Theory

Now the orbi-simplex picture is the orbit space of the action of G on the simplex whose vertices are the elements of S.

Simplices:



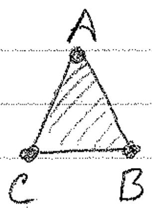
0 simplex
1 - simplex
2 - simplex
3 - simplex

As long as S is 3 pts we can keep drawings on the board.

So let S be the 3 elt group. ∴ G is a subgroup of S_3 (3!)

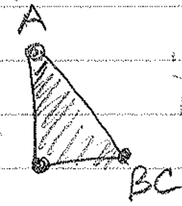
Define G to be the transformations that are invertible of the set $\{A, B, C\}$ that preserve A.

So construct a simplex of elts of S (if $|S| = n$ then we claim the n-simplex)



} this is a topological object

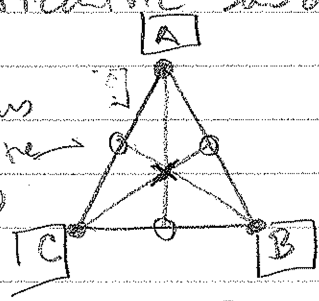
The orbit space is the quotient space by identifying points that are in the same orbit.

So the orbi-simplex is 

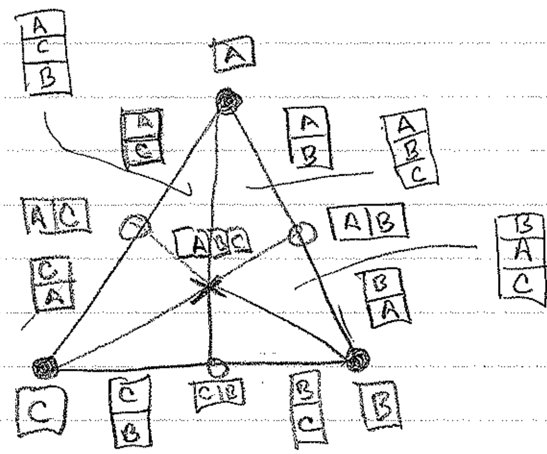
; this is a topological space.

- Baricentric subdivision: more corners give me elts of the group

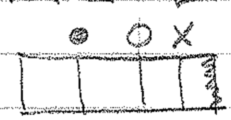
baricenters (centers of the edges)



The labeling corresponds to a Young tableaux; Young diagrams



note that $\boxed{n}\boxed{m} = \boxed{m}\boxed{n}$

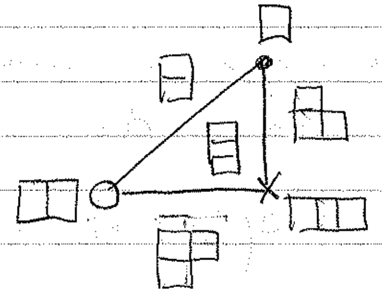
Rule: 

split; stacks

Faces: 6-subfaces correspond to

(4)

Each fold is an action, For the full group:



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