

We are talking about groupoid cardinality again

$$\underline{\text{Tame Groupoid}} \xrightarrow{\text{"Groupoid Cardinality"}} \underline{\mathbb{R}^+}$$

This is a decategorification process given by

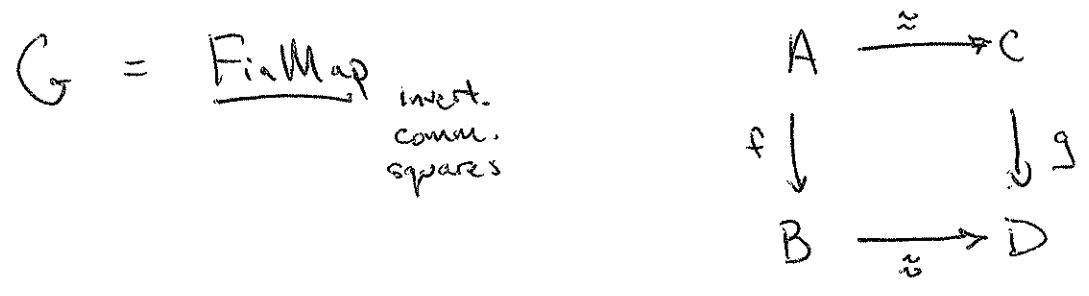
$$G \longmapsto \sum_{x \text{ in } \text{PI}_0(G)} \frac{1}{|\text{PI}_1(G, x)|}$$

Last time JB posed a puzzle to find the groupoid cardinality of the groupoid of finite sets.

$$\underline{\text{FinSet}} \xrightarrow{\text{bijections}} E$$

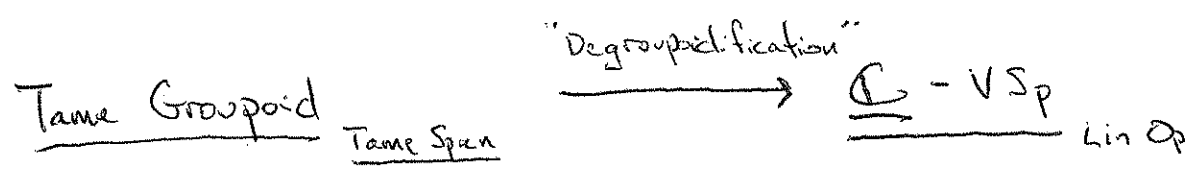
$$\underline{\text{2- Colored Fin Set}} \xrightarrow{\text{color-preserving bijections}} E^{-2}$$

Here is another puzzle:



An interesting open problem is to find an "interesting" groupoid whose cardinality is 2π .

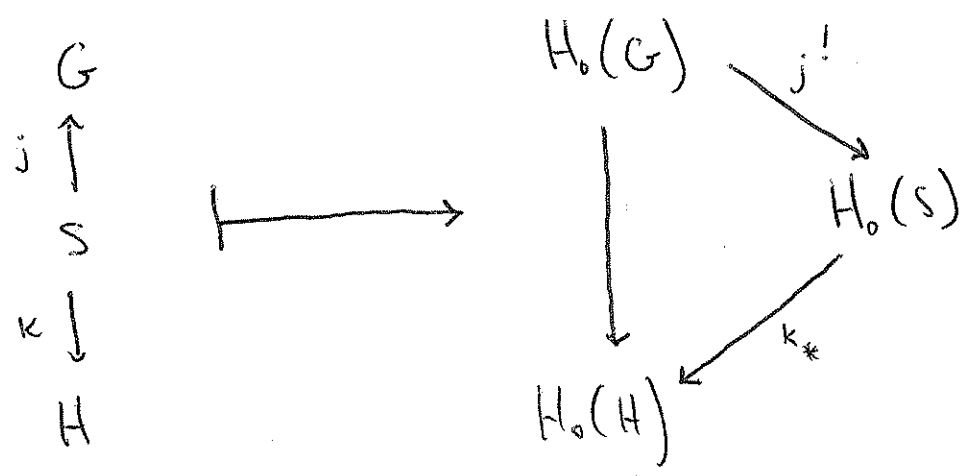
We know want to piggyback on our decategorification process.



We will be using the process of groupoid cardinality at the level of Tame Span in Lin Op.

$$G \longmapsto H_0(G) = \underline{\mathbb{C}} \left[\begin{array}{l} \text{set of iso} \\ \text{classes of} \\ G\text{-objects} \end{array} \right]$$

For now, we will take all groupoids to be finite.



$$H_0(G) \xrightarrow{j!} H_0(S)$$

$$\begin{array}{l}
 Y \longmapsto \text{sum} \\
 \text{(component of } G) \quad \times \quad |X| \cdot X \\
 \text{component} \\
 \text{of } S \\
 \text{in } j^{-1}(Y) \\
 \hline
 |Y|
 \end{array}$$

$$\text{Hom}_{\underline{\mathbb{C}}[G]}(\underline{\mathbb{C}}[X], \underline{\mathbb{C}}[Y])$$

X, Y finite G -sets
 G finite group

$$\begin{array}{ccc} (X \times Y) // G & & H_0((X \times Y) // G) \\ \parallel & & \parallel \end{array}$$

"Groupoid of elements of the G -set $X \times Y$ "

$$\underline{\mathbb{C}} \left[\begin{array}{l} \text{iso classes in the gpd} \\ \text{of elements of } G\text{-set } X \times Y \end{array} \right]$$

We want to show how spans get turned into linear operators. There is a useful map called "composition X, Y, Z "

$$\text{Hom}_{\underline{\mathbb{C}}[G]}(\underline{\mathbb{C}}[X], \underline{\mathbb{C}}[Y]) \otimes \text{Hom}_{\underline{\mathbb{C}}[G]}(\underline{\mathbb{C}}[Y], \underline{\mathbb{C}}[Z])$$



$$\text{Hom}_{\underline{\mathbb{C}}[G]}(\underline{\mathbb{C}}[X], \underline{\mathbb{C}}[Z])$$

We consider the span

$$(X \times Y) // G \times (Y \times Z) // G$$



$$(X \times Y \times Z) // G$$



$$(X \times Z) // G$$