Lecture 2 Jim Dolan 

# Geometric Representation Theory

A group equipped with an action by transformation groups is a representation.

Representations can arise as groups of transformations preserving some structure.

Ex: Diffeomorphism group / Differentiable structure

"Every transformation group is the group of something-o-morphisms for some something"
- and this something is essentially unique.

Groups describe symmetries, which are in some sense dual to structure.

Symmetry is a negative description, whereas structure is a more positive description

| symmetry | structure |
|---|---|
| entropy | information |
| relativity | invariance |

It is the job of logic to put some kind of structure on a set.

"Axiomatic Theory" — Ex. Euclidean geometry

Types — point, line

Abstract Predicates — "point P lies on line L"

Axioms about the predicates — distinct points lie on a unique line

"Model of the theory" is "a concrete realization of abstract types, predicates, & axioms"
– the Euclidean plane, for example.

"Orbi-simplex" picture of a transformation group $G \subseteq S$!

We impose some strong conditions for now

- finitary transformation group
  (group & set being acted on are both finite)

- complete axiomatic theory with an axiom stating that the "universe" of the model is bounded by N

(When we remove completeness, we switch from a group to a groupoid.)

So the orbi-simplex picture is

"the orbit space of the action of $G$ on the simplex whose vertexes are the elements of $S$."

0 - simplex

1 - simplex

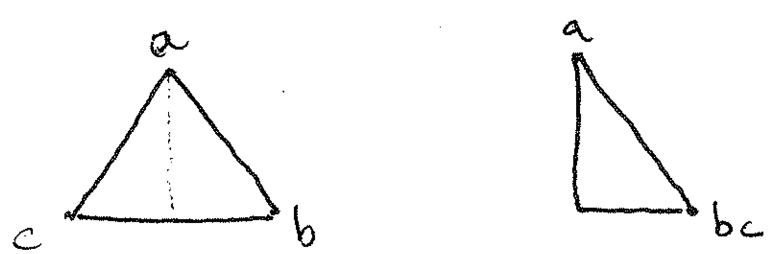
2 - simplex

3 - simplex

⋮

Let's look at an example with $G \subsetneq 3!$

$G$ = invertible transformations of $\{a, b, c\}$ that preserve $a$




We have introduced a new vertex here which we would like to label.

We draw the barycentric subdivisions.