Last time we stated a theorem saying that for a finite group G

G-equivariant linear operators between permutation representations	are equivalent to	linear combinations of G-orbits on cartesian products of G-sets

The left-hand side will be our tentative definition of a "Hecke operator". The right-hand side can be interpreted as "Geometrico-logical relationships between types of geometrical figures."

Example:

G = Isometries of a cube, $|G| = 48$

G-Set = Corners of cube

G-Set = Edges of cube

What are the relationships?

(1). corner lies on an edge

(2). corner & edge lie on some face (& no better)

(3). corner & edge lie on cube (& no better)

We can compose operators which suggests that we should be able to "compose" logical relationships.

$$\begin{tikzcd} \overset{X}{\text{Corner}} \arrow[r, "\substack{\text{Corner lies}\\ \text{on}\\ \text{edge}}"] \arrow[rr, bend right] & \overset{Y}{\text{Edge}} \arrow[r, "\substack{\text{Edge contains}\\ \text{corner}}"] & \overset{Z}{\text{Corner}} \end{tikzcd}$$

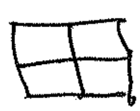
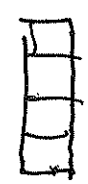
Corner X = corner Z

or

Corners X & Z lie on some edge

In this situation, we are multiplying two basis elements of an algebra, and we do <u>not</u> get a basis element back.

Now we look at examples of this sort corresponding to things John has been talking about.

Fix a Young diagrams D_1 = , D_2 = 

$G = 4!$ G-Set = D_2-flags on $\{a, b, c\}, d\}$

(specifically $\{a, b, c\}!$)

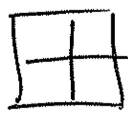
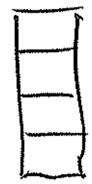
But we will come back to this example.

First consider

$$G = GL(4, \mathbb{F}_q)$$

First G-Set $=$ D_1 - flags on $(\mathbb{F}_q)^4$

Second G-Set $=$ D_2 - flags on $(\mathbb{F}_q)^4$

with $D_1 =$  "line" & $D_2 =$  "Point on a line on a plane"

What are the relationships?

P

L $\qquad L'$

PL

1 $\quad L = L'$

2 $\quad P \le L' \le PL$

3 A $\quad P \le L'$

B $\quad L' \le PL$

4 $\quad$ "L' touches L"

5 $\quad$ Generic