In lecture 7, we left off talking about the Gram-Schmidt process

	1	1	1	1	1
	1	2	2	3	4
	1	2	3	4	6
	1	3	4	7	12
	1	4	6	12	24

This process will give us a "better" (orthonormal) basis. So we will have a change of basis matrix.

old

new					
	1	0	0	0	0
	-1	1	0	0	0
	0	-1	1	0	0
	1	-1	-1	1	0
	-1	2	1	-3	1

The "old" are flag representations, the "new" are irreps.

The 1's on the diagonal correspond to the unique "skinny" representation trying to get out of the "fat" representation.

And an inverse change of basis matrix

New

Old					
	1				
	1	1			
	1	1	1		
	1	2	1	1	
	1	3	2	3	1

The old Young diagrams above are representations of the following dimensions:

1, 4, 6, 12, 24

& the dimensions of the "new" irreducible representations are:

1, 3, 2, 3, 1

We want a system of notation to describe the things represented by the numbers in our original matrix. Let's start with the number 7.

Ordered pairs

$$\begin{array}{c|ccc} & 2 & 1 & 1 \\ \hline 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{array}$$

"same second"

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

"same first"

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

"~~[illegible]~~"

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

"no overlap"

$$\begin{bmatrix} 2 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

"same"

$$\begin{bmatrix} 2 & & \\ & & 1 \\ & 1 & \end{bmatrix}$$

"reverse"