

Announcement: Topos Institute  
(David Spivak & Brendan Fong)

11/12  
Tuesday

## Hyperbolic Trig Functions

can you find  $H: S \rightarrow \text{Set}$  s.t.

$$|H|(x) = \sin(x) \quad \text{or} \quad |H|(x) = \cos(x) ?$$

No, because the power series have negative coeffs.

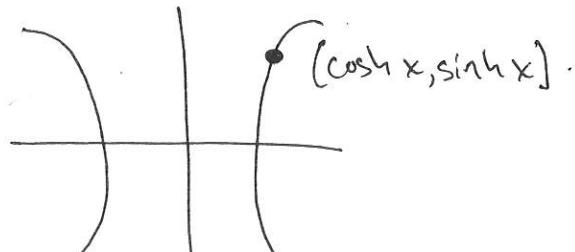
$$\sin(x) = \sum \frac{(-1)^{2n+1}}{(2n+1)!} x^{2n+1}, \quad \cos(x) = \sum \frac{(-1)^{2n}}{(2n)!} x^{2n}.$$

( $\sec x$  &  $\tan x$  have all positive, but for later.)  
→ look up "zigzag secant & tangent".

But trig functions are really just funny ways of "processing" exponentials:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad \sin(x) = \frac{e^{ix} - e^{-ix}}{2}$$

give coordinates for the circle, while



$$x^2 - y^2 = 1$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

parameterize the hyperbola.

$$\cosh(x) = \sum \frac{1}{(2n)!} x^{2n}$$

$$\sinh(x) = \sum \frac{1}{(2n+1)!} x^{2n+1}$$

these have a chance of being generating functions

so,  $\text{Cosh}: S \rightarrow \text{Set}$  is "having even cardinality  
and  $\text{Sinh}: S \rightarrow \text{Set}$  is "having odd cardinality"

hence, both are properties.

we have:

$$\text{Cosh} + \text{Sinh} \cong \text{Exp}$$

"any function is a sum of an

'even' one and an 'odd' one — literally, like

just as  $\sin' = \cos$        $\sinh' = \cosh$

$$\cos' = -\sin \quad \cosh' = \sinh$$

$$\sin^2 x + \cos^2 x = 1, \quad \cosh^2 x = 1 + \sinh^2 x$$

so, do we have  $\text{Cosh}' \cong \text{Sinh}$  ?

yes, because  $(|x| \text{ odd} \Leftrightarrow |x+1| \text{ even})$ .

similarly,  $\text{Sinh}' \cong \text{Cosh}$ .

Now what about...

$$\cosh^2 x = \sinh^2 x + 1 ?$$

(given two species  $G, H : S \rightarrow S$ , have products.)

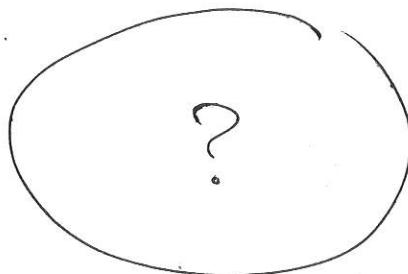
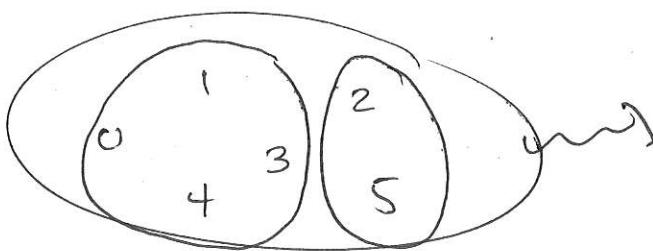
so we're checking whether

$$\text{Cosh} \cdot \text{Cosh} \cong \text{Sinh} \cdot \text{Sinh} + 1$$

ns.

(being even(s)  $\Leftrightarrow$  being odd(s) or being  $\emptyset$ )

so "yes". — but need to check naturality.



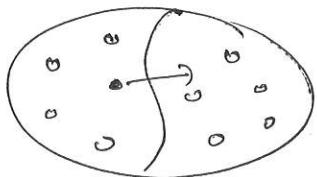
Cosh · Cosh

Sinh · Sinh

have to pick an element, but this will not be natural under relabelling.

hence there is no natural isomorphism.

but there are various ways to save the day:  
if we were using pointed sets (equipped with distinguished element), we would have a specified one to move across the partition (to turn from two evens to two odds, or vice versa)



let's think of how  
to do this.

if  $H: S \rightarrow \text{Set}$ , let  $\tilde{H}: S \rightarrow \text{Set}$ :

$$\tilde{H}(x) = \left\{ \begin{array}{l} \text{ways of choosing a } x \in X \\ \text{and putting an } H\text{-structure on } X \end{array} \right\}$$

(this will give  $\tilde{\text{Cosh}}^2 \cong \tilde{\text{Sinh}}^2 + \tilde{A}_0$ )

can do  $\tilde{H} \cong H \times \text{id}$ , or  $\tilde{H} \cong A.H'$ .

$$(\text{aka } \tilde{H} \cong X \frac{\partial}{\partial x} H)$$

You can show that  $\tilde{G} \cong \tilde{H} \Rightarrow |G| = |H| +$

• seminar •

Lenses

" $\text{lens}_F$ "

$$(F: C^{\text{op}} \rightarrow \text{Cat}) \rightarrow (\int F : \text{Cat})$$
$$(C \xrightarrow{f} C', F(f)(x') \xrightarrow{\cong} x)$$

given  $F: C^{\text{op}} \rightarrow \text{Cat}$ , then:

- ①  $(\int F)^{\text{op}}$ ,
- ② contragrlst of  $C^{\text{op}} \xrightarrow{F} \text{Cat} \xrightarrow{\text{op}} \text{Cat}$
- ③ explicit construction

⊗ ringed spaces

$$\text{Hom}\left((c), (c')\right) = \left\{ \begin{array}{l} c \rightarrow c' \\ c \times c' \rightarrow x \end{array} \right\}$$

Lens  $\xrightarrow{ff}$  Bundlset

$$= \text{lens}_{(S/\text{Set})}$$

$$\left( \begin{matrix} c \\ c \end{matrix} \right) \xrightarrow{x} \left( \begin{matrix} x \\ d \end{matrix} \right) = \left\{ \begin{array}{l} c \rightarrow \\ c \times x \rightarrow d \end{array} \right\}$$

$$\dashv/\text{Set}: \text{Set}^{\text{op}} \rightarrow \text{Cat}$$

$$\dashv \boxed{\begin{matrix} c \\ c \end{matrix}} \rightarrow d$$

Sustainability Seminar Series (Dec. 4)

Extension CoC mixer for Riverside (Dec. 5)

- CARB & UCR (Spring 2021)

certifications, testing, monitoring + research

\* Creating the Sustainability Institute + oh wow

Faculty Research Summit ~ (Jan. 30, Botanical Garden)

\* Bending the Curve (projects) Berlin

Brian Siana — climate, physics + energy.

• Working Groups (Engagement? or zw)

Decomposing Species, contd.

11/1  
Thurs

(+) suppose  $H:S \rightarrow \text{Set}$ . then  $H$  is indecomposable

1.  $H$  is supported on  $n$ -orbit sets ( $H(x)=0$ )

2.  $\tilde{H}:S_n \rightarrow \text{Set}$  is indecomposable  $\rightarrow$  acts transitively (exactly one orbit)

$\hookrightarrow$  (iff  $\tilde{H}(u)=S_n/K$  for some  $K \leq S_n$ )

so, we get indecomposable species

by choosing  $n \in \mathbb{N}$  and a subgroup  $K \leq S_n$

— but we don't know if this uniquely specifies.

\* two indec. species isomorphic  $\Leftrightarrow K, K'$  conjugate

hence, classifying indecomposable species  
is the same as classifying subgroups of  
symmetric groups, up to conjugation.

(+)  $\text{AH} : \text{Set}^S$  ( $H \cong \sum_{\alpha} H_{\alpha} \wedge H_{\alpha} : \text{Set}^S$  indecomposable)  
and this is unique up to iso.

so: what are the subgroups of  $S_n$  just like  
relabeling  
up to conjugation?

Alas, nobody knows. ("like decomp of primes")  
— still ongoing.

n

#conj. classes of subgrps of  $S_n$

0

1

1

1

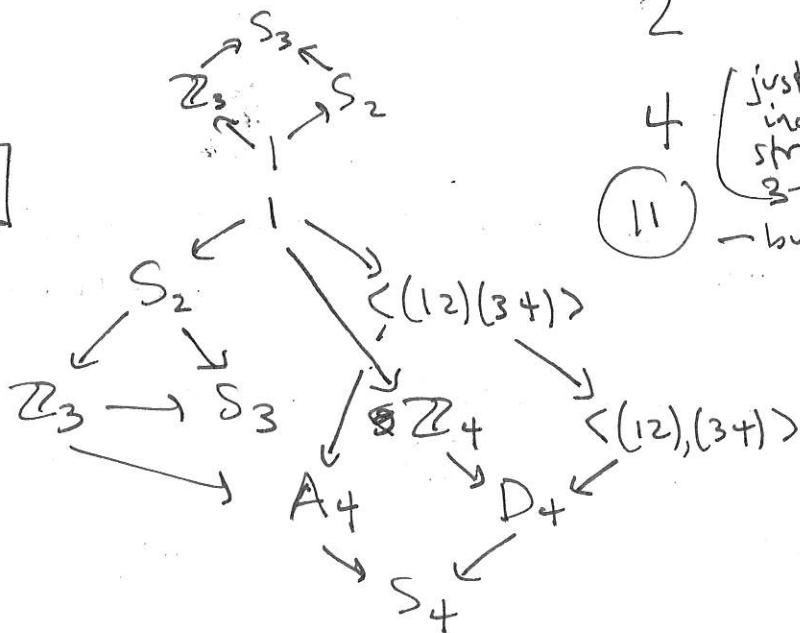
2

2

3

4

8



11

just four  
indecomposable  
structures on  
3-elmt sets  
— but what are they?

So, what are the four "basic" structures we can put on a 3-element set?

let's take  $[\mathbb{Z}_3 \hookrightarrow S_3] \rightsquigarrow H: S \rightarrow \text{Set}$

$$H(X) = \begin{cases} \circ & |X| \neq 3 \\ "2" & |X| = 3 \end{cases} \quad \begin{array}{l} \text{(2 } \xrightarrow{\text{wesets}} \text{ conjugacy classes)} \\ \text{of } \mathbb{Z}_3 \text{ in } S_3 \end{array}$$

where " $2$ " =  $\{\text{cyclic orderings of } X\}$ .

these are "the same" as elements of  $S_3/\mathbb{Z}_3$

and for  $[S_2 \hookrightarrow S_3] \rightsquigarrow I: S \rightarrow \text{Set}$

$$I(X) = \begin{cases} \circ & |X| \neq 3 \\ Y & |X| = 3 \end{cases}, \quad \begin{array}{l} \text{for } Y = \{\text{pick two elmts}\} \\ \text{and cyclic order}\} \\ \cong \{\text{points of } X\} \end{array}$$

what about for  $[\mathbb{Z}_2 \times \mathbb{Z}_2 \hookrightarrow S_4]$ ?



more generally, consider functors  $H: X \rightarrow Y$  from a groupoid to a category with (finite) coproducts

(+)  $H \in Y^X$  is indecomposable iff:

1. let  $X_\alpha$  be the <sup>connected</sup> components of groupoid  $X$   
then  $H$  is supported on one component  $X_\alpha$