spin — reps of $SU(2)$

5/12/03

Standard Model patterns:

strong force — weak isospin — hypercharge

① 1) Gauge bosons live in adjoint rep of $G = \underbrace{SU(3)}_{\text{strong force}} \times \underbrace{SU(2)}_{\text{weak isospin}} \times \underbrace{U(1)}_{\text{hypercharge}}$.

2) The Higgs is weird — it has everything to do w/ breaking the electroweak symmetry grp. (It hasn't been seen.)

3) Fermions come in 3 generations w/ <u>same</u> 16-dim'l rep F of G. (Nobody knows why!)

4) Fermions consist of • <u>leptons</u> (trivial rep of $SU(3)$) — don't feel strong force) • <u>quarks</u> (fund rep of $SU(3)$ on $\mathbb{C}^3$ — do feel strong force)

* quarks are held together in "bags" called protons, neutrons, etc.

→ red, green, blue — because acting on $\mathbb{C}^3$.

• leptons have integral hypercharge, quarks have $\mathbb{Z} + \frac{1}{3}$ hypercharge

5) Left-handed fermions transform in fund. rep of $SU(2)$ on $\mathbb{C}^2$. But right-handed ones transform in trivial rep of $SU(2)$ — don't feel that part of the electroweak force (no weak isospin)

6) Average of hypercharges of right-handed leptons is hypercharge of left-handed one. Same for quarks.

② $\dim_{\mathbb{C}} F = 16$ Fermion rep in Standard Model

So, what sort of spinors give a 16-dim'l complex v. space?

Complex spinors: Want a complex Clifford algebra

$$\mathbb{C}_n^0 = \mathbb{C}[16] \quad \text{or} \quad \mathbb{C}_n^0 = \mathbb{C}[16] \oplus \mathbb{C}[16]$$

$$\mathbb{C}_1^0 \cong \mathbb{C}_0 = \mathbb{C}, \quad \underset{\cong\,\mathbb{C}_2^0}{\mathbb{C}_1} = \mathbb{C}\oplus\mathbb{C}, \quad \underset{\cong\,\mathbb{C}_3^0}{\mathbb{C}_2} = \mathbb{C}[2], \quad \underset{\cong\,\mathbb{C}_4^0}{\mathbb{C}_3} = \mathbb{C}[2]\oplus\mathbb{C}[2]$$

$$\ldots \quad \underset{\cong\,\mathbb{C}_9^0}{\mathbb{C}_8} = \mathbb{C}[16], \quad \underset{\cong\,\mathbb{C}_{10}^0}{\mathbb{C}_9} = \mathbb{C}[16]\oplus\mathbb{C}[16]$$

So we could have

$$F = S_9^{\mathbb{C}} \quad \text{or} \quad F = S_{10}^{\mathbb{C}+} \text{ or } S_{10}^{\mathbb{C}-}$$

So we could try to treat F as the left-handed spinor rep of $\mathfrak{so}(10) \subseteq \mathbb{C}_{10}^{\circ}$.

Lie alg of Spin(10)

We're hoping to have F be a rep. of Spin(10).

Can we extend F from being a rep of $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$ to a rep. of $\mathfrak{so}(10)$?
Yes!

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1) \cong \mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$$

skew-adjoint 3x3 complex matrices — skew-adj. 2x2 $\mathbb{C}$ matrices

We can think of $\mathfrak{u}(3) \oplus \mathfrak{u}(2)$ as a 5×5 matrix w/ a 3x3 & 2x2 block:

$$\begin{pmatrix} \begin{matrix} - & - & - \\ - & - & - \\ - & - & - \end{matrix} & 0 \\ 0 & \begin{matrix} - & - \\ - & - \end{matrix} \end{pmatrix} \subseteq \mathfrak{u}(5)$$

so let $\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$ be the traceless 5x5 matrices of this form: i.e. lying in $\mathfrak{u}(3) \oplus \mathfrak{u}(2)$.

If you are in $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$, you're in $\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$, but it's not <u>all</u> of $\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$.

ie $\quad \mathfrak{su}(3) \oplus \mathfrak{su}(2) \subsetneq \mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$.

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) = \left\{ \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \,\middle|\, \begin{array}{l} \operatorname{tr} A = 0 \\ \operatorname{tr} B = 0 \end{array} \right\} \subset \mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$$

but $\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$ has dim. one <u>more</u> than $\mathfrak{su}(3) \oplus \mathfrak{su}(2)$ since we only need

$$\operatorname{tr} A + \operatorname{tr} B = 0$$

(so certainly, $\operatorname{tr} A = \operatorname{tr} B = 0$ works, but we could also have $\operatorname{tr} A = -\operatorname{tr} B$)

This gives a 1-dim'l Lie sub-algebra:

$$\left\{ \alpha \begin{pmatrix} i/3 & 0 & 0 & & \\ 0 & i/3 & 0 & & 0 \\ 0 & 0 & i/3 & & \\ & & & -i/2 & 0 \\ & 0 & & 0 & -i/2 \end{pmatrix} ; \alpha \in \mathbb{R} \right\} \cong \mathfrak{u}(1)$$

The bracket of 2 of these guys is zero, so an abel. Lie alg.

Everything in this Lie subalg. commutes w/ everything in $\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2))$, so this means we have a <u>direct</u> <u>sum</u>

$$\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2)) = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1).$$

In fact —

$$\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1) \cong \mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2)) \subseteq \mathfrak{su}(5) \subseteq \mathfrak{su}(10)$$

since traceless skew-adj. transformations of $\mathbb{C}^5$ are among those of $\mathbb{R}^{10}$.

Punchline: If we take the left-handed spinor rep ($S_{10}^{\mathbb{C}+}$ or $S_{10}^{\mathbb{C}-}$) of $\mathfrak{so}(10)$ and restrict it ($S_{10}^{\mathbb{C}+}$ or $S_{10}^{\mathbb{C}-}$) from $\mathfrak{so}(10)$ to $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$, we get F!

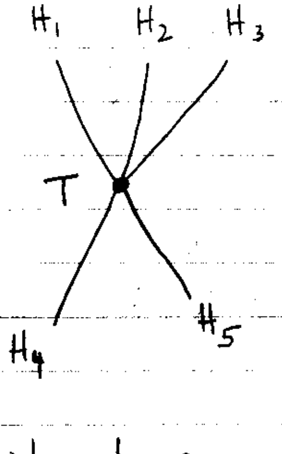
There's a "grand unified theory" where the "true" symmetry group is $\mathrm{Spin}(10)$, not $SU(3) \times SU(2) \times U(1)$.

Theories of Particle Physics:

General Setup:

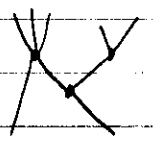
1) A group $I\mathrm{Spin}(3,1) \times G$
2) a list of irreps of this group – "elementary particles"
3) a list of intertwining operators between tensor products of these irreps — "interactions"

$T: H_1 \otimes H_2 \otimes H_3 \longrightarrow H_4 \otimes H_5$ can be drawn as:



and it describes a process turning 3 particles into 2.

We know how to compose & tensor these to get Feynman diagrams (aka string diagrams).

ex) 

Examples:

"free"

① Free electromagnetic field (no particles) – just electromagnetism, not interacting w/ anything!

1) Here our group is just ISpin(3,1) i.e., G is trivial group.

2) our 1-irrep is: {massless spin-1 rep} = $\left\{\begin{array}{l}\text{space of solns of} \\ \text{vacuum Maxwell eqns}\end{array}\right\}$

This one irrep corresponds to a photon.

3) No interactions.

This describes the photon: a massless, zero charge spin-1 particle called γ.

A state of one photon is a vector in

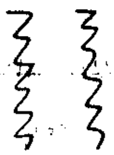
$$H_\gamma = \{\text{massless spin-1 rep}\}$$

With this raw material, we can only draw Feynman diagrams like:

$1 : H_\gamma \longrightarrow H_\gamma$ looks like:

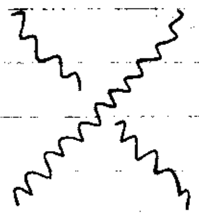
"one photon"

$1 : H_\gamma \otimes H_\gamma \longrightarrow H_\gamma \otimes H_\gamma$ looks like:



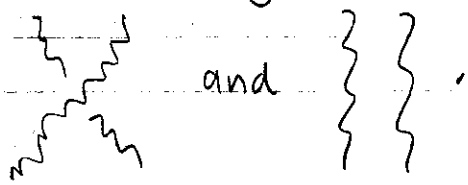
"two photons"

$$S : H_\gamma \otimes H_\gamma \longrightarrow H_\gamma \otimes H_\gamma$$
$$\psi \otimes \varphi \longmapsto \varphi \otimes \psi$$



Crossing doesn't matter since we have a symmetric monoidal category.

In fact, photons are "indistinguishable bosons" meaning we can't distinguish between



What people do is replace $H_\gamma \otimes H_\gamma$ by

$$S^2 H_\gamma = H_\gamma \otimes H_\gamma \Big/ \langle \psi \otimes \varphi - \varphi \otimes \psi \rangle$$

linear span of these vectors

so $\psi \otimes \varphi$ gets identified w/ $\varphi \otimes \psi$.

i.e. we identify $\psi \otimes \varphi$ and $\varphi \otimes \psi$, so photons have no "labels".

On $S^2 H_\gamma$ we have

People go ahead and consider the "bosonic Fock space"

$$K_\gamma = \underset{S^0 H_\gamma}{\underset{\shortparallel}{\mathbb{C}}} \oplus \underset{S^1 H_\gamma}{\underset{\shortparallel}{H_\gamma}} \oplus S^2 H_\gamma \oplus S^3 H_\gamma \oplus \ldots = \bigoplus_{n \geq 0} S^n H_\gamma$$

where $S^n H_\gamma$ is a quotient space of $\overbrace{H_\gamma \otimes \cdots \otimes H_\gamma}^{n \text{ times}}$ by the span of vectors

$$\psi_1 \otimes \cdots \otimes \psi_n - \psi_{\sigma(1)} \otimes \cdots \otimes \psi_{\sigma(n)} \quad \sigma \in S_n$$

Recall: $\begin{cases} \oplus \text{ corresponds to "or" in QM.} \\ \otimes \text{ corresponds to "and"} \end{cases}$

A state in Fock space describes no photons ($\mathbb{C}$) or one photon (H_γ) or two photons ($S^2 H_\gamma$) or...

Basis vector $1 \in \mathbb{C} \subseteq K_\gamma$ is called the "<u>vacuum</u>" — no photons.

(2) Free Dirac field — describes electrons & positrons (their antiparticles... i.e. anti-electrons), but not interacting at all.

The electron has mass .511 MeV — "million electron volts" is the energy that an electron picks up through a potential of 10^6 volts, so we can get a mass by dividing by c^2.

So — MeV/c^2 is a unit of mass which particle physicists simply call MeV.

A GeV is a thousand MeV.

MeV ~ 2 electron masses
GeV ~ 1 proton mass

The electron has charge -1 & spin $-1/2$.

The positron is the same, but with charge $+1$.

We call the electron e and positron $\bar{e}$.

$1, PT$

$\Uparrow$ has 2 components!

1) Group $= ISpin(3,1)$

2) $H_{e,\bar{e}} = \{$massive spin-$\frac{1}{2}$ rep$\}$

$$= \{\psi : \mathbb{R}^{3,1} \longrightarrow \underset{\rho_{3,1}^{\mathbb{C}}}{\mathbb{C}^4} \mid (\not{\partial} + m)\psi = 0 \text{ where } m = .511 \text{ MeV}\}$$

3) No interactions.

Note: $\mathbb{C}^4$ since electrons & positrons can spin right & left handed, so 4 ways.

$\mathbb{C}^4$ has basis:

$$e^L = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad e^R = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad \bar{e}^L = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad \bar{e}^R = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

In hmwk — we have $m=0$, so solns of $\not{\partial}\psi = 0$ give only 2 of these: e^L, $\bar{e}^R$

* PT to an electron gives a positron! So we can't write this rep as an electron part & positron part!

$H_{e,\bar{e}}$ is an irrep of $ISpin(3,1)$ but not of $ISpin_0(3,1)$ (identity component) which doesn't contain PT, as a rep of this smaller group

$$H_{e\bar{e}} = H_e \oplus H_{\bar{e}}.$$

Recall - symmetric algs — polys of commuting variables $xy = yx$

Electron/positrons are indistinguishable fermions, so Hilbert space for 2 of them is not

$$H_{e\bar{e}} \otimes H_{e\bar{e}}$$

but

$$\Lambda^2 H_{e\bar{e}} = H_{e\bar{e}} \otimes H_{e\bar{e}} \big/ \langle \psi \otimes \varphi + \varphi \otimes \psi \rangle$$

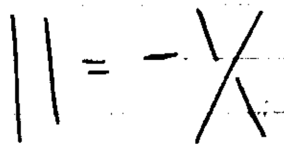
2nd exterior power

since $\psi \otimes \varphi$ is indistinguishable from $-\varphi \otimes \psi$.

Clifford algs — good for fermions
Weyl algs — good for bosons

We describe a system of arbitrarily many electron/positrons by the <u>Fermionic Fock space</u>

$$K_{e\bar{e}} = \mathbb{C} \oplus H_{e\bar{e}} \oplus \Lambda^2 H_{e\bar{e}} \oplus \dots = \bigoplus_{n \geq 0} \Lambda^n H_{e\bar{e}}$$



ℓ_2 is like $\mathbb{C} \oplus \mathbb{C} \oplus \ldots$

But an algebraist would mean only finitely many nonzero of $\circlearrowleft$.

5/13/03 Particle Physics

<u>Recall:</u> Group $= \underset{I,\,PT}{ISpin(3,1)} \times G$

① Free electromagnetic field

- $G = 1$ (trivial group)
- $H_\gamma = \{$massless spin-1 rep$\}$ (photon: γ)
- no interactions

The photon is a <u>boson</u>:

this is defn. of boson

or

"bosonic Fock space"

$$K_\gamma = \overline{SH_\gamma} \quad \text{completion of symmetric space}$$

$$= \overline{\mathbb{C} \oplus H_\gamma \oplus S^2 H_\gamma \oplus \ldots}$$

symmetric powers

② Free Dirac field

- $G = 1$
- $H_{e\bar{e}} = \{$massive spin-½ rep$\}$

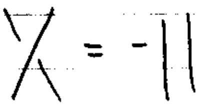
e = electron
$\bar{e}$ = positron

mass of electron: $m_e = .511$ MeV

MeV $\approx 1.78 \times 10^{-27}$ grams

- no interactions

The electron-positron is a <u>fermion</u>: (each is a fermion)



or

$$K_{e\bar{e}} = \overline{\Lambda H_{e\bar{e}}} = \mathbb{C} \oplus H_{e\bar{e}} \oplus \Lambda^2 H_{e\bar{e}} \oplus \ldots$$

exterior powers

Recall – symmetric alg is alg of polys (which commute)

As a rep of $I\mathrm{Spin}_0(3,1)$, $H_{e\bar{e}}$ is reducible:

$$H_{e\bar{e}} = H_e \oplus H_{\bar{e}}$$

since 1) $\Lambda(V \oplus W) \cong \Lambda V \otimes \Lambda W$ fermionic
2) $S(V \oplus W) \cong SV \otimes SW$ bosonic

need to complete these to get Hilbert spaces

$\Lambda(V \oplus W) \sim$ electrons or positrons

We have $K_{e\bar{e}} = K_e \otimes K_{\bar{e}}$ $\qquad K_e = \overline{\Lambda H_e}$, $K_{\bar{e}} = \overline{\Lambda H_{\bar{e}}}$

1) says "A bunch of (electron or positron)s is (a bunch of electrons) and (a bunch of positrons)"

Λ – "bunch"

$\oplus$ – disjoint union ; $\otimes$ – cartesian product

We draw

$1 : H_e \longrightarrow H_e$ as 

and

$1 : H_{\bar{e}} \longrightarrow H_{\bar{e}}$ as  "a positron is an electron going backwards in time and mirror reversed"

Note: PT takes $H_{\bar{e}}$ to H_e, H_e to $H_{\bar{e}}$.

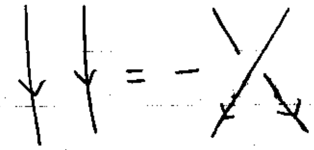
$PT : H_e \longrightarrow H_{\bar{e}}$, $\quad PT : H_{\bar{e}} \longrightarrow H_e$

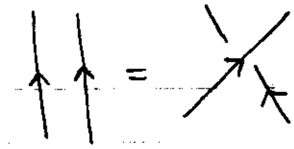
T by itself also switches these two.

$H_{e\bar{e}}$ is really a rep of $I\mathrm{Pin}(3,1)$.

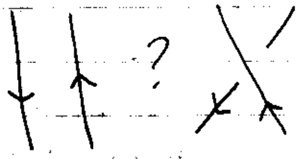
Note: P acts nontrivially on H_e ... it maps it to itself, but isn't identity.

So we have:





We don't have an eqn bet. these two



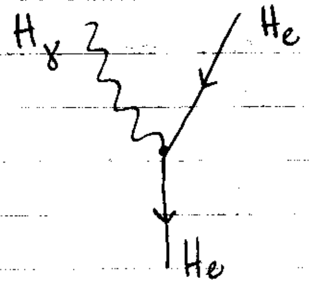
because these operators have different source & target.

③ QED - quantum electrodynamics
"the theory of electrons & positrons interacting w/ photons"

1) $G = 1$ (trivial grp)

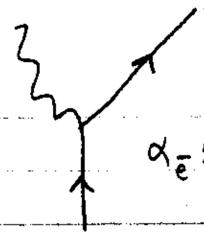
2) H_γ - photon, $H_{e\bar{e}}$ - electron/positron $H_{e\bar{e}} = H_e \otimes H_{\bar{e}}$

3) $\alpha : H_\gamma \otimes H_{e\bar{e}} \longrightarrow H_{e\bar{e}}$ there's a particular formula for this involving the number $\alpha = \frac{1}{137}$



electron absorbs photon

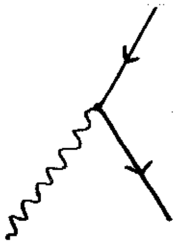
we also get:



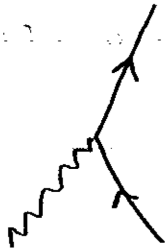
$\alpha_e : H_\gamma \otimes H_e \longrightarrow H_e$

$\alpha_{\bar{e}} : H_\gamma \otimes H_{\bar{e}} \longrightarrow H_{\bar{e}}$

Given this α, we automatically get other interactions:



$H_e \longrightarrow H_\gamma \otimes H_e$ and



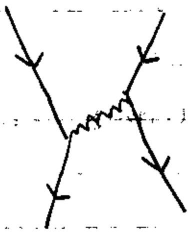
$H_{\bar{e}} \longrightarrow H_\gamma \otimes H_{\bar{e}}$

and

$H_e \otimes H_{\bar{e}} \longrightarrow H_\gamma$ and

$H_\gamma \longrightarrow H_{\bar{e}} \otimes H_e$

In QED we can have complicated processes like



"photon exchange"

$$\begin{array}{c} H_e \otimes H_e \\ \big\downarrow \\ H_e \otimes H_e \end{array}$$

this is NOT the identity

Actually, QED has $U(1)$ symmetry; i.e. we can make H_γ, $H_{e\bar{e}}$ into reps of $\tilde{I}Spin(3,1) \times U(1)$ and then

$\alpha : H_\gamma \otimes H_{e\bar{e}} \longrightarrow H_{e\bar{e}}$ (and all its spinoffs) turns out to

be an intertwining operator. Given $e^{i\theta} \in U(1)$ here's how it acts on $H_{e\bar{e}}$: given $\psi \in H_{e\bar{e}}$ we have

$$\psi : \mathbb{R}^{3,1} \longrightarrow \mathbb{C}^4 \qquad \psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix} \begin{array}{l} \Big] \text{ electron} \\ \Big] \text{ positron} \end{array}$$

and $\rho(e^{i\theta})\psi \in H_{e\bar{e}}$ is defined by

$$\rho(e^{i\theta})\psi \in H_{e\bar{e}}$$

$$(\rho(e^{i\theta})\psi)(x) = \begin{pmatrix} e^{-i\theta}\psi_1(x) \\ e^{-i\theta}\psi_2(x) \\ e^{i\theta}\psi_3(x) \\ e^{+i\theta}\psi_4(x) \end{pmatrix} \qquad x \in \mathbb{R}^{3,1}$$

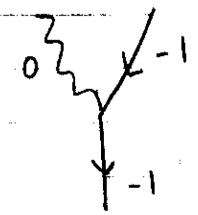
can't tell the difference bet. i & $-i$ (Galois theory!)

Different reps of $U(1)$ — integers = charge: $\rho_n(e^{i\theta})x = e^{in\theta}x$, $n \in \mathbb{Z}$, $x \in \mathbb{C}$, is the <u>charge-n rep</u> of $U(1)$ on $\mathbb{C}$.

Electrons have charge -1, positrons have charge $+1$.

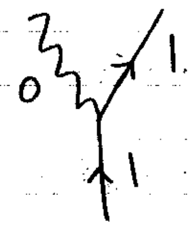
quarks — have charge $1/3$ of electron.
Found in protons & neutrons.
photon has electric charge 0. so rep of $U(1)$ is trivial.

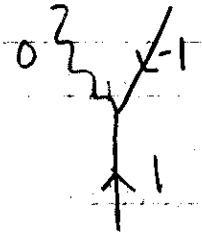
Photons have charge 0 so H_γ becomes the trivial rep of $U(1)$.



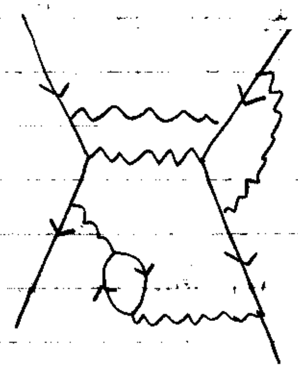
charge is conserved by this process.

In Fall — we worked out that this operator is an intertwining operator for $U(1)$ $\Leftrightarrow$ charge is conserved.
Same for



We can't have:  if we want our interactions to be intertwiners for $U(1)$.

We can get all sorts of fancy processes:



④ Heisenberg's Theory of Strong Interactions:

There are other particles in ordinary matter:

- Proton: p has charge $+1$, spin $1/2$
  mass 938.3 MeV

- Neutron: n has charge 0, spin $1/2$
  mass 939.6 MeV

Heisenberg hypothesized that these were both states of one particle, the <u>nucleon</u>.

Problem: different masses, different charges

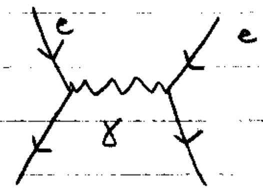
so can't be same irrep of $U(1)$.

Fallback positron: Try to understand the nucleon ignoring electromagnetism & hope that electromagnetic effects explain mass difference

protons & neutrons form nucleus: 

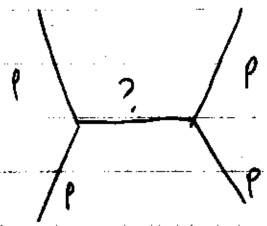
But electromagnetism is <u>NOT</u> what's holding these together! These have <u>same</u> charge (+ protons) so should repel. So it must be some stronger force.

This is not too weak of a position, since we know some force much stronger than electromagnetism must hold nucleons together in the nuclei.
The "<u>strong</u> <u>interaction</u>"



this gives repulsive electromagnetic force

Yukawa guessed that maybe the strong interaction is due to some particle:



etc.

Yukawa called this particle the <u>meson</u>, which must have a mass ~100 MeV, i.e. between electron mass & nucleon mass to explain observed range of strong force.

People found this:

<u>Muon</u>: μ charge -1, spin $1/2$
mass: 105.6 MeV

Problem: Not blocked by lead.

Actually the muon is almost like an electron but ~200 times heavier.

* This was the 1st particle of the 2nd generation to be discovered.

"Who ordered this?" I.I. Rabi

Then they found <u>pions</u>:

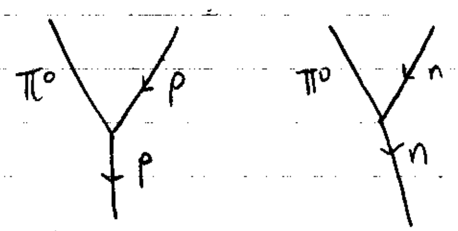
antiparticles

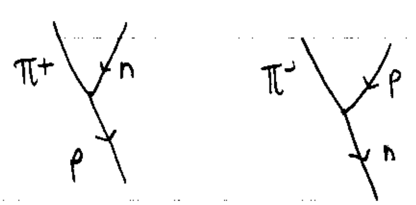
π^+ charge $+1$, spin 0, mass 139.6 MeV
π^0 charge 0, spin 0, mass 135.0 MeV (own antiparticle)
π^- charge -1, spin 0, mass 139.6 MeV

(Heinsenberg guessed these were all states of same particle.)

These <u>do</u> interact strongly w/ nucleons.

We see:





Heisenberg's theory:

nucleon $N = \begin{pmatrix} p \\ n \end{pmatrix} \in \mathbb{C}^2$

pion $\pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \in \mathbb{C}^3 \cong su(2) \otimes \mathbb{C} \cong sl(2, \mathbb{C})$

$su(2)$ - 3 dim'l $\mathbb{R}$ v. space

Make up a symmetry grp of which these are irreps.

Heisenberg postulated $G = SU(2)$ as symmetries of the strong force, w/ nucleon in spin-½ (fund.) rep and the pion in spin-1 (adjoint) rep.

i.e. really we have basis vectors

$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{C}^2$ "isospin-up"

$n = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in \mathbb{C}^2$ "isospin-down"

$\pi^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \pi^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \pi^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

This works nicely:

$$\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{\pi^+} \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{n} = \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{p}$$

π^+, π^- raise & lower charge of your particle,
π^0 does nothing to charge!

$\pi^+ n = p$ &, etc – giving observed interactions!

i.e. the intertwiner

$$\mathfrak{sl}(2,\mathbb{C}) \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2$$
$$T \otimes \psi \longmapsto T\psi$$

describes how nucleons can absorb pions!

So:

1) $G = SU(2)$

2) $\underbrace{H_N}_{\text{nucleon}} = \{\text{massive spin-}1/2\} \otimes \underbrace{\mathbb{C}^2}_{\text{nucleon}}$

$$H_\pi = \{\text{massive spin-}0\} \otimes \underbrace{\mathfrak{su}(2) \otimes \mathbb{C}}_{\substack{\cong \mathbb{C}^3 \\ \cong \mathfrak{sl}(2,\mathbb{C})}}$$

3) Interaction

$$\begin{array}{ccc} H_\pi \otimes H_N & \longrightarrow & H_N \\ \cong & \nearrow T & \\ \{\text{massive spin-}0\} \otimes \mathfrak{sl}(2,\mathbb{C}) \otimes \{\text{massive spin-}1/2\} \otimes \mathbb{C}^2 & & \end{array}$$

where T is built from $\mathfrak{sl}(2,\mathbb{C}) \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2$

$\{\text{massive spin-}0\} \otimes \{\text{massive spin-}1/2\} \longrightarrow \{\text{massive spin-}1/2\}$