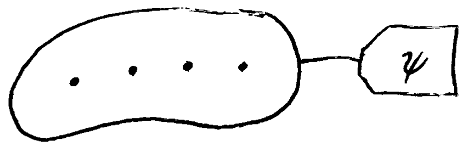
25 May 2004

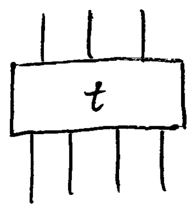
Feynman Diagrams

We've seen that we can draw an object of:

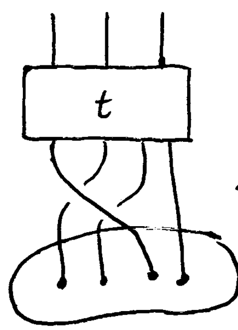
- the stuff type Ψ as



- the stuff operator T as:

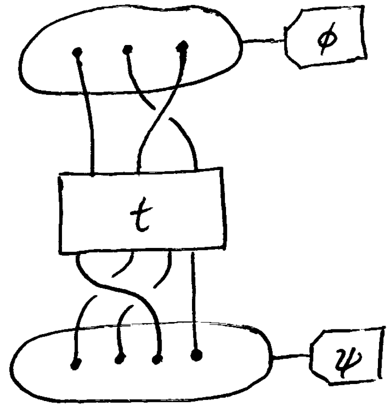


- the stuff type $T\Psi$ as:



the isomorphism α in def. of "weak pullback"

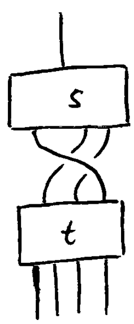
• the inner product $\langle \Phi, T\Psi \rangle$ as



This should remind you of Dirac's Bra-Ket notation:

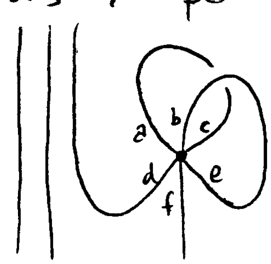
$$\langle \Phi | T | \Psi \rangle := \langle \Phi, T\Psi \rangle$$

Given two stuff operators S & T we can draw an object of ST as:



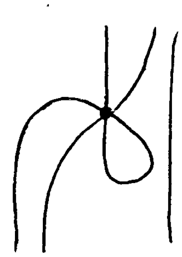
We've looked at two stuff operators in particular:

Φ^n here a typical object (for $n=6$) looks like:



A Feynman diagram with one n-valent vertex with totally ordered incidences

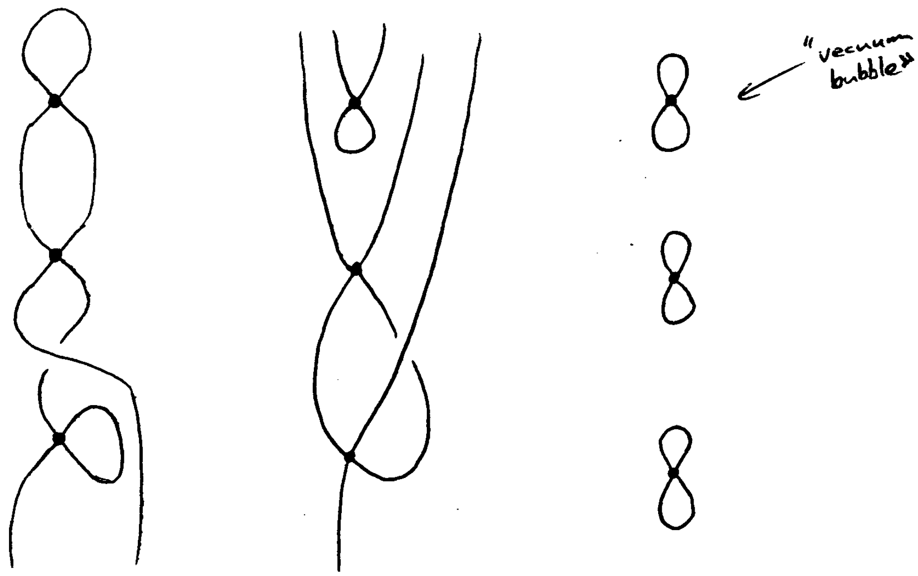
$\dfrac{\Phi^n}{n!}$ has typical object ($n=6$):



A Feynman diagram with one n-valent vertex.

These pictures are pictures of extra stuff that can be put on a pair of finite sets!

Here are some objects in $\frac{\Phi^4}{4!} \frac{\Phi^4}{4!} \frac{\Phi^4}{4!}$:



Now... let's study the time evolution in the perturbed harmonic oscillator. Let's see how to calculate

$$\langle z^k, e^{-itH} z^\ell \rangle$$

where

$$H = H_0 + V$$

$$V = \frac{\varphi^m}{m!}$$

Recall:

$$\langle z^k, e^{-itH} z^\ell \rangle = \sum_{n=0}^{\infty} \int_{0 \le t_1 \le \cdots \le t_n \le t} (-i)^n \langle z^k, e^{-i(t-t_n)H_0} V \cdots V e^{-it_1 H_0} z^\ell \rangle \, dt_1 \cdots dt_n$$

Calculating $\langle z^k, e^{-i(t-t_n)H_0} V e^{-i(t_n - t_{n-1})H_0} V \cdots V e^{-it_1 H_0} z^\ell \rangle$

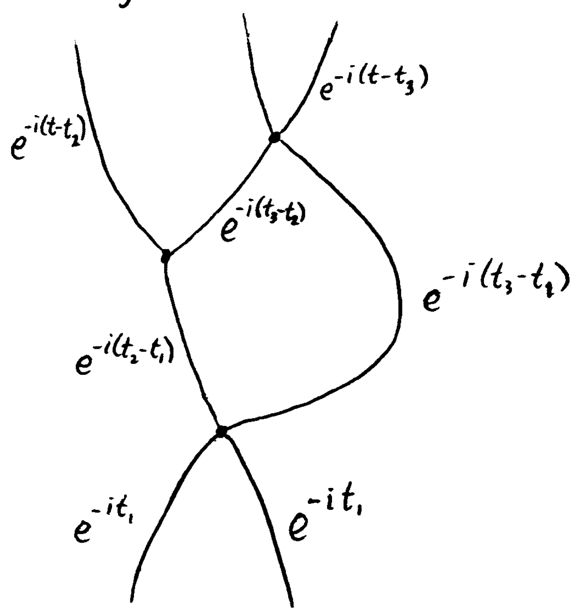
is almost like calculating

$$\langle z^k, VV \cdots V z^\ell \rangle$$

$$\|$$

$$\left| \left\langle Z^k, \frac{\Phi^m}{m!} \cdots \frac{\Phi^m}{m!} Z^\ell \right\rangle \right|$$

except to keep track of the "$e^{-i(t_p - t_{p-1})H_0}$" terms we label each edge by a phase: we label any edge from the pth vertex to the qth vertex $(p \le q)$ by $e^{-i(t_q - t_p)}$, e.g.



(or e^{-it_p} if the edge comes from bottom to the pth vertex or $e^{-i(t-t_q)}$ if it goes from the qth vertex to the top) In short: label each edge with e^{-iT} where T is the amount of time that passes along that edge. These phases are called <u>propagators</u>.

We calculate

$$\langle z^k, e^{-i(t-t_n)H_0} V \cdots V e^{-it_1 H_0} z^\ell \rangle$$

by summing over Feynman diagrams, each weighted by the product of all these phases (& divided by the size of their symmetry group, as usual in groupoid cardinality).
Why? Short answer: each "quantum" has energy 1, so applying $e^{-i(t_p - t_{p-1})H_0}$ to a state with E of the quanta (z^E) multiplies it by $e^{-i(t_p-t_{p-1})E}$.
Here we're getting that effect by attaching a phase to each edge (corresponding to a quantum).

27 May 2004

Feynman Diagrams

Let's see why our Feynman diagram recipe for calculating transition amplitudes like

$$\langle z^k, e^{-i(t-t_n)H_0} V e^{-i(t_n - t_{n-1})H_0} \cdots e^{-i(t_2-t_1)H_0} V e^{-it_1 H_0} z^\ell \rangle$$

agrees with a "direct" computation. We'll look at some examples coming from the homework, where we did

$$\langle z^2, \frac{\varphi^3}{3!} \frac{\varphi^3}{3!} z^2 \rangle$$

But now consider

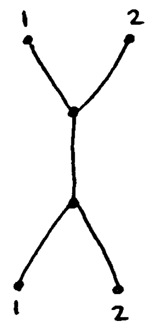
$$\langle z^2, e^{-i(t-t_2)H_0} \frac{\varphi^3}{3!} e^{-i(t_2-t_1)H_0} \frac{\varphi^3}{3!} e^{-it_1 H_0} z^2 \rangle$$

This will be a sum of many terms, since $\varphi^3 = (a+a^*)^3$, and each term corresponds to one or more Feynman diagrams. Let's look at a couple.

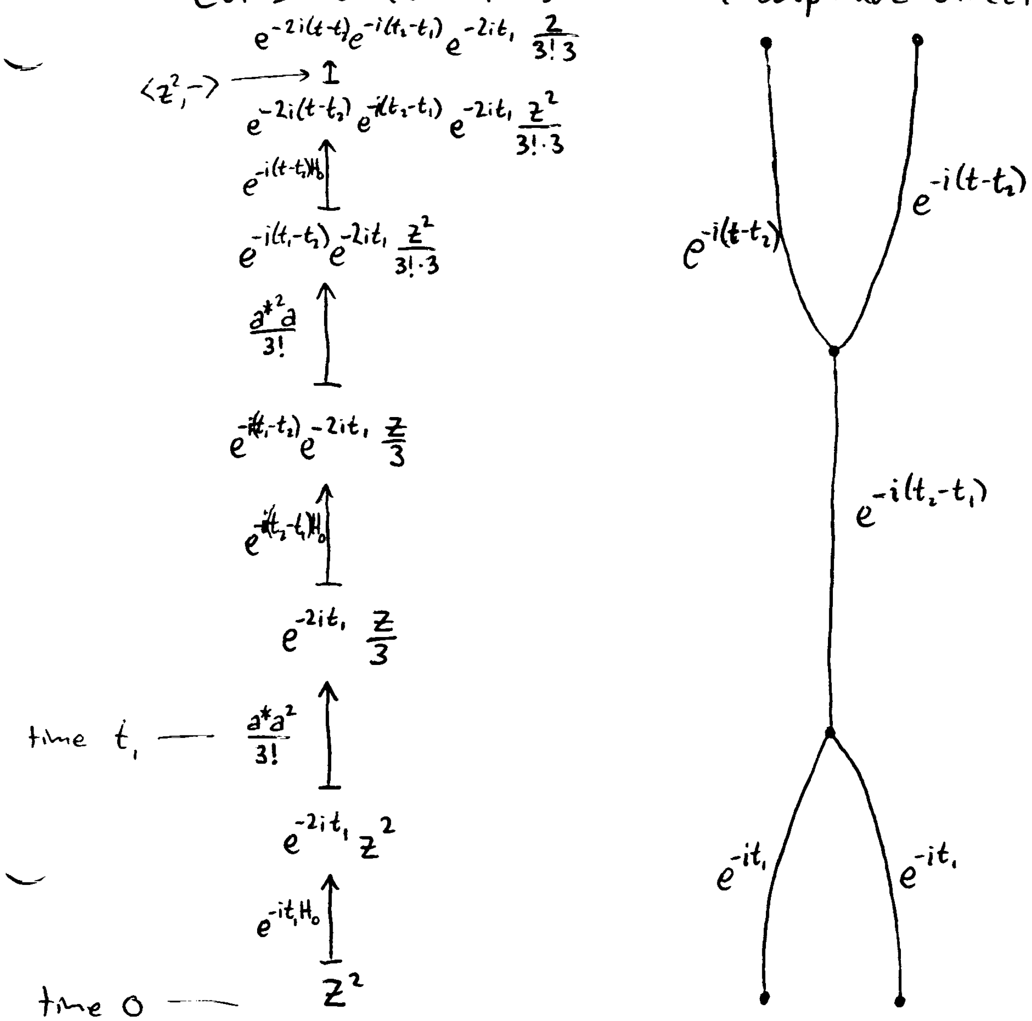
Example 1:

$$\left\langle z^2, e^{-i(t-t_2)H_0} \frac{a^{*2}a}{3!} e^{-i(t_2-t_1)H_0} \frac{a^* a^2}{3!} e^{-it_1 H_0} z^2 \right\rangle$$

corresponding to:



Let's calculate this transition amplitude directly & using Feynman diagrams:



The phases work out right: product of phases over all edges is

$$e^{-2i(t-t_2)} e^{-i(t_2-t_1)} e^{-2it_1}$$

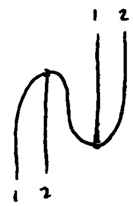
since there were:

2 quanta from time 0 to t_1

Example 2:

$$\left\langle z^2,\ e^{-i(t-t_2)H_0}\,\frac{a^3}{3!}\,e^{-i(t_2-t_1)H_0}\,\frac{a^{*3}}{3!}\,e^{-it_1H_0}\,z^2\right\rangle$$

corresponds to many Feynman diagrams, including:



$$e^{-2i(t-t_2)}\,e^{-5i(t_2-t_1)}\,e^{-2it_1}\,\frac{5\cdot4\cdot3}{3!\,3!}\,2!$$

$$\Big\uparrow \ \langle z^2, - \rangle$$

$$e^{-2i(t-t_2)}\,e^{-5i(t_2-t_1)}\,e^{-2it_1}\,\frac{5\cdot4\cdot3}{3!\,3!}\,z^2$$

$$\Big\uparrow \ e^{-i(t-t_2)H_0}$$

$$e^{-5i(t_2-t_1)}\,e^{-2it_1}\,5\cdot4\cdot3\,\frac{z^2}{(3!)^2}$$

time t_2 —— $\Big\uparrow \ \frac{a^3}{3}$

$$e^{-5i(t_2-t_1)}\,e^{-2it_1}\,\frac{z^5}{3!}$$

$$\Big\uparrow \ e^{i(t_2-t_1)H_0}$$

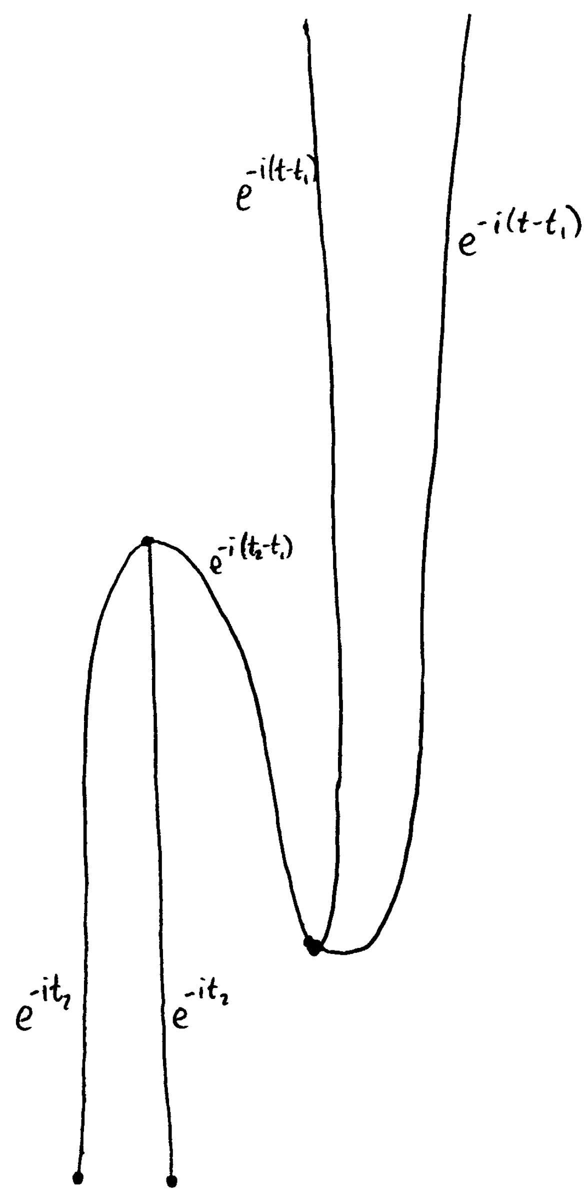
$$e^{-2it_1}\,\frac{z^5}{3!}$$

time t_1 —— $\Big\uparrow \ \frac{a^{*3}}{3!}$

$$e^{-2it_1}\,z^2$$

$$\Big\uparrow \ e^{-it_1H_0}$$

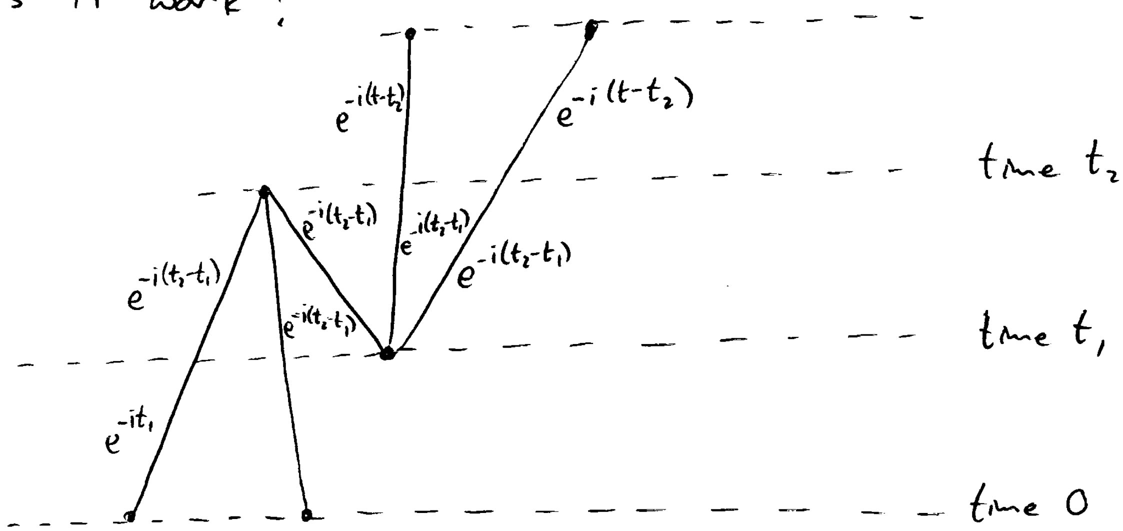
time 0 —— z^2



Even in this example the product of phase labellings on edges matches our "direct" calculation:

$$e^{-2it}e^{3it_1}e^{-3it_2} = e^{-2i(t-t_2)}e^{-5i(t_2-t_1)}e^{-2it_1}.$$

Why does it work?



Break each edge by horizontal lines at times $0, t_1, t_2, t$, and factor its phase as above!