Recall: given smooth categories C & D, a smooth anafunctor $F: C \to D$ consists of:

1) An open cover U_i of $Ob(C)$
2) Smooth functors
$$F_i : C|_{U_i} \longrightarrow D$$
3) Smooth natural isomorphisms
$$\begin{tikzcd} C|_{U_{ij}} \arrow[r, bend left=50, "F_j"{name=U}] \arrow[r, bend right=50, "F_i"'{name=L}] & D \arrow[from=U, to=L, Rightarrow, "g_{ij}"] \end{tikzcd}$$

such that

4) the cocycle condition
$$g_{ij} g_{jk} = g_{ik}$$
holds on $C|_{U_{ijk}}$.

So: what's a smooth anafunctor
$$F: \mathrm{Disc}(M) \longrightarrow G \ ?$$

1) An open cover U_i of $Ob(\mathrm{Disc}(M)) = M$
2) smooth functors $F_i : \mathrm{Disc}(U_i) \longrightarrow G$ — but there's only one, since $\mathrm{Disc}(U_i)$ is discrete & G has just one object.

3) Smooth natural isomorphisms

$$\begin{tikzcd} \mathrm{Disc}(U_{ij}) \arrow[r, bend left, "F_j"] \arrow[r, bend right, "F_i"'] \arrow[r, phantom, "\Downarrow g_{ij}"] & G \end{tikzcd}$$

For each object in $\mathrm{Disc}(U_{ij})$, i.e. $x \in U_{ij}$, this gives an isomorphism $g_{ij}(x) : \underset{\underset{*}{\parallel}}{F_j(x)} \longrightarrow \underset{\underset{*}{\parallel}}{F_i(x)}$ where $* \in G$ is unique.

In short, $g_{ij} \in G$. Why is g_{ij} natural?

$$\begin{tikzcd} F_j(x) \arrow[r, "F_j(1_x)"] \arrow[d, "g_{ij}(x)"'] & F_j(x) \arrow[d, "g_{ij}(x)"] \\ F_i(x) \arrow[r, "F_i(1_x)"'] & F_i(x) \end{tikzcd}$$

must commute, and obviously does!

4) And we require $g_{ij} g_{jk} = g_{ik}$

All this is just a Čech 1-cocycle! So:

$$\{\text{smooth anafunctors } F : \mathrm{Disc}(M) \to G\} \cong \{\text{Čech 1-cocycles}\}$$

so we can cook up a definition of "ananatural transformation" s.t.

$$\frac{\{\text{smooth anafunctor } F : \mathrm{Disc}(M) \to G\}}{\text{ananatural isomorphism}} \cong \check{H}^1(M, G)$$

In general:

Def: Given smooth anafunctors $C \underset{F'}{\overset{F}{\rightrightarrows}} D$

a smooth ananatural transformation
$$\begin{tikzcd} C \arrow[r, bend left, "F"{name=U}] \arrow[r, bend right, "F'"'{name=L}] & D \arrow[from=U, to=L, Rightarrow, "f"] \end{tikzcd}$$
is

a collection of smooth natural transformations

$$\begin{tikzcd} C|_{U_i} \arrow[r, bend left, "F_i"{name=U}] \arrow[r, bend right, "F_i'"'{name=L}] & D \arrow[from=U, to=L, Rightarrow, "f_i"] \end{tikzcd}$$

such that

$$f_i \, g_{ij} = g_{ij}' \, f_j \qquad \forall i,j$$

where g_{ij}, g_{ij}' come from the anafunctors F, F', in the way we described.

In summary:

$$\frac{\{G\text{-bundles over } M\}}{\text{isomorphism}} \cong \check{H}^1(M,G) \cong \frac{\{\text{smooth anafunctors } F: \mathrm{Disc}(M) \to G\}}{\text{ananatural isomorphism}}$$

but in fact:

$$\frac{\left\{\begin{matrix} G \text{ bundles with} \\ \text{connection over } M \end{matrix}\right\}}{\text{isomorphism}} \cong \frac{\left\{\begin{matrix} \text{smooth anafunctors} \\ \mathrm{hol}: PM \longrightarrow G \end{matrix}\right\}}{\text{ananatural isomorphism}}$$

this deserves to be called "$\check{H}^1(PM, G)$" — now we're really doing Čech cohomology of a smooth category!

More generally given two smooth 1-categories C & D, let's define

$$\check{H}^1(C,D) = \frac{\{\text{smooth anafunctors } F: C \to D\}}{\text{ananatural isomorphism}}$$

Then replace 1 by "n"!