Or better:

"the groupoid of F-structured finite sets labelled by objects of Z_0."

Yes! Let's see how this works. Recall the weak quotient

$$\frac{F_n \times Z_0^n}{n!}$$

has the same objects as $F_n \times Z_0^n$, so ~~its~~ an object of this is:

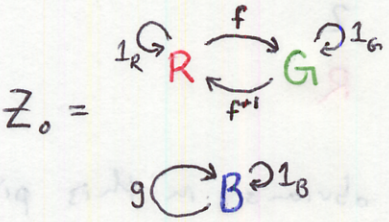
an ordered pair consisting of
an F-structure on the n-elt set
& an n-tuple of objects in Z_0

i.e.

an F-structure on n & a way of labelling its elements by objects of Z_0.

Example:

F = being a totally ordered set

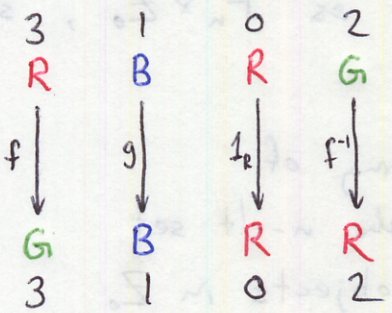
$Z_0 =$ 

$n = \{0, 1, 2, 3\} = 4$

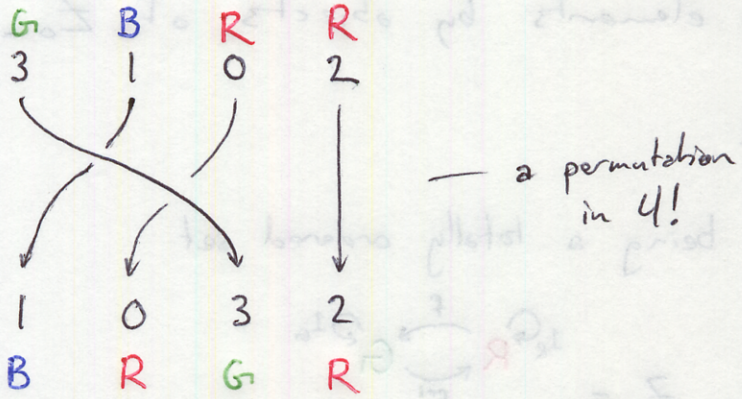
An object of $\frac{F_4 \times Z_0^4}{4!}$:

3	1	0	2
R	B	R	G

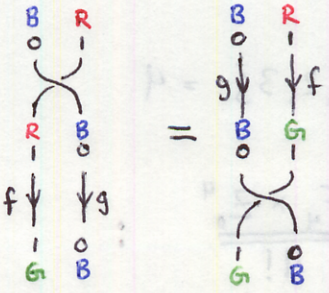
The morphisms of $\frac{F_n \times Z_0^n}{n!}$ are generated by morphisms in $F_n \times Z_0^n$ & morphisms coming from elts of the permutation group $n!$, & satisfying certain relations we called (1) & (2) last time. In our example, a typical morphism in $F_n \times Z_0^n$ is



while a typical morphism coming from 4! is:



The relations (1) & (2) are obvious in this picture, e.g.:



is an example of relation (2).