Schur Functors

We have indicated that Schur functors are some kind of "categorified polynomials".

Polynomials in x w. integer coefficients

$\boxed{\text{Free comm. ring on one generator } x}$

Universal property of object in the category Comm Ring.

Free monoid on on one generator x	Free comm monoid on one generator x	Free ring on one generator x	Free commutative ring on one generator x
$\mathbb{N}$	$\mathbb{N}$	$\mathbb{Z}[x]$	$\mathbb{Z}[x]$

"Universal property"

way of defining (or characterizing, up to isomorphism) an object in a given category, by describing how to map out of (or alternatively into) the object.

Every object has a universal property

$$C \xrightarrow{\text{"Actual universal property"}} \text{World of potential universal properties of } C\text{-objects}$$

Or really:

$$C \underset{\text{full \& faithful}}{\xrightarrow{\text{" Yoneda embedding "}}} \underline{\mathrm{Set}}^{C^{op}}$$

$\parallel$

potential right-universal properties

We are talking about left universal properties.

$$\underbrace{\{\mathbb{N} \longrightarrow M\}}_{=\, u(M)}$$

some other monoid

we only care about what happens to the one generator

$C = \mathrm{Monoid}^{op}$

$$C \xrightarrow{Y_C} \underline{\mathrm{Set}}^{C^{op}}$$

$$\mathbb{N} \longmapsto C^{op} \longrightarrow \underline{\mathrm{Set}}$$

$$M \longmapsto \hom(\mathbb{N}, M)$$

"Functional calculus"

class of "nice" functions

class of "nice" operators

"Apply a nice function to a nice operator to get another nice operator."

Monoid = set with "multiplication"

Monoidal category = category with "tensor product"

Universal problems:

Free monoidal cat on one object X

Free braided monoidal cat on one object

Free monoidal cat <u>with colimits</u> on one object X.

"with addition"