

Monomials \longrightarrow Polynomials

Monoid \longrightarrow Ring

We have complicated freely generated expressions such as

$$(3x+1)(2x+5) + (x+3)$$

but distributivity allows us to view addition as a finishing touch.

Cat \xrightarrow{F} Cat w. colimits
 $C \longmapsto F(C) = \text{Set}^{C^{\text{op}}}$

Set $\xrightarrow{\text{"Free"}} \text{Comm Mon}$
 $S \longmapsto \mathbb{N}^S$

Let's look at an example:

$$C = \text{FinSet}_0$$

We saw this C when we were studying the free symmetric monoidal category.

$$F(C) = \text{Set}^{\text{FinSet}_0} = \text{"formal sums of finite sets"}$$

$$\text{FinSet}_0 \longrightarrow \text{Set}$$

$$\mathbb{N} \longmapsto 2 \cdot \mathbb{N}$$

or

$$\mathbb{N} \longmapsto \text{"2-colorings of } \mathbb{N}\text{"} = 2^{\mathbb{N}}$$

The power series for $\mathbb{N} \longmapsto 2 \cdot \mathbb{N}$ is

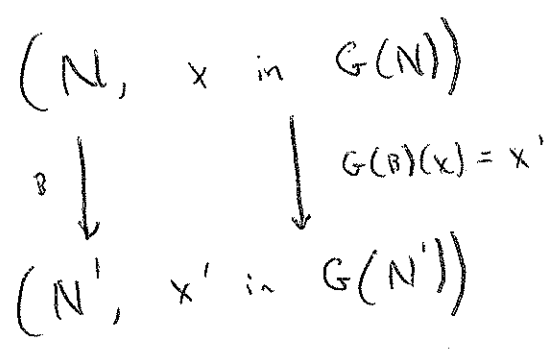
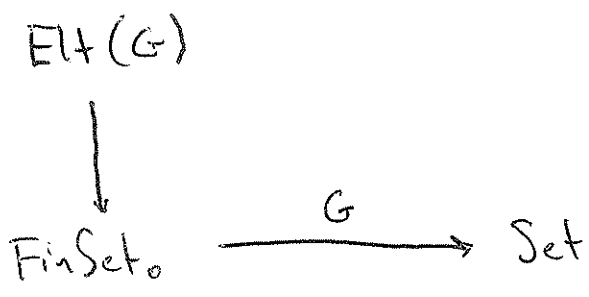
$$2x + \frac{4}{2}x^2 + \frac{6}{6}x^3 + \frac{8}{24}x^4 + \dots$$

$$= \sum_{\mathbb{N}} \frac{2^{\mathbb{N}}}{\mathbb{N}!} x^{\mathbb{N}}$$

For $\mathbb{N} \longmapsto 2^{\mathbb{N}}$ we have

$$1 + 2x + \frac{4}{2}x^2 + \frac{8}{6}x^3 + \dots$$

$$= \sum_{\mathbb{N}} \frac{2^{\mathbb{N}}}{\mathbb{N}!} x^{\mathbb{N}}$$



Since we are dividing by factorials in our power series we have a special kind of colimits:

- "CO-invariants of a group action"
- "co-equalizer"

See Joyal, "Especies de structure"
Species of structure