Last time we were talking about

$$\text{Set}^{\text{FinSet}_0},$$

categorified "polynomials in one variable",

$$\mathbb{N}^{\mathbb{N}}$$

$\text{Vect}^{\text{FinSet}_0}$ is the same as representations of the symmetric groups.

We can tensor representations V of $m!$ & W of $n!$ to get a representation of $(m+n)!$

$$V \tilde{\otimes} W$$

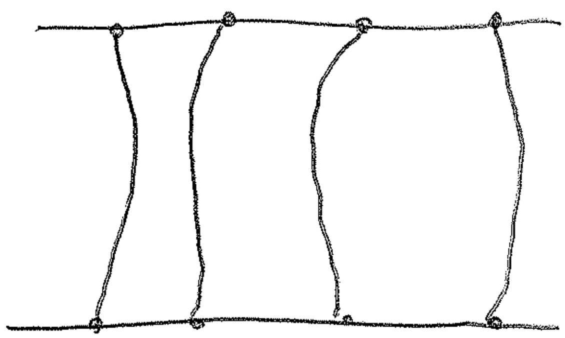
We have parallel examples which we can use as "exponents"

$$\mathrm{FinSet}_0$$

$$\mathrm{Planar\ FinSet}_0$$

$$\mathrm{Linear\ FinSet}_0$$

The morphisms in $\mathrm{Linear\ FinSet}_0$ look like



$\mathrm{Vect}^{\mathrm{Planar\ FinSet}_0}$ is representations of the braid group.

We can also look at

$$\mathrm{Vect}^{\mathrm{Linear\ FinSet}_0}$$

We can talk about the "representation theory of finite groups $GL(n, F_q)$"

$$\mathrm{Vect}^{\text{Fin Dim } F_q\text{-Vect}_0}$$

This has to do with "q-deformation" & "braiding", which we have discussed before.

So there is a connection between

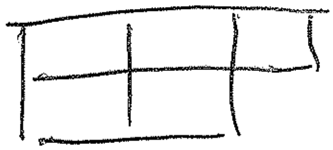
$$\mathrm{Vect}^{\text{Planar Fin Set}_0} \quad \& \quad \mathrm{Vect}^{\text{Fin Dim } F_q\text{-Vect}_0}$$

We recall that Young diagrams are a notation for irreducible reps of symmetric groups.

$m = 5$ $\quad V$ a rep of $1!$

$$(V \tilde{\otimes} V \tilde{\otimes} V)/3! \;\tilde{\otimes}\; (V \tilde{\otimes} V)/2!$$

This is the representation we get from S_5 acting on flags of type



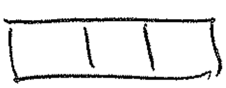
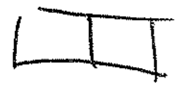
This Young diagram is also notation for a representation of $GL(5, \mathbb{F}_q)$.

Given a rep V of $GL(m, \mathbb{F}_q)$ & W of $GL(n, \mathbb{F}_q)$, we can get a rep

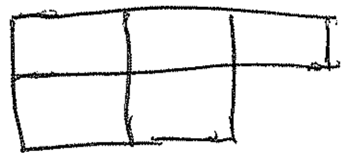
$$V \tilde{\otimes} W$$

of $GL(m+n, \mathbb{F}_q)$.

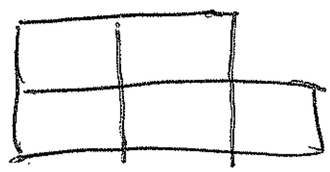
For example, if $m = 3$ & $n = 2$, we have

 & 

and tensoring we get



and to begin with this should be different from



We want to understand these things more concretely.

$$\mathrm{FinSet}_0 \underset{G}{\overset{F}{\rightrightarrows}} \mathrm{Set}$$

$$\mathrm{FinSet}_0 \xrightarrow{F \,\hat{\otimes}\, G} \mathrm{Set}$$

$$n \longmapsto \left\{ \begin{array}{l} \text{Ways of splitting } n \text{ into two} \\ \text{parts with an } F\text{-structure on} \\ 1^{\text{st}} \text{ part \& } G\text{-structure on } 2^{\text{nd}} \text{ part} \end{array} \right\}$$

An F-structure on n is just an element of $F(n)$.

Example:

$$F(n) = 2^n$$

$$G(n) = 3^n$$

$$[F \tilde{\otimes} G](n) = 5^n$$

As power series:

$$\left(1 + \frac{2}{1!}x + \frac{4}{2!}x^2 + \frac{8}{3!}x^3 + \cdots\right)\left(1 + \frac{3}{1!}x + \frac{9}{2!}x^2 + \cdots\right)$$

$$= \left(1 + \frac{5}{1!}x + \frac{25}{2!}x^2 + \cdots\right)$$