TCU MATHEMATICS
GREEN HONORS CHAIR LECTURES

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Sept. 8, 1 PM	My Favorite Numbers: 5
Sept. 8, 7 PM*	Zooming Out In Time: A History of Climate Change
Sept. 9, 1 PM	My Favorite Numbers: 8
Sept. 9, 4 PM	Fundamental Physics: Where We Stand Today
Sept. 10, 1 PM	My Favorite Numbers: 24

Texas Christian University, Fort Worth, TX

The talk on Sept. 8 at 7 PM will be in EDU 130-Palko Hall - refreshments to follow
All other talks in Tucker Technology Center 139, preceded by refreshments in TTC-300
Short abstracts can be found on the back of this sheet - more info online at
http://www.math.tcu.edu

My Favorite Numbers: 5

Different numbers have different personalities. The number 5 is quirky and intriguing, thanks in large part to its relation with the golden ratio, the "most irrational" of irrational numbers. The plane cannot be tiled with regular pentagons, but there exist quasiperiodic planar patterns with pentagonal symmetry of a statistical nature, first discovered by Islamic artists in the 1600s, later rediscovered by the mathematician Roger Penrose in the 1970s, and found in nature in 1984.

The Greek fascination with the golden ratio is probably tied to the dodecahedron. Much later, the symmetry group of the dodecahedron was found to give rise to a 4-dimensional regular polytope, the 120-cell, which in turn gives rise to the Poincaré homology sphere and the root system of the exceptional Lie group E_8. So, a wealth of exceptional objects arise from the quirky nature of 5-fold symmetry.

Zooming Out In Time: A History of Climate Change

How can we detect and understand oncoming crises in time to avert them? Sometimes we must "zoom out": expand our perspective and find similar situations in the distant past. A good example is climate change. What can a few degrees of warming do? To answer this, we need to know some history: how the Earth's climate has changed over the last 65 million years.

My Favorite Numbers: 8

The number 8 plays a special role in mathematics due to the "octonions", an 8-dimensional number system where one can add, multiply, subtract and divide, but where the commutative and associative laws for multiplication — ab = ba and (ab)c = a(bc) — fail to hold. The octonions were discovered by Hamilton's friend John Graves in 1843 after Hamilton told him about the "quaternions". While much neglected, they stand at the crossroads of many interesting branches of mathematics and physics.

For example, superstring theory works in 10 dimensions because 10 = 8+2: the 2-dimensional worldsheet of a string has 8 extra dimensions in which to wiggle around, and the theory crucially uses the fact that these 8 dimensions can be identified with the octonions. Or: the densest known packing of spheres in 8 dimensions arises when the spheres are centered at certain "integer octonions", which form the root lattice of the exceptional Lie group E_8. The octonions also explain the curious way in which topology in dimension n resembles topology in dimension n+8.

Fundamental Physics: Where We Stand Today

Since the discovery of the W and Z particles over twenty years ago, no truly novel prediction of fundamental theoretical physics has been confirmed by experiment, except perhaps Guth's inflationary cosmology. On the other hand, observations in astronomy have revealed shocking new facts which our theories do not really explain: most of our universe consists of "dark matter" and "dark energy". Where does fundamental physics stand today, and why has theory become divorced from experiment?

My Favorite Numbers: 24

The numbers 12 and 24 play a central role in mathematics thanks to a series of "coincidences" that is just beginning to be understood. One of the first hints of this fact was Euler's bizarre "proof" that

$$1 + 2 + 3 + 4 + \ldots = -1/12$$

which he obtained before Abel declared that "divergent series are the invention of the devil". Euler's formula can now be understood rigorously in terms of the Riemann zeta function, and in physics it explains why bosonic strings work best in 26=24+2 dimensions. The fact that

$$1^2 + 2^2 + 3^2 + \ldots + 24^2$$

is a perfect square then sets up a curious link between string theory, the Leech lattice (the densest known way of packing spheres in 24 dimensions) and a group called the Monster. A better-known but closely related fact is the period-12 phenomenon in the theory of "modular forms". We shall do our best to demystify some of these deep mysteries.