MATH 009C **FINAL** **August 11, 2018**

Name: ______________________________ **Score:** _______ / 100

Student ID: __________________________

DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO

	1	2	3	4	5	6	7	8	9	Total
✓										80
Score										
Pts. Possible	10	10	10	10	10	10	10	10	10	85

INSTRUCTIONS FOR STUDENTS

- Questions are on both sides of the paper. This is an 9 question exam.
- Students have 2 hours to complete the exam.
- The test will be out of **80** points (8 questions). You may attempt a 9^{th} question, which will have a maximum of 5 possible points. The highest possible score is therefore **85** points.
- In the row with the ✓, mark with a ✓ the problems you want graded for credit, and **EC** for your extra credit problem. **If you do not mark the boxes, problems 1-8 will be graded for credit regardless of which ones you complete, and 9 will be your extra credit.**
- You may complete parts of problems, as partial credit will be given based on correctness, completeness, and ideas that are leading to the correct solutions.
- **PLEASE SHOW ALL WORK**. **Any unjustified claims will receive no credit. This means you need to state which test you are using for series questions!** Clearly box your final answer.
- No notes, textbooks, phones, calculators, etc. are allowed for the exam.
- The back of the test can be used for scratch work.

GOOD LUCK!

FORMULAS:

Common Taylor Series	Common Taylor Series
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, for all $\lvert x\rvert < 1$	$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, for all $x \in \mathbb{R}$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, for all $x \in \mathbb{R}$	$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, for all $x \in \mathbb{R}$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$, for $x \in (-1, 1]$	$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$, for $\lvert x\rvert \leq 1$
$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, for $\lvert x-a\rvert < R$	$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n$, for $\lvert x\rvert < 1$

1) (5 pts.) (a) Determine whether the sequence converges or diverges:

$$a_n = \frac{\arctan(n)}{e^{-n} - 1}.$$

(5 pts.) (b) Determine whether the sequence converges or diverges:

$$a_n = \frac{(-1)^n + n}{(-1)^n - n}.$$

2) (5 pts.) Determine whether the series is convergent or divergent. If it is convergent, find the sum.

$$\sum_{n=1}^{\infty} \arctan(n).$$

(5 pts.) (b) Determine whether the series is convergent or divergent. If it is convergent, find the sum.

$$\sum_{n=1}^{\infty} 2^{1-n}3^{-3n}.$$

3) (10 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=3}^{\infty} \frac{\ln(n)}{n}.$$

4) (10 pts.) Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{5n}{n^2 + |\sin(n)|}.$$

5) (10 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}.$$

6) (10 pts.) Determine whether the series is absolutely convergent, conditionally convergent, or divergent:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[6]{n^6 - 1}}.$$

7) (10 pts.) Find the radius of convergence and interval of convergence for the following power series. (*Note*: This is known as the *Bessel function of order 2*.

$$\sum_{n=0}^{\infty} \frac{(-1)^n \, x^{2(n+1)}}{n! \, (n+2)! \, 2^{2(n+1)}}$$

8) (10 pts.) Find the Taylor polynomial of degree 3, centered at the point $a = 0$ for the function $f(x) = e^{-x}$ using the definition of Taylor series. **Note: Do NOT use the substitution method, as you will receive no credit. You must use the definition.**

9) (5 pts.) (a) Compute the following integral using Taylor series.

$$\int e^{-x^2}\ dx$$

(5 pts.) (b) Find the Taylor series centered at $a = 0$ for the function

$$f(x) = \frac{\arctan(x) - x}{x^2}$$

THIS PAGE IS LEFT BLANK FOR ANY SCRATCH WORK

END OF TEST