MATH 009C - Summer 2018 - Table for Infinite Series Tests

Test	Series Type	Convergence or Divergence	Required Conditions/Helpful Notes
Divergence Test	$\sum a_n$	divergent if $\lim\limits_{n\to\infty} a_n \neq 0$	Test tells you **nothing** if $\lim\limits_{n\to\infty} a_n = 0$, use another test.
Geometric Series	$\sum\limits_{n=1}^{\infty} ar^{n-1}$ or $\sum\limits_{n=0}^{\infty} ar^n$	convergent to $\frac{a}{1-r}$ for $\|r\| < 1$ divergent for $\|r\| \geq 1$	Use for the two comparison tests for choice of $\sum b_n$.
p-series	$\sum\limits_{n=1}^{\infty} \dfrac{1}{n^p}$	convergent for $p > 1$ divergent for $p \leq 1$	Use for the two comparison tests for choice of $\sum b_n$.
Integral Test	$\sum\limits_{n=\alpha}^{\infty} a_n \quad \alpha \in \mathbb{N}$	convergent if $\int_{\alpha}^{\infty} f(x)dx$ converges divergent if $\int_{\alpha}^{\infty} f(x)dx$ diverges	Must have $a_n = f(n)$, and f must be positive, continuous, and decreasing for $x \geq \alpha$, where α is a positive integer. You must also be able to integrate $f(x)$.
Comparison Test	$\sum a_n$ and $\sum b_n$ $a_n > 0$ and $b_n > 0$	If $a_n \leq b_n$ and $\sum b_n$ converges $\Rightarrow \sum a_n$ converges If $a_n \geq b_n$ and $\sum b_n$ diverges $\Rightarrow \sum a_n$ diverges	Try geometric or p-series for choice of $\sum b_n$. You must prove the inequality, and state why $\sum b_n$ is convergent or divergent.
Limit Comparison Test	$\sum a_n$ and $\sum b_n$ $a_n > 0$ and $b_n > 0$	$\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = c$	If $c > 0$, then both series converge or both series diverge. If $\sum b_n$ converges and $c = 0$, then $\sum a_n$ converges. If $\sum b_n$ diverges and $c = \infty$, then $\sum a_n$ diverges.
Ratio Test	$\sum a_n$ and $\lim\limits_{n\to\infty} \left\| \dfrac{a_{n+1}}{a_n} \right\| = L$	convergent for $L < 1$ divergent for $L > 1$ inconclusive for $L = 1$	Use when a_n has factorials or powers of n. Use for power series problems for radius of convergence.
Root Test	$\sum a_n$ and $\lim\limits_{n\to\infty} (\|a_n\|)^{\frac{1}{n}} = L$	convergent for $L < 1$ divergent for $L > 1$ inconclusive for $L = 1$	Use when a_n has powers or functions of n.
Alternating Series Test	$\sum\limits_{n=1}^{\infty} (-1)^{n+1} b_n$ with $b_n > 0$	convergent if a_n is decreasing, and $\lim\limits_{n\to\infty} b_n = 0$	Use only for alternating series, must have $(-1)^n$, $(-1)^{n-1}$, or $(-1)^{n+1}$.