LAST NAME: FIRST NAME:

MATH 009C - Summer 2018

Worksheet 1: June 26, 2018

1. Compute the following limit:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Solution:

Note that if we "plug in" infinity, we get the result of 1^∞, which is indeterminate. Therefore, since we have this indeterminate form, and the limit is of a function to the power of a function, we use the exponential/logarithm trick

$$\begin{aligned}
\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n &= \lim_{n\to\infty} e^{\ln\left(\left(1+\frac{1}{n}\right)^n\right)} \\
&= \lim_{n\to\infty} e^{n\ln\left(1+\frac{1}{n}\right)} = \lim_{n\to\infty} e^{\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}} \\
&= e^{\lim_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}} \overset{\text{L'H}}{=} e^{\lim_{n\to\infty}\frac{\frac{-\frac{1}{n^2}}{1+\frac{1}{n}}}{-\frac{1}{n^2}}} \\
&= e^{\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}} \\
&= e
\end{aligned}$$

2. Compute the following limit:

$$\lim_{n\to\infty} n^{\frac{1}{1+\ln(n)}}$$

Solution:

Note that if we "plug in" infinity, we get the result of ∞^0, which is indeterminate. Therefore, since we have this indeterminate form, and the limit is of a function to the power of a function, we use the exponential/logarithm trick

$$\begin{aligned}
\lim_{n\to\infty} n^{\frac{1}{1+\ln(n)}} &= \lim_{n\to\infty} e^{\ln\left(n^{\frac{1}{1+\ln(n)}}\right)} \\
&= \lim_{n\to\infty} e^{\frac{\ln(n)}{1+\ln(n)}} = e^{\lim_{n\to\infty}\frac{\ln(n)}{1+\ln(n)}} \\
&\overset{\text{L'H}}{=} e^{\lim_{n\to\infty}\frac{\frac{1}{n}}{\frac{1}{n}}} \\
&= e
\end{aligned}$$

Please, show all work.

3. Determine if the integral is convergent or divergent:

$$\int_1^{\infty} \frac{1}{\sqrt{x^6+1}}\,dx$$

Solution:

Use the (Direct or Limit) Comparison Test for Improper Integrals. Note that

$$\begin{aligned} 0 < x^6 < x^6+1 \quad &\Rightarrow \quad \sqrt{x^6} < \sqrt{x^6+1} \\ &\Rightarrow \quad \frac{1}{\sqrt{x^6+1}} < \frac{1}{\sqrt{x^6}} = \frac{1}{x^3} \quad \text{for all } x > 1 \end{aligned}$$

So,

$$0 < \int_1^{\infty} \frac{1}{\sqrt{x^6+1}}\,dx < \int_1^{\infty} \frac{1}{x^3}\,dx = \lim_{t\to\infty} -\frac{1}{2x^2}\Bigg|_1^t = \lim_{t\to\infty} -\frac{1}{2t^2} + \frac{1}{2} = \frac{1}{2} < \infty$$

The improper integral is convergent by the Direct Comparison Test.

Please, show all work.

4. Eliminate the parameter for the following parameterized curve. Sketch the curve and use arrows to denote the direction.

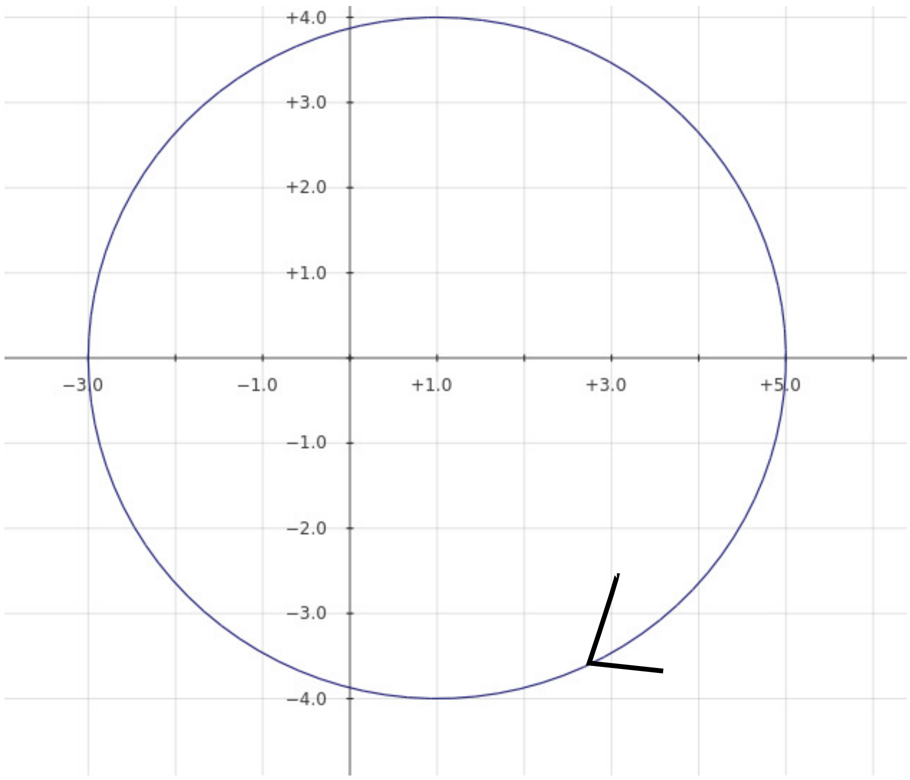
$$x = 1 + 4\sin(t), \quad y = 4\cos(t), \quad 0 < t < 2\pi$$

Solution:

Notice that we can do some algebra and trigonometric identities to eliminate the parameter:

$$\begin{aligned} x &= 1 + 4\sin(t) \\ x - 1 &= 4\sin(t) \\ &\Rightarrow \quad (x-1)^2 + y^2 = 16\sin^2(t) + 16\cos^2(t) \\ &\Rightarrow \quad (x-1)^2 + y^2 = 16 \end{aligned}$$

which gives us a circle with radius 4, centered at the point $(1, 0)$. The restriction of $0 < t < 2\pi$ tells us we go around the circle 1 time. The direction is clockwise, as at $t = 0$, we are at $(1, 4)$; then at $t = \frac{\pi}{2}$, we are at $(5, 0)$. The arrow designates the direction of motion.



Please, show all work.