LAST NAME: FIRST NAME:

MATH 009C - Summer 2018

Worksheet 5: July 24, 2018

1. For parts (a) and (b), determine whether the series converges or diverges. If the series converges, find its limit. For part(c), find a rational number that gives the decimal expansion.

(a) $\displaystyle\sum_{n=1}^{\infty} 2^{1-n}3^{-3n}9^{n-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$

(c) $3.\overline{1415} = 3.1425142514251425\ldots$

Please, show all work.

2. A casino offers the following game to play: a fair coin is tossed until tails turns up for the first time. If this occurs on the first toss, then you receive 2 dollars; if you get heads on the first and tails on the second toss you receive $2^2 = 4$ dollars; in general, if tails turns up for the first time on the n^{th} toss you receive 2^n dollars.

Hints to set up the problem to use series: For part (a), if X is the amount of money you would pay to play this game, the expected value of X is given by

$$\mathbb{E}[X] = x_1p_1 + x_2p_2 + x_3p_3 + \ldots = \sum_{n=1}^{\infty} x_np_n$$

where the x_n represent outcomes (how much money you win on the n^{th} toss) and p_n represents the probability of event x_n occurring. For part (d), X is the amount of games you play, x_n is represents the toss number you are on, and p_n represents the probability of getting a tails on that toss number.

Answer and explain your solutions to the following questions:

(a) What would be a fair price to pay the casino for playing the game? In other words, what is $\mathbb{E}[X]$ for our game?

(b) What does your result from part (a) mean in real life terms?

(c) What is the probability that the game stops in a finite number of tosses?

(d) What is the expected length of the game? In other words, what is the expected number of flips before you get tails?

Please, show all work.

3. Use the Integral Test to determine if the following series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{1+n^2}$$

Please, show all work.