Math 10B: Multi-var. Cal. 2
Spring 2018
Quiz 1 (010)
04/19/2018
Time Limit: 30 Minutes

Name (Print): _____
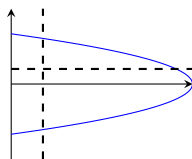
Discussion TA: _____

Discussion time: _____

1. (5 points) Let D be the region bounded by the y-axis and the parabola $x = -9y^2 + 4$. Set up the iterated integral of

$$\iint_D 3\cos(y)e^{xy-x^2+y^3}\, dx\, dy$$

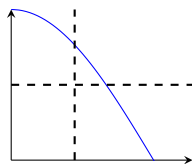in both ways. You DO NOT need to evaluate it.

Solution:

$$\iint_D 3\cos(y)e^{xy-x^2+y^3}\, dx\, dy = \int_{-2/3}^{2/3}\int_0^{-9y^2+4} 3\cos(y)e^{xy-x^2+y^3}\, dx\, dy$$

$$= \int_0^4 \int_{-\sqrt{4-x}/3}^{\sqrt{4-x}/3} 3\cos(y)e^{xy-x^2+y^3}\, dy\, dx.$$

2. (5 points) Let D be the region bounded by the x-axis ($0 \le x \le \pi/2$), y-axis and $y = \cos(x)$. Set up the iterated integral of

$$\iint_D \ln\left(\cos\left(\frac{x+y}{3\pi}\right)\right)\, dx\, dy$$
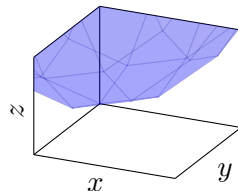
in both ways. You DO NOT need to evaluate it.

Solution:

$$\iint_D \ln\left(\cos\left(\frac{x+y}{3\pi}\right)\right)\, dx\, dy = \int_0^1 \int_0^{\arccos y} \ln\left(\cos\left(\frac{x+y}{3\pi}\right)\right)\, dx\, dy$$

$$= \int_0^{\pi/2} \int_0^{\cos x} \ln\left(\cos\left(\frac{x+y}{3\pi}\right)\right)\, dy\, dx.$$

3. (5 points) Let W be the region bounded by $z = 0$, $x = 0$, $x = 1$, $y = 0$, $y = 1$, and $z = x^2 + y^2$. Please write a double integral formula **AND** a triple integral formula to compute the volume of W. You can use any order you want and you DON'T need to evaluate the formulas.
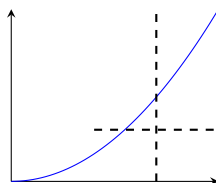
Solution:

$$Vol(W) = \int_0^1 \int_0^1 (x^2 + y^2)\, dx\, dy = \int_0^1 \int_0^1 \int_0^{x^2+y^2} 1\, dz\, dx\, dy.$$

4. (5 points) Change the order of integration, and evaluate it in that order.

$$\int_0^1 \int_0^{x^2} (x + y)\, dy\, dx.$$

Solution:

$$\int_0^1 \int_0^{x^2} (x + y)\, dy\, dx = \int_0^1 \int_{\sqrt{y}}^1 (x + y)\, dx\, dy = \int_0^1 \left(\frac{x^2}{2} + xy \right)\Big|_{\sqrt{y}}^1 dy$$

$$= \int_0^1 \left(\frac{1}{2} + y \right) - \left(\frac{y}{2} + \sqrt{y}\,y \right) dy = \int_0^1 \left(\frac{1}{2} + \frac{y}{2} - y^{3/2} \right) dy$$

$$= \frac{1}{2}y + \frac{1}{2}\frac{y^2}{2} - \frac{y^{5/2}}{5/2}\Big|_0^1 = \frac{1}{2} + \frac{1}{4} - \frac{2}{5} = \frac{7}{20}.$$