Math 10B: Multi-var. Cal. 2
Spring 2018
Quiz 2 (010)
05/24/2018
Time Limit: 30 Minutes

Name (Print): ____________________

Discussion TA: ____________________

Discussion time: ____________________

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1. (8 points) Let S be the cylinder $x^2 + y^2 = 1$ with $0 \le z \le 1$ with an outer normal vector. Evaluate
$$\iint_S (2x\mathbf{i} - 2y\mathbf{j} + z^2\mathbf{k}) \cdot d\mathbf{S}.$$

Solution: Use cylindrical coordinates. The parametrization of S is given by
$$x = \cos\theta, \quad y = \sin\theta, \quad z = z, \quad \text{for } 0 \le \theta \le 2\pi,\ 0 \le z \le 1.$$
Then
$$\mathbf{T}_\theta = (-\sin\theta, \cos\theta, 0), \quad \mathbf{T}_z = (0, 0, 1),$$
and
$$\mathbf{T}_\theta \times \mathbf{T}_z = (\cos\theta, \sin\theta, 0).$$
Test the orientation. Let $\theta = 0$, $z = 0$, we get $(1, 0, 0)$ at $(1, 0, 0)$. It is an outer normal vector. Therefore the normal vector should be
$$\mathbf{n} = (\cos\theta, \sin\theta, 0).$$
Then
$$\begin{aligned}\iint_S (2x\mathbf{i} - 2y\mathbf{j} + z^2\mathbf{k}) \cdot d\mathbf{S} &= \int_0^1 \int_0^{2\pi} (2x, -2y, z^2) \cdot (\cos\theta, \sin\theta, 0)\, d\theta\, dz \\ &= \int_0^1 \int_0^{2\pi} (2\cos\theta, -2\sin\theta, z^2) \cdot (\cos\theta, \sin\theta, 0)\, d\theta\, dz \\ &= \int_0^1 \int_0^{2\pi} (2\cos^2\theta - 2\sin^2\theta)\, d\theta\, dz = \int_0^1 dz \int_0^{2\pi} 2\cos(2\theta)\, d\theta \\ &= 1 \cdot \sin(2\theta)\big|_0^{2\pi} = 0.\end{aligned}$$

2. (6 points) Let S be defined by
$$x = u\cos(v), \quad y = u\sin(v), \quad z = v$$
for $0 \le u \le 1$ and $0 \le v \le 2\pi$. Evaluate
$$\iint_S \sqrt{x^2 + y^2 + 1}\, dS.$$

Solution:

$$\mathbf{T}_u = (\cos v, \sin v, 0), \quad \mathbf{T}_v = (-u\sin v, u\cos v, 1).$$

Then

$$\mathbf{T}_u \times \mathbf{T}_v = (\sin v, -\cos v, u\cos^2 v + u\sin^2 v) = (\sin v, -\cos v, u).$$

Therefore

$$\|\mathbf{T}_u \times \mathbf{T}_v\| = \sqrt{\sin^2 v + (-\cos v)^2 + u^2} = \sqrt{1+u^2}.$$

Then

$$\iint_S \sqrt{x^2+y^2+1}\, dS = \int_0^{2\pi}\int_0^1 \sqrt{u^2\cos^2 v + u^2\sin^2 v + 1}\sqrt{1+u^2}\, du\, dv$$
$$= \int_0^{2\pi}\int_0^1 (u^2+1)\, du\, dv = \int_0^1 (u^2+1)\, du \int_0^{2\pi} dv = \frac{8}{3}\pi.$$

3. (6 points) Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Let the curve be the straight line from $(0,0,0)$ to $(5,5,5)$. Evaluate

$$\int_C \mathbf{F}\cdot d\mathbf{s}.$$

Solution: C is part of a straight line. Therefore we may assume the parametrization is

$$x = at+b, \quad y = ct+d, \quad z = et+f, \quad 0 \le t \le 1.$$

Then since C going through $(0,0,0)$ and $(5,5,5)$, we have

$$a=5, \quad b=0, \quad c=5, \quad d=0, \quad e=5, \quad f=0.$$

Then the parametrizaiton $\mathbf{c}(t)$ of C is given by

$$x = 5t, \quad y = 5t, \quad z = 5t, \quad \text{for } 0 \le t \le 1.$$

Then $\mathbf{c}'(t) = (5,5,5)$. Therefore

$$\int_C F\cdot d\mathbf{s} = \int_0^1 (5t\mathbf{i} + 5t\mathbf{j} + 5t\mathbf{k})\cdot(5,5,5)\, dt = \int_0^1 (25t+25t+25t)\, dt = \int_0^1 75t\, dt = \frac{75}{2}.$$