MATH 010B - Spring 2018

Worked Problems - Section 5.2

1. Compute the following integral if $R = [0,1] \times [0,1]$.

$$\iint_R 8(xy)^2 \cos\left(x^3\right)\ dA$$

Solution: We can use the trick (Fubini's Theorem) here to do this integral.

$$\begin{aligned}
\iint_R 8(xy)^2 \cos\left(x^3\right)\ dA &= \iint 8x^2y^2 \cos\left(x^3\right)\ dA \\
&= \int_0^1 \int_0^1 8x^2y^2 \cos\left(x^3\right)\ dx\ dy \\
&= \left(8 \int_0^1 x^2 \cos\left(x^3\right)\ dx\right) \cdot \left(\int_0^1 y^2\ dy\right) \\
&= \left(\frac{8}{3} \sin\left(x^3\right)\Big|_0^1\right) \cdot \left(\frac{1}{3} y^3\Big|_0^1\right) \\
&= \frac{8}{3} \sin(1) \left(\frac{1}{3}\right) \\
&= \frac{8\sin(1)}{9}
\end{aligned}$$

□

2. Compute the following iterated integral if $R = [0,1] \times [0,1]$.

$$\iint_R 5\sin(x+y)\ dA$$

Solution: Here we just compute the iterated integral directly.

$$\begin{aligned}
\int_0^1 \int_0^1 (5\sin(x+y))\ dx\ dy &= 5 \int_0^1 \int_0^1 \sin(x+y)\ dx\ dy \\
&= 5 \int_0^1 \int_y^{1+y} \sin(u)\ du\ dy \qquad u = x+y,\ du = dx \\
&= 5 \int_0^1 -\cos(u)\big|_y^{1+y}\ dy \\
&= 5 \int_0^1 (-\cos(1+y) + \cos(y))\ dy \\
&= 5\left(-\sin(1+y) + \sin(y)\right)\big|_0^1 \\
&= 5(-\sin(2) + \sin(1) - (-\sin(1) + 0)) \\
&= 10\sin(1) - 5\sin(2)
\end{aligned}$$

□

3. Compute the following iterated integral if $R = [0, 5] \times [-1, 1]$.

$$\iint_R \frac{yx^3}{y^2+2}\ dy\ dx$$

***Solution*:** Here we can use Fubini's Theorem, then compute two single integrals

$$\begin{aligned}
\iint_R \frac{yx^3}{y^2+2}\ dy\ dx &= \int_0^5 \int_{-1}^1 \frac{yx^3}{y^2+2}\ dx\ dy \\
&= \left(\int_0^5 x^3\ dx\right) \cdot \left(\int_{-1}^1 \frac{y}{y^2+2}\ dy\right) \\
&= \left(\frac{1}{4}x^4\Big|_0^5\right) \cdot \left(\frac{1}{2}\ln(y^2+2)\Big|_{-1}^1\right) \\
&= \frac{5^4}{4} \cdot \left(\frac{1}{2}(\ln(3) - \ln(3))\right) \\
&= 0
\end{aligned}$$

□

4. Compute the volume of the solid bounded by the xz-plane, the yz-plane, the xy-plane, the planes $x = 1$ and $y = 1$, and the surface $z = 6x^2 + 7y^4$.

***Solution*:** The first 3 restrictions tell us that the solid must be in the first octant, the region where x, y, and z are all positive. In addition, the planes $x = 1$ and $y = 1$ restrict us to the rectangle $R = [0, 1] \times [0, 1]$, and the solid is bounded above the rectangle by the surface $z = 6x^2 + 7y^4$. Therefore, we have all we need to compute the integral

$$\begin{aligned}
\iint_R f(x,y)\ dA &= \int_0^1 \int_0^1 6x^2 + 7y^4\ dx\ dy \\
&= \int_0^1 \left(2x^3 + 7xy^4\Big|_{x=0}^{x=1}\right)\ dy \\
&= \int_0^1 \left(2 + 7y^4\right)\ dy \\
&= \left(2y + \frac{7}{5}y^5\Big|_0^1\right) \\
&= 2 + \frac{7}{5} \\
&= \frac{17}{5}
\end{aligned}$$

□

5. Compute the volume of the solid bounded by the graph $z = x^2 + y$, the rectangle $R = [0,1] \times [1,3]$, and the "vertical sides" of R.

***Solution*:** All of the relevant information is provided, R gives us the bounds on the integral, and the surface is given, thus we proceed as in the previous example

$$\begin{aligned}
\iint_R f(x,y)\, dA &= \int_0^1 \int_1^3 \left(x^2 + y\right)\, dy\, dx \\
&= \int_0^1 \left(yx^2 + \frac{1}{2}y^2\right)\Bigg|_{y=1}^{y=3} dx \\
&= \int_0^1 \left(3x^2 + \frac{9}{2} - x^2 - \frac{1}{2}\right)\, dx \\
&= \int_0^1 \left(2x^2 + 4\right)\, dx \\
&= \left(\frac{2}{3}x^3 + 4x\right)\Bigg|_0^1 \\
&= \frac{2}{3} + 4 \\
&= \frac{14}{3}
\end{aligned}$$

□