

MATH 046 020-QUIZ 1, SPRING 2018

Name: KEY

1 (4 pts). Model a population $P(t)$, if its rate of growth is proportional to the amount at time t . Assume the rate constant is $r > 0$.

rate of growth proportional to $P(t) \Rightarrow \frac{dP}{dt} = rP > 0$

$$\Rightarrow \boxed{\begin{matrix} P'(t) = rP(t) \\ r > 0 \end{matrix}}$$

Solution $P(t) = Ce^{rt}$
to ODE C constant

2 (6 pts). Determine if the equation $y' = xy + 1$ is linear, separable, or exact. For exactness, use the differential form $(xy + 1)dx - dy = 0$. Explain your reasoning.

Linear: $y' = xy + 1 \Rightarrow y' - xy = 1$ $\begin{matrix} p(x) = -x \\ q(x) = 1 \\ y' + p(x)y = q(x) \end{matrix}$
 \Rightarrow ODE is linear.

Separable: $y' = \frac{A(x)}{B(y)}$, need to find $A(x)$ and $B(y)$
but there is no way to write $xy + 1$ as $\frac{A(x)}{B(y)}$
 \Rightarrow ODE is not separable.

Exact: $(xy + 1)dx - dy = 0 \Rightarrow \begin{matrix} M(x,y) = xy + 1 \\ N(x,y) = -1 \end{matrix}$

$$\Rightarrow \frac{\partial M}{\partial y} = x, \quad \frac{\partial N}{\partial x} = 0$$

Not equal, so ODE is not exact.