MATH 046 020-QUIZ 1, SPRING 2018

Name: KEY

1 (6 pts). Determine if the equation $y' = -\frac{2y}{x}$ is linear, separable, or exact. For exactness, use the differential form $2xydx + x^2dy = 0$. Explain your reasoning.

Linear: $y' = \frac{-2y}{x} \Rightarrow y' + \frac{2}{x}y = 0$ $p(x) = \frac{2}{x}$, $q(x) = 0$, $y' + p(x)y = q(x)$

$\Rightarrow$ ODE is linear.

Separable: $y' = \frac{A(x)}{B(y)}$, need to find $A(x)$ and $B(y)$

$-\frac{2y}{x} = \frac{-\frac{2}{x}}{\frac{1}{y}} \Rightarrow A(x) = -\frac{2}{x}$, $B(y) = \frac{1}{y}$

$\Rightarrow$ ODE is separable.

Exact: $(2xy)dx + x^2dy = 0$ $M(x,y) = 2xy$, $N(x,y) = x^2$ $\Rightarrow$ $\frac{\partial M}{\partial y} = 2x$, $\frac{\partial N}{\partial x} = 2x$

$\Rightarrow$ ODE is exact

2 (4 pts). Model a population $P(t)$, if its rate of growth is propotional to the amount at time t. Assume the rate constant is $r > 0$.

rate of growth proportional to $P(t)$ $\Rightarrow \frac{\frac{dP}{dt}}{P} = r > 0$

$\Rightarrow \boxed{\begin{matrix} P'(t) = rP(t) \\ r > 0 \end{matrix}}$

Solution to ODE $P(t) = Ce^{rt}$, c constant