MATH 046 020-QUIZ 3, SPRING 2018

Name: KEY

1 (5 pts). Find the general solution of the ODE:

$$y' = 3y + e^x$$

$\Rightarrow y' - 3y = e^x$

$\Rightarrow y'e^{-3x} - 3e^{-3x}y = e^x e^{-3x}$

$\Rightarrow \frac{d}{dx}\left[ye^{-3x}\right] = e^{-2x}$

$\Rightarrow \int \frac{d}{dx}\left[ye^{-3x}\right]dx = \int e^{-2x}dx$

$\Rightarrow ye^{-3x} = -\frac{1}{2}e^{-2x} + C$

$\Rightarrow \boxed{y(x) = -\frac{1}{2}e^{x} + Ce^{3x}}$

2 (5 pts). Suppose you opened a savings account with the annual interest rate 2% and deposited \$10000 at the time you opened it. Suppose the interest compund continuously in time and you deposit \$800 into this account annully. What's yoru net profit for this account after 10 years? *You may use* $e^{0.2} = 1.2$.

$A(t)$ = amount in account $\quad A(0) = 10000$

r = interest rate $\quad r = 0.02$

$$\Rightarrow \frac{dA}{dt} = rA + 800 \Rightarrow \frac{dA}{dt} - rA = 800$$

$$\Rightarrow \frac{dA}{dt} e^{-rt} - re^{-rt} A = 800 e^{-rt}$$

$$\Rightarrow \frac{d}{dt}\left[Ae^{-rt}\right] = 800 e^{-rt}$$

$$\Rightarrow \int \frac{d}{dt}\left[Ae^{-rt}\right] dt = \int 800 e^{-rt}\, dt$$

$$\Rightarrow Ae^{-rt} = -\frac{800}{r} e^{-rt} + C$$

$$\Rightarrow A(t) = -\frac{800}{r} + Ce^{rt}$$

$$A(0) = -\frac{800}{r} + Ce^{0}$$

$$10000 = -\frac{800}{r} + C$$

$$\Rightarrow C = 10000 + \frac{800}{r}$$

$$\Rightarrow A(t) = -\frac{800}{r} + \left(10000 + \frac{800}{r}\right) e^{rt}$$

$$\Rightarrow \boxed{A(t) = 10000\, e^{0.02t} + \frac{800}{0.02}\left(e^{0.02t} - 1\right)}$$

$$\boxed{A(10) = 10000\, e^{0.02(10)} + \frac{800}{0.02}\left(e^{0.02(10)} - 1\right)}$$

$$\text{Net Profit} = A(10) - A(0)$$