MATH 046 020-QUIZ 5, SPRING 2018

Name: KEY

1 (6 pts). Consider the second order linear ODE $y'' - 4y = 0$. Show that

(1) $\{e^{2x}, e^{-2x}\}$ are two solutions of the given equation.

(2) $\{e^{2x}, e^{-2x}\}$ are linearly independent.

Then find the general solution of the given equation by using the information above.

1) (1) $y'' - 4y = 0$

$(e^{2x})'' - 4e^{2x} \stackrel{?}{=} 0$

$4e^{2x} - 4e^{2x} = 0$ ✓

$y'' - 4y = 0$

$(e^{-2x})'' - 4e^{-2x} = 0$

$4e^{-2x} - 4e^{-2x} = 0$ ✓

(2) $f = e^{2x}$, $g = e^{-2x}$

$\Rightarrow W = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = \begin{vmatrix} e^{2x} & e^{-2x} \\ 2e^{2x} & -2e^{-2x} \end{vmatrix} = -2-2 = -4 \neq 0$

$\Rightarrow$ linearly independent

Since $y_1(x) = e^{2x}$ and $y_2(x) = e^{-2x}$ are linearly independent, and the ODE $y'' - 4y = 0$ is linear, then the linear combination of solutions is also a solution, hence

$$\boxed{y(x) = c_1 e^{2x} + c_2 e^{-2x}}$$

2 (4 pts). What can one say about the general solution of $y'' + 16y = 0$ if two particular solutions are known to be $y_1 = \sin 4x$ and $y_2 = \cos 4x$?

Assume $y_1(x) = \sin(4x)$ and $y_2(x) = \cos(4x)$ are solutions

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \sin(4x) & \cos(4x) \\ 4\cos(4x) & -4\sin(4x) \end{vmatrix} = -4\sin^2(4x) - 4\cos^2(4x)$$

$$= -4 \neq 0$$

$\Rightarrow y_1$ and y_2 are linearly independent

Since $y'' + 16y = 0$ is linear, and $W \neq 0$, then the linear combination of solutions is a solution, then

$$\boxed{y(x) = c_1 \sin(4x) + c_2 \cos(4x)}$$