Name: __KEY__

1 (5 pts). Find the solution of the IVP below:
$$y'' - y' - 20y = 0; \quad y(0) = 1, \quad y'(0) = 0.$$

$$y'' - y' - 20y = 0$$

$$\Rightarrow \lambda^2 - \lambda - 20 = 0$$

$$\Rightarrow (\lambda - 5)(\lambda + 4) = 0$$

$$\lambda = 5, -4$$

$$\Rightarrow y(x) = c_1 e^{5x} + c_2 e^{-4x} \quad \text{and} \quad y'(x) = 5c_1 e^{5x} + (-4)c_2 e^{-4x}$$

Using $y(0) = 1, \ y'(0) = 0$

$$\Rightarrow \quad c_1 + c_2 = 1 \qquad \Rightarrow \quad 9c_1 = 4$$

$$5c_1 - 4c_2 = 0 \qquad \qquad c_1 = \frac{4}{9} \Rightarrow c_2 = \frac{5}{9}$$

$$\Rightarrow \boxed{y(x) = \frac{4}{9}e^{5x} + \frac{5}{9}e^{-4x}}$$

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2 (5 pts). Find the general solution of the equation:

$$y'' - 4y' + 5y = 0$$

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$$\lambda^2 - 4\lambda + 5 = 0$$

$$\lambda = \frac{4}{2} \pm \frac{\sqrt{16 - 4(1)(5)}}{2} = 2 \pm \frac{\sqrt{-4}}{2} = 2 \pm i$$

$$\Rightarrow \boxed{y(x) = e^{2x}\left(c_1 \cos(x) + c_2 \sin(x)\right)}$$