MATH 046 020-QUIZ 7, SPRING 2018

Name: KEY

1 (5 pts). Find the general solution of the equation:

$$y'' - 4y' + 4y = 4x$$

$$\left.\begin{aligned} y_h'' - 4y_h' + 4y_h &= 0 \\ \lambda^2 - 4\lambda + 4 &= 0 \\ (\lambda - 2)^2 &= 0 \\ \lambda &= 2, 2 \end{aligned}\right\} \Rightarrow y_h(x) = (c_1 + c_2 x)e^{2x}$$

There is no duplication with $\phi(x) = 4x$

$\Rightarrow y_p(x) = Ax + B$; $y_p'(x) = A$; $y_p''(x) = 0$

$\Rightarrow y_p'' - 4y_p' + 4y_p = 4x$

$\Rightarrow -4A + 4Ax + 4B = 4x$

$$\Rightarrow \left.\begin{aligned} 4Ax &= 4x \\ 4B - 4A &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} 4A &= 4 \\ 4B - 4A &= 0 \end{aligned} \Rightarrow \begin{aligned} A &= 1 \\ B &= 1 \end{aligned}$$

$\Rightarrow y_p(x) = x + 1$

$\Rightarrow y(x) = y_h(x) + y_p(x)$

$$\Rightarrow \boxed{y(x) = (c_1 + c_2 x)e^{2x} + x + 1}$$

2 (5 pts). Find the general solution of the equation:

$$\ddot{x} + x = 4(\sin t + \cos t)$$

$x_h'' + x_h = 0$

$\lambda^2 + 1 = 0$

$\Rightarrow \lambda = \pm i \Rightarrow x_h(t) = c_1 \cos(t) + c_2 \sin(t)$

Note there is duplication between $x_h(t)$ and the choice $A\cos(t) + B\sin(t)$. So we multiply by a factor of t so that

$x_p(t) = t(A\cos(t) + B\sin(t)) = At\cos(t) + Bt\sin(t)$

$x_p'(t) = (A+Bt)\cos(t) + (B-At)\sin(t)$

$x_p''(t) = (2B-At)\cos(t) - (2A+Bt)\sin(t)$

$\Rightarrow x_p'' + x_p = 4\sin(t) + 4\cos(t)$

$\Rightarrow (2B-At)\cos(t) - (2A+Bt)\sin(t) + At\cos(t) + Bt\sin(t) = 4\sin(t) + 4\cos(t)$

$\Rightarrow 2B\cos(t) - 2A\sin(t) = 4\sin(t) + 4\cos(t)$

$\Rightarrow \begin{matrix} 2B = 4 \\ -2A = 4 \end{matrix} \Rightarrow \begin{matrix} B = 2 \\ A = -2 \end{matrix} \Rightarrow x_p(t) = -2t\cos(t) + 2t\sin(t) = 2t(\sin(t) - \cos(t))$

$\Rightarrow x(t) = x_h(t) + x_p(t)$

$\Rightarrow \boxed{x(t) = c_1\cos(t) + c_2\sin(t) + 2t(\sin(t) - \cos(t))}$