MATH 046 020-QUIZ 7, SPRING 2018

Name: KEY

1 (5 pts). Find the general solution of the equation:

$$y'' - 6y' + 9y = 4x + 3$$

$$\left.\begin{aligned} y_h'' - 6y_h' + 9y_h &= 0 \\ \lambda^2 - 6\lambda + 9 &= 0 \\ (\lambda - 3)^2 &= 0 \\ \lambda &= 3, 3 \end{aligned}\right\} \Rightarrow y_h(x) = (c_1 + c_2 x)e^{3x}$$

There is no duplication with $\phi(x) = 4x + 3$

$$\Rightarrow y_p(x) = Ax + B\,; \quad y_p'(x) = A\,; \quad y_p''(x) = 0$$

$$\Rightarrow y_p'' - 6y_p' + 9y_p = 4x + 3$$

$$\Rightarrow 0 - 6A + 9Ax + 9B = 4x + 3$$

$$\Rightarrow 9Ax + 9B - 6A = 4x + 3$$

$$\Rightarrow \left.\begin{aligned} 9A &= 4 \\ 9B - 6A &= 3 \end{aligned}\right\} \Rightarrow \begin{aligned} A &= \frac{4}{9} \\ B &= \frac{51}{81} = \frac{17}{27} \end{aligned}$$

$$\Rightarrow \boxed{y(x) = y_h(x) + y_p(x) = (c_1 + c_2 x)e^{3x} + \frac{4}{9}x + \frac{17}{27}}$$

2 (5 pts). Find the general solution of the equation:

$$\ddot{x} + x = 2(\sin t + \cos t)$$

$$\left.\begin{aligned} x_h'' + x_h &= 0 \\ \lambda^2 + 1 &= 0 \end{aligned}\right\} \Rightarrow x_h(t) = C_1\cos(t) + C_2\sin(t)$$

$\Rightarrow \lambda = \pm i$

Note there is duplication between $x_h(t)$ and $\phi(x) = 2\sin(t) + 2\cos(t)$, so we multiply by a factor of t, so that

$$x_p(t) = t(A\cos(t) + B\sin(t)) = At\cos(t) + Bt\sin(t)$$

$$x_p'(t) = (A + Bt)\cos(t) + (B - At)\sin(t)$$

$$x_p''(t) = (2B - At)\cos(t) - (2A + Bt)\sin(t)$$

$$\Rightarrow x_p'' + x_p = 2\sin(t) + 2\cos(t)$$

$$\Rightarrow (2B - At)\cos(t) - (2A + Bt)\sin(t) + At\cos(t) + Bt\sin(t) = 2\sin(t) + 2\cos(t)$$

$$\Rightarrow 2B\cos(t) - 2A\sin(t) = 2\sin(t) + 2\cos(t)$$

$$\Rightarrow \begin{aligned} 2B &= 2 \\ -2A &= 2 \end{aligned} \Rightarrow \begin{aligned} A &= -1 \\ B &= 1 \end{aligned} \Rightarrow x_p(t) = -t\cos(t) + t\sin(t) = t(\sin(t) - \cos(t))$$

$$\Rightarrow x(t) = x_h(t) + x_p(t)$$

$$\Rightarrow \boxed{x(t) = C_1\cos(t) + C_2\sin(t) + t(\sin(t) - \cos(t))}$$