MATH 046 - Spring 2018

Worked Problems - Chapter 2

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1. Assume $M(t)$ represents the mass of an element in kg. Suppose research has shown that the instantaneous rate of decay of this element in $\dfrac{kg}{yr}$ is proportional to the amount present: $\frac{dM}{dt} \propto M(t)$. Set up a model for this relationship.

***Solution*:** To set up the model, note that instantaneous rate of change refers to $\dfrac{dM}{dt}$ and amount of material is given by $M(t)$. The proportionality relationship between the rate of change and the amount of material implies that

$$\frac{dM}{dt} \propto M(t) \quad \Rightarrow \quad \frac{\frac{dM}{dt}}{M(t)} = k \quad \Rightarrow \quad \frac{dM}{dt} = kM(t)$$

where by "proportional" we mean the ratio is equal to a constant, call it k. The we just rearrange the pieces to get a differential equation. Here, the constant k in units of $\dfrac{1}{yr}$, is called a proportionality constant. So the model we are looking for is given as $\dfrac{dM}{dt} = kM(t)$, where we must note that $k < 0$, because $M(t)$ is decreasing over time, so the rate of change must be negative. The function $M(t) > 0$, since it is the amount of material, which cannot be negative. Therefore the constant k must be negative. $\square$

2. Derive a model of the population growth of a bacteria culture, $P(t)$, if its rate of growth is proportional to the number of bacteria present at time t.

***Solution*:** To set up the model, we essentially follow the same line of reasoning as the problem above.

$$\frac{dP}{dt} \propto P(t) \quad \Rightarrow \quad \frac{\frac{dP}{dt}}{P(t)} = k \quad \Rightarrow \quad \frac{dP}{dt} = kP(t)$$

where k is a constant. Note that here we are talking about population growth, so the rate of change $\dfrac{dP}{dt}$ must be increasing, therefore $k > 0$ since $P(t)$ must be greater than or equal to zero as well. $\square$