MATH 046 - Spring 2018

Worked Problems - Chapter 3

1. Write the following differential equation in standard form

$$e^{y'+y} = x$$

***Solution*:** Note that our text defines standard form of a first order ODE to be given as

$$y' = f(x, y)$$

where the ODE is in terms of the unknown function $y(x)$, and independent variable x. Note that the y' is alone on the left-hand side of the ODE. So we just rewrite the ODE using algebraic tricks to isolate y'.

$$\begin{aligned} e^{y'+y} &= x \\ \ln\left(e^{y'+y}\right) &= \ln(x) \\ y' + y &= \ln(x) \\ y' &= -y + \ln(x) \end{aligned}$$

□

NOTE: For our class, the professor has stated you only need to classify the ODEs as: separable, linear, or exact. We will not be considering homogeneous or Bernoulli equations here. The definitions are

- **Linear**: If we can write the ODE as $y' + p(x)y = q(x)$. Note that *linear* means linear in terms of y and y', not in terms of x. So, $p(x)$ and $q(x)$ can be nonlinear functions.
- **Separable**: If we have the ODE $M(x, y)\ dx = N(x, y)\ dy$, a separable equation means that we can write $M(x, y)$ as only a function of x and $N(x, y)$ as only a function of y, ie. $M(x, y) = A(x)$ and $N(x, y) = B(y)$.
- **Exact**: If we have the ODE $M(x, y)\ dx = N(x, y)\ dy$, the ODE is exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

2. Classify the following ODE

$$y' = xy \qquad x\ dx - \frac{1}{y}\ dy = 0$$

***Solution*:** We just go through and check each one of the definitions above. For linear:

$$y' = xy \quad \Rightarrow \quad y' - xy = 0 \quad \Rightarrow \quad p(x) = -x,\ q(x) = 0.$$

Therefore the ODE is linear. For separable:

$$x\ dx - \frac{1}{y}\ dy = 0 \quad \Rightarrow \quad M(x, y) = x,\ N(x, y) = -\frac{1}{y}.$$

Therefore the ODE is separable. For exact:

$$x\ dx - \frac{1}{y}\ dy = 0 \quad \Rightarrow \quad M(x, y) = x,\ N(x, y) = -\frac{1}{y} \quad \Rightarrow \quad \frac{\partial M}{\partial y} = 0,\ \frac{\partial N}{\partial x} = 0.$$

Therefore the ODE is exact. □

3. Classify the following ODE

$$y' = 2xy + x \qquad \left(2xye^{-x^2} + xe^{-x^2}\right)\, dx - e^{-x^2}\, dy = 0$$

***Solution*:** We just go through and check each one of the definitions above. For linear:

$$y' = 2xy + x \quad \Rightarrow \quad y' - 2xy = x \quad \Rightarrow \quad p(x) = -2x,\ q(x) = x.$$

Therefore the ODE is linear. For separable, there is no way for us to algebraically move the y in $2xye^{-x^2}$, therefore, we cannot fulfill the definition. So the ODE is not separable. For exact:

$$\left(2xye^{-x^2} + xe^{-x^2}\right)\, dx - e^{-x^2}\, dy = 0 \quad \Rightarrow \quad M(x,y) = 2xye^{-x^2} + xe^{-x^2},\ N(x,y) = -e^{-x^2}$$

$$\Rightarrow \quad \frac{\partial M}{\partial y} = 2xe^{-x^2},\ \frac{\partial N}{\partial x} = -e^{-x^2}(-2x) = 2xe^{-x^2}.$$

Therefore the ODE is exact. □