MATH 046 - Spring 2018

Worked Problems - Chapter 6

NOTE: The sign and/or constants being multiplied in front of the arbitrary constant C don't matter, as multiplying a constant by a minus sign or another constant is still just a constant. So it is fine if your constant looks different, as long as the rest of the solution is correct.

1. Solve the given differential equation

$$\frac{dy}{dx} + 5y = 0$$

***Solution*:** The ODE is first order linear, so use the integrating factor, or find the right term to multiply to use reverse product rule

$$\begin{aligned} \frac{dy}{dx} + 5y &= 0 \\ \frac{dy}{dx}e^{5x} + 5e^{5x}y &= 0 \\ \frac{d}{dx}\left[ye^{5x}\right] &= 0 \\ \int \frac{d}{dx}\left[ye^{5x}\right]\, dx &= \int 0\, dx \\ ye^{5x} &= C \\ y(x) &= Ce^{-5x} \end{aligned}$$

□

2. Solve the given differential equation

$$\frac{dy}{dx} - 7y = e^{x}$$

***Solution*:** The ODE is first order linear, so use the integrating factor, or find the right term to multiply to use reverse product rule

$$\begin{aligned} \frac{dy}{dx} - 7y &= e^{x} \\ \frac{dy}{dx}e^{-7x} - 7e^{-7x}y &= e^{x}e^{-7x} \\ \frac{d}{dx}\left[ye^{-7x}\right] &= e^{-6x} \\ \int \frac{d}{dx}\left[ye^{-7x}\right]\, dx &= \int e^{-6x}\, dx \\ ye^{-7x} &= C - \frac{1}{6}e^{-6x} \\ y(x) &= Ce^{7x} - \frac{1}{6}e^{x} \end{aligned}$$

□

3. Solve the given differential equation

$$\frac{dy}{dx} + x^2 y = x^2$$

***Solution*:** The ODE is first order linear, so use the integrating factor, or find the right term to multiply to use reverse product rule

$$\begin{aligned}
\frac{dy}{dx} + x^2 y &= x^2 \\
\frac{dy}{dx}e^{\frac{x^3}{3}} + x^2 e^{\frac{x^3}{3}} y &= x^2 e^{\frac{x^3}{3}} \\
\frac{d}{dx}\left[ye^{\frac{x^3}{3}}\right] &= x^2 e^{\frac{x^3}{3}} \\
\int \frac{d}{dx}\left[ye^{\frac{x^3}{3}}\right]\, dx &= \int x^2 e^{\frac{x^3}{3}}\, dx \\
ye^{\frac{x^3}{3}} &= C + e^{\frac{x^3}{3}} \\
y(x) &= Ce^{-\frac{x^3}{3}} + 1
\end{aligned}$$

□

4. Solve the given differential equation

$$\frac{dy}{dx} - \frac{3}{x^2} y = \frac{1}{x^2}$$

***Solution*:** The ODE is first order linear, so use the integrating factor, or find the right term to multiply to use reverse product rule

$$\begin{aligned}
\frac{dy}{dx} - \frac{3}{x^2} y &= \frac{1}{x^2} \\
\frac{dy}{dx}e^{\frac{3}{x}} - \frac{3}{x^2} e^{\frac{3}{x}} y &= \frac{1}{x^2} e^{\frac{3}{x}} \\
\frac{d}{dx}\left[ye^{\frac{3}{x}}\right] &= \frac{1}{x^2} e^{\frac{3}{x}} \\
\int \frac{d}{dx}\left[ye^{\frac{3}{x}}\right]\, dx &= \int \frac{1}{x^2} e^{\frac{3}{x}}\, dx \\
ye^{\frac{3}{x}} &= C - \frac{1}{3} e^{\frac{3}{x}} \\
y(x) &= Ce^{-\frac{3}{x}} - \frac{1}{3}
\end{aligned}$$

□

5. Solve the given initial value problem

$$\frac{dq}{dt} + q = 4\cos(t) \qquad q(0) = 1$$

***Solution*:** The ODE is first order linear, so use the integrating factor, or find the right term to multiply to use reverse product rule

$$\begin{aligned}
\frac{dq}{dt} + q &= 4\cos(t) \\
\frac{dq}{dt}e^t + e^t q &= 4\cos(t)e^t \\
\frac{d}{dt}\left[qe^t\right] &= 4e^t\cos(t) \\
\int \frac{d}{dt}\left[qe^t\right]\ dt &= \int 4e^t\cos(2t)\ dt \\
qe^t &= C + \frac{4}{5}e^t(2\sin(2t) + \cos(2t)) \\
q(t) &= Ce^{-t} + \frac{4}{5}(2\sin(2t) + \cos(2t)) && \text{general solution} \\
q(0) &= Ce^0 + \frac{4}{5}(2\sin(0) + \cos(0)) \\
1 &= C + \frac{4}{5} \\
C &= \frac{1}{5} \\
q(t) &= \frac{1}{5}e^{-t} + \frac{4}{5}(2\sin(2t) + \cos(2t)) && \text{solution to IVP}
\end{aligned}$$

□