MATH 046 - Spring 2018

Worked Problems - Chapter 9

1. Solve the following differential equation

$$y'' - y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' - y &= 0 \\ \lambda^2 - 1 &= 0 \\ (\lambda + 1)(\lambda - 1) &= 0 \\ \lambda &= \pm 1 \end{aligned}$$

So we have distinct real roots. Therefore, the general solution is $y(x) = c_1 e^{x} + c_2 e^{-x}$. □

2. Solve the following differential equation

$$y'' - y' - 30y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' - y' - 30y &= 0 \\ \lambda^2 - \lambda - 30 &= 0 \\ (\lambda - 6)(\lambda + 5) &= 0 \\ \lambda &= 6, -5 \end{aligned}$$

So we have distinct real roots. Therefore, the general solution is $y(x) = c_1 e^{6x} + c_2 e^{-5x}$. □

3. Solve the following differential equation

$$y'' - 2y' + y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' - 2y' + y &= 0 \\ \lambda^2 - 2\lambda + 1 &= 0 \\ (\lambda - 1)^2 &= 0 \\ \lambda &= 1, 1 \end{aligned}$$

So we have repeated real roots. Therefore, the general solution is $y(x) = (c_1 + c_2 x)e^{x}$. □

4. Solve the following differential equation

$$y'' + y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' + y &= 0 \\ \lambda^2 + 1 &= 0 \\ \lambda^2 &= -1 \\ \lambda &= 0 \pm i \end{aligned}$$

So we have complex roots. Therefore, the general solution is $y(x) = e^0(c_1 \cos(x) + c_2 \sin(x)) = c_1 \cos(x) + c_2 \sin(x)$. □

5. Solve the following differential equation

$$y'' + 2y' + 2y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' + 2y' + 2y &= 0 \\ \lambda^2 + 2\lambda + 2 &= 0 \\ \lambda &= \frac{-2}{2} \pm \frac{\sqrt{2^2 - 4(1)(2)}}{2} \\ &= -1 \pm i \end{aligned}$$

So we have complex roots. Therefore, the general solution is $y(x) = e^{-x}(c_1 \cos(x) + c_2 \sin(x))$. □

6. Solve the following differential equation

$$y'' - 7y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' - 7y &= 0 \\ \lambda^2 - 7 &= 0 \\ \lambda &= \pm\sqrt{7} \end{aligned}$$

So we have distinct real roots. Therefore, the general solution is $y(x) = c_1 e^{-\sqrt{7}x} + c_2 e^{\sqrt{7}x}$. □

7. Solve the following differential equation

$$y'' - 3y' - 5y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' - 3y' - 5y &= 0 \\ \lambda^2 - 3\lambda - 5 &= 0 \\ \lambda &= \frac{-(-3)}{2} \pm \frac{\sqrt{(-3)^2 - 4(1)(-5)}}{2} \\ &= \frac{3}{2} \pm \frac{\sqrt{29}}{2} \end{aligned}$$

So we have distinct real roots. Therefore, the general solution is $y(x) = c_1 e^{\left(\frac{3}{2}+\frac{\sqrt{29}}{2}\right)x} + c_2 e^{\left(\frac{3}{2}-\frac{\sqrt{29}}{2}\right)x}$. □

8. Solve the following differential equation

$$y'' + y' + \frac{1}{4}y = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' + y' + \frac{1}{4}y &= 0 \\ \lambda^2 + \lambda + \frac{1}{4} &= 0 \\ \left(\lambda + \frac{1}{2}\right)^2 &= 0 \\ \lambda &= -\frac{1}{2}, -\frac{1}{2} \end{aligned}$$

So we have repeated real roots. Therefore, the general solution is $y(x) = (c_1 + c_2 x)e^{-\frac{1}{2}x}$. □

9. Solve the following differential equation

$$y'' + 25y' = 0$$

***Solution*:** The ODE is a linear second order ODE with constant coefficients, so we use the characteristic equation

$$\begin{aligned} y'' + 25y' &= 0 \\ \lambda^2 + 25\lambda &= 0 \\ \lambda(\lambda + 25) &= 0 \\ \lambda &= 0, -25 \end{aligned}$$

So we have distinct real roots. Therefore, the general solution is $y(x) = c_1 + c_2 e^{-25x}$. □