

Problem Set 4

Toggling Lockers/Train Problem

1) Suppose you are in a high school hallway lined with 100 closed lockers. And suppose that you are really bored and have nothing better to do, so you decide to toggle the lockers.

You begin by opening all 100 lockers. After this, you then close every 2nd locker (so the 2nd, 4th, 6th, 8th, and 100th are all closed). Then, you go to every third locker and open it if it is closed or close it if it is open (lets call this toggling the locker). Continue toggling every n th locker on pass number n . You proceed to toggle every n th locker on pass number n . So, for example, on pass number 16 you will toggle every 16th locker. After 100 passes, where you toggle only locker number 100, how many lockers are open?

Explain your reasoning.

2) You are given 8 pennies, 7 of which weigh exactly the same, but one penny weighs less than the other 7. You also have a judge scale. Find the one penny that weighs the least in less than 3 steps.

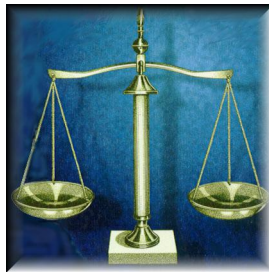


FIGURE 1. Judge scale

3) Find the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

for the function $f(x) = \frac{1}{x+1}$.