

These notes cover the lecture on § 6.4 - Sine and Cosine Functions (specifically cosine), and the graphs in § 6.5 - Tangent, Cotangent, Cosecant, and Secant. These are some guidelines to the graphs if you need to sketch them. Examples are given for the cosine graphs, as being able to sketch these using transformations will be required knowledge.

### 1. § 6.4 - SINE AND COSINE FUNCTIONS

**Definition 1.1.** We can define the **cosine** function as a horizontal shift of  $\sin(x)$ , since we know the graph of this function already:

$$f(x) = \cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

where the horizontal shift is  $\frac{\pi}{2}$  units to the left. The graph is given in the plot below. Note the zeros are at  $\frac{\pi}{2}$  and  $\frac{3\pi}{2}$ .

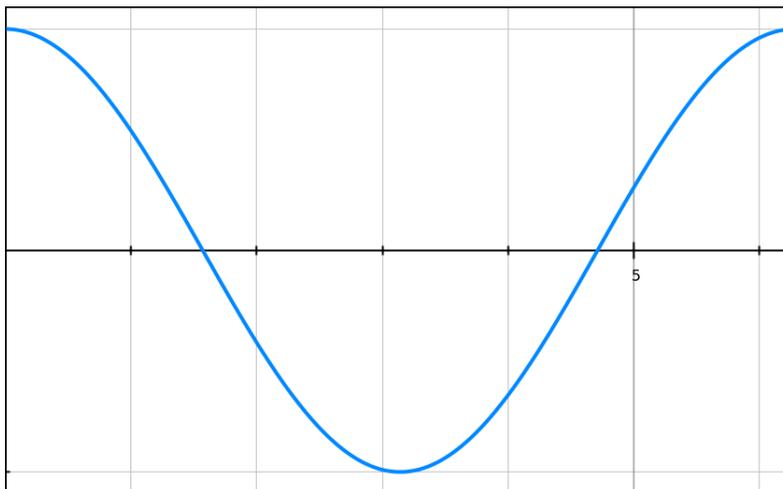


FIGURE 1. Graph of  $f(x) = \cos(x)$

**Definition 1.2.** In general, we can write a trig function, say sine, as

$$y(t) = A \sin(\omega t - c)$$

where  $A$  = amplitude,  $\omega$  = frequency, and  $\frac{c}{\omega} = \phi$  = phase shift. The period,  $T$  is given by the formula  $T = \frac{2\pi}{\omega}$  for sin and cos.

**Example 1.** Sketch a complete graph of  $f(x) = -4 \cos(2x) - 2$

*Solution:* We want to do this graph by the graph transformations that are given in a previous lecture. First we calculate 3 quantities: Amplitude, Period, and vertical shift. Recall that the function has the general form:  $f(x) = A \cos(\omega x) + b$ . So then we have that  $|A| = 4$  is the amplitude,  $P = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ , and  $b = -2$ . From this, we can deduce the graph. Starting out with the parent graph above in figure 1, we apply transformations as follows:

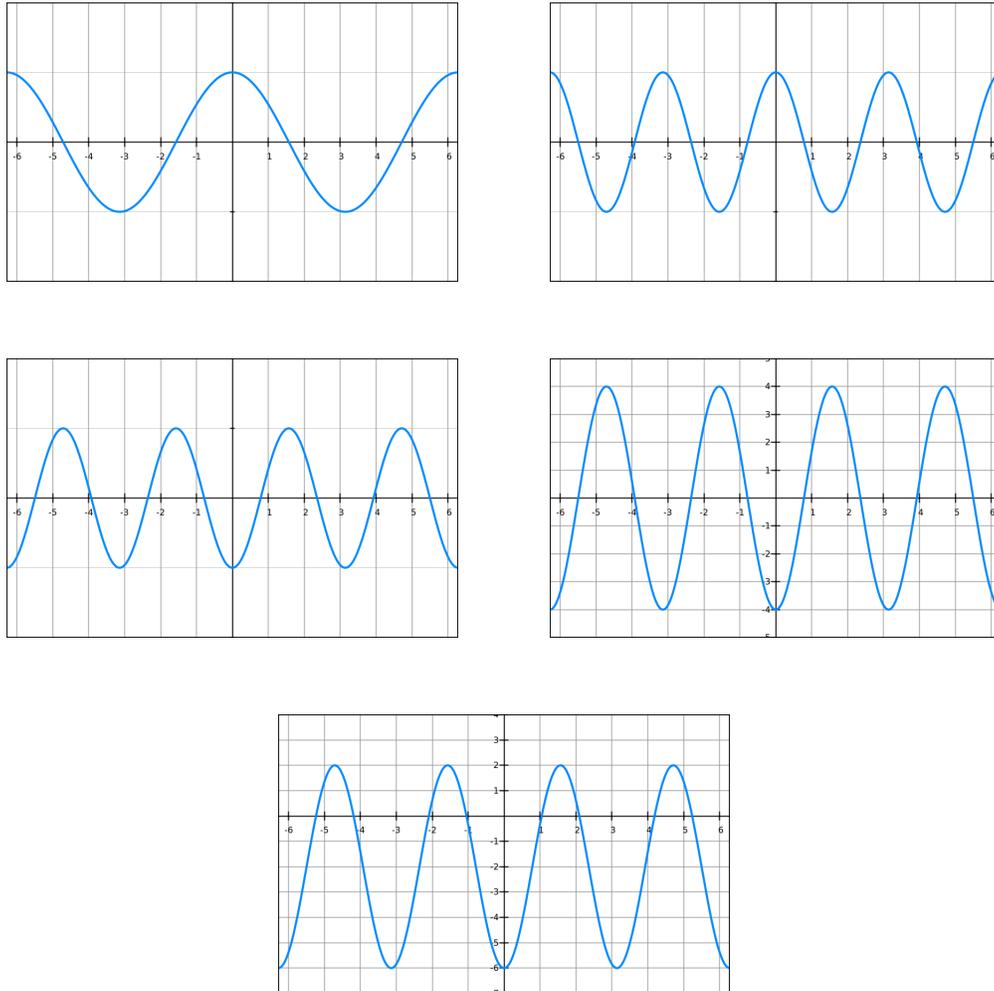


FIGURE 2. Graph Transformations: (1)  $f(x) = \cos(x)$ , (2)  $f(x) = \cos(2x)$ , (3)  $f(x) = -\cos(2x)$ , (4)  $f(x) = -4\cos(2x)$ , (5)  $f(x) = -4\cos(2x) - 2$ . From left to right and top to bottom.

□

**Example 2.** Sketch a complete period:  $y = 4 \sin(4x) + 1$

**Example 3.** Sketch a complete period:  $y = 2 \cos(6x) - 3$

**Example 4.** Sketch a complete period:  $y = 3 \sin(2x) + 1$

**Definition 1.3.** We can define the **tangent** function using  $\sin(x)$  and  $\cos(x)$ , since we know the function values of these trig functions already:

$$f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

So we will have vertical asymptotes where  $\cos(x) = 0$  and zeros where  $\sin(x) = 0$ . The graph is shown below for the zoomed in version and the periodic zoomed out version.

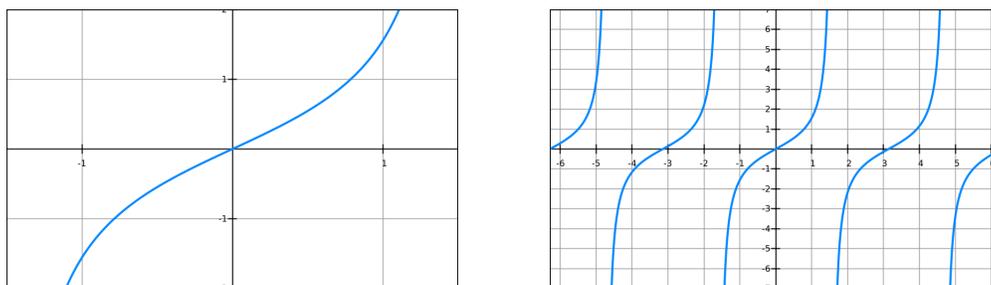


FIGURE 3. Graph of  $f(x) = \tan(x)$  zoomed in on left. Graph of  $f(x) = \tan(x)$  zoomed out on right.

**Definition 1.4.** We can define the **cotangent** function using  $\sin(x)$  and  $\cos(x)$ , since we know the function values of these trig functions already:

$$f(x) = \cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

So we will have vertical asymptotes where  $\sin(x) = 0$  and zeros where  $\cos(x) = 0$ . The graph is shown below for the periodic zoomed out version.

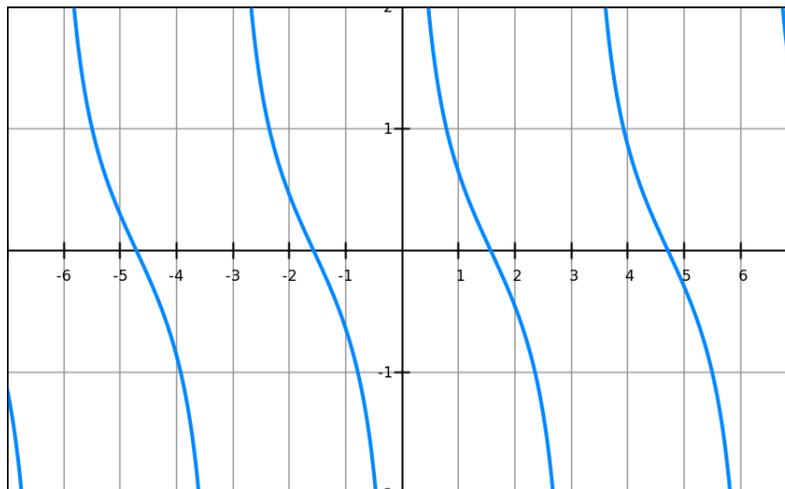


FIGURE 4. Graph of  $f(x) = \cot(x)$  zoomed out.

**Definition 1.5.** We can define the **cosecant** function using  $\sin(x)$ , since we know the function values of this trig functions already:

$$f(x) = \csc(x) = \frac{1}{\sin(x)}$$

So we will have vertical asymptotes where  $\sin(x) = 0$ . The graph is shown below for the periodic zoomed out version.

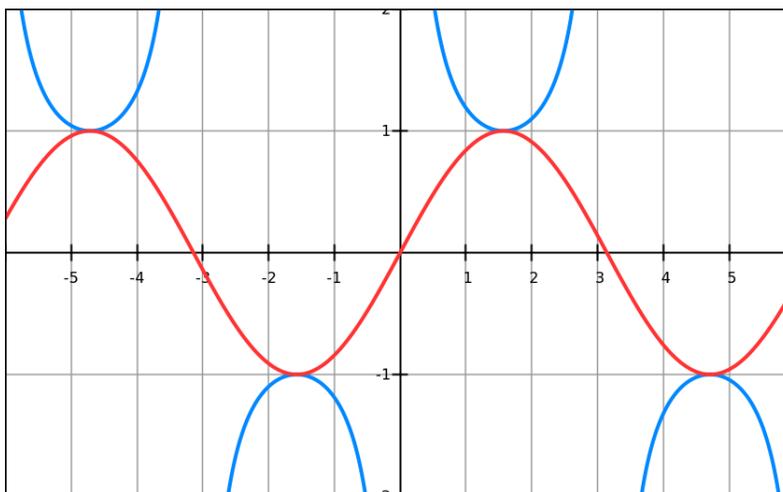


FIGURE 5. Graph of  $f(x) = \csc(x)$  in blue. Graph of  $f(x) = \sin(x)$  in red.

**Definition 1.6.** We can define the **secant** function using  $\cos(x)$ , since we know the function values of this trig functions already:

$$f(x) = \sec(x) = \frac{1}{\cos(x)}$$

So we will have vertical asymptotes where  $\cos(x) = 0$ . The graph is shown below for the periodic zoomed out version.

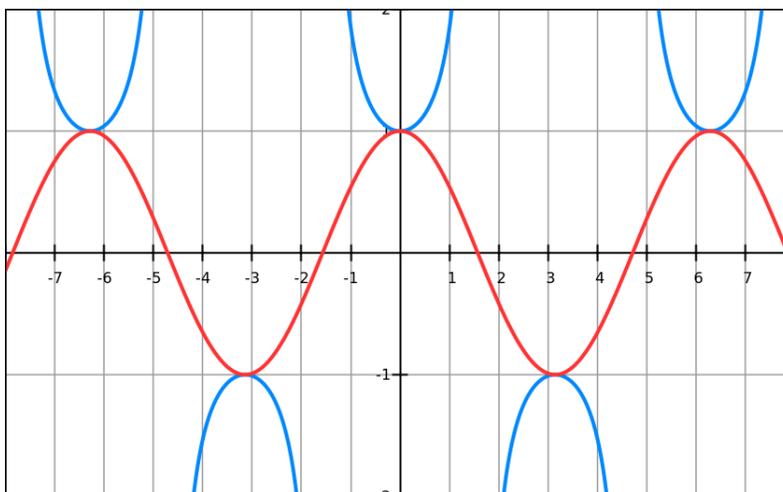


FIGURE 6. Graph of  $f(x) = \sec(x)$  in blue. Graph of  $f(x) = \cos(x)$  in red.