Section 1.1 - Limits

Consider $y = \frac{\sin x}{x}$. When x is near 1, where is y "close" to?

Look at graph, one can see $y \approx \frac{\sin(1)}{1} \approx .84$

$\Rightarrow$ x "near" 1 $\Rightarrow$ y near 0.84.

What happens when $y = \frac{\sin x}{x} = \frac{\sin 0}{0} \rightarrow \frac{0}{0}$??

Ex) $\lim\limits_{x \to 1} f(x) = L \Rightarrow \lim\limits_{x \to 1} \frac{\sin x}{x} \approx 0.84$

Do a chart to see

x	$\frac{\sin x}{x}$
-0.1	0.998
-0.01	0.999
0	undef
0.01	0.999
0.1	0.998

Ex) $\lim\limits_{x \to 3} \frac{x^2 - x - 6}{6x^2 - 19x + 3}$

Ex) $f(x) = \begin{cases} x+1 & x<0 \\ -x^2+1 & x>0 \end{cases}$ $\lim\limits_{x \to 0} f(x) = ?$

Limits fail to exist if

1. $f(x)$ approaches different values on either side of c as $x \to c$
2. $f(x)$ grows without upper or lower bounds
3. The function may oscillate as $x \to c$

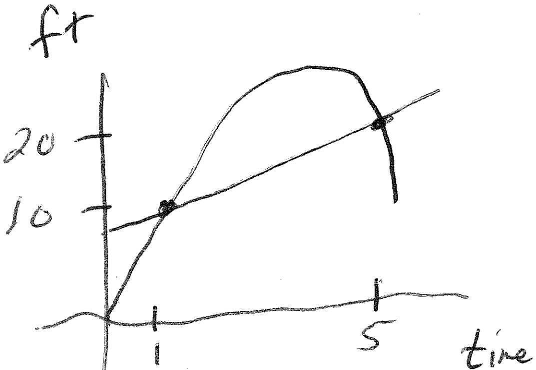
Ex) $\lim\limits_{x \to 1} f(x)$ for $f(x) = \begin{cases} x^2 - 2x + 3 & x \leq 1 \\ x & x > 1 \end{cases}$

Ex) $\lim_{x \to 1} \frac{1}{(x-1)^2}$

Ex) $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

Limits of difference quotients

What is $\lim_{h \to 0} \frac{f(h+1)-f(1)}{h}$?

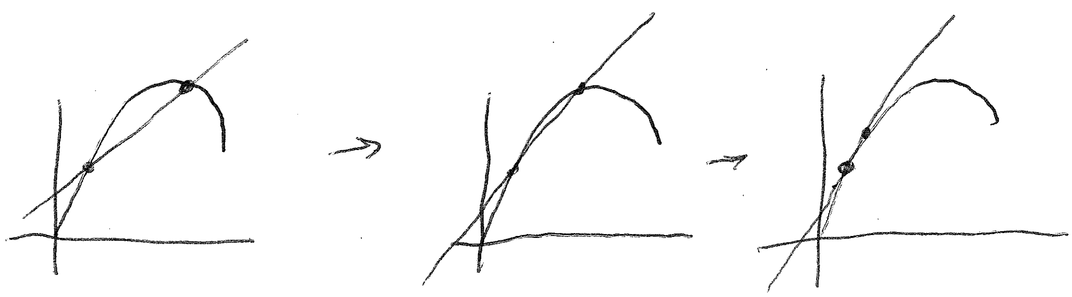


$$\frac{f(5)-f(1)}{5-1} = \frac{20-10}{5-1} = \frac{10}{4} = 2.5 \ \text{ft/s}$$

$\Rightarrow$ Average velocity

Now what about $\lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$?

Pictorially:



Section 1.2 - ε & δ definition of Limit

Definition: Let I be an open interval containing c, and let f be defined on I, except possibly at c. The limit of $f(x)$ as x approaches c is L, denoted by

$$\lim_{x \to c} f(x) = L$$

means that given any $\varepsilon > 0$, there exists $\delta > 0$ s.t. for all $x \neq c$ if $|x-c| < \delta$, then $|f(x) - L| < \varepsilon$

Ex) Show $\lim_{x \to 2} x^2 = 4$